



Mechanisms of nonlinear frequency broadening and time-domain relay dissipation in an integrated NES-ABH absorber for structural vibration damping

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ABSTRACT

Conventional nonlinear energy sinks (NESs) may suffer from delayed targeted energy transfer and degraded dissipation efficiency under large-amplitude excitation, while standalone acoustic black hole (ABH) absorbers are often limited by insufficient low-frequency energy reception. To overcome these complementary limitations, this paper proposes an integrated nonlinear-continuum NES-ABH absorber, in which the lumped mass of a conventional NES is replaced by a two-dimensional plate-type ABH structure. A coupled dynamic model of a simply supported plate with the NES-ABH attachment is established using classical plate theory, modal superposition, and interfacial displacement compatibility. The proposed configuration is compared with standalone NES and standalone ABH absorbers under different impulsive excitation levels. The results show that the NES-ABH absorber preserves the nonlinear energy transfer capability of the NES while introducing broadband multimodal dissipation through the ABH. Under large excitation, the NES induces frequency broadening of the plate vibration energy, and the ABH receives and dissipates the broadened components through multiple structural modes. Energy analysis further reveals a relay-type sequential dissipation mechanism: the ABH dominates the early high-energy stage, whereas the NES becomes more effective after the system energy decreases to a suitable range for targeted energy transfer. Based on this mechanism, a multimodal ABH design principle is proposed to balance low-order modal matching and broadband modal reception. Numerical and experimental results demonstrate that the proposed NES-ABH absorber improves robustness against varying excitation amplitudes and alleviates the performance degradation of conventional NESs under large-energy inputs.

1. Introduction

Structural vibration widely exists in aerospace, rail transit, mechanical equipment, and civil engineering. Sustained vibration may induce fatigue damage, reduce operational accuracy and service life, increase noise radiation, and even lead to structural instability. Therefore, efficient vibration reduction has long been an important topic in structural dynamics and vibration control [1,2]. Compared with active and semi-active control, passive control techniques still have broad application potential in practical engineering due to their independence from external energy, simple structural forms, high reliability, and low maintenance requirements [2–4].

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Among various passive vibration control methods, nonlinear energy sinks (NESs) [4–6] and acoustic black hole (ABH) structures [7] are two representative approaches. The NES is based on nonlinear energy pumping and targeted energy transfer (TET). By attaching an essentially nonlinear oscillator to the primary structure, vibration energy can be irreversibly transferred from the primary structure to the NES and then dissipated through damping [4,5,8,9]. Unlike linear tuned mass dampers, NESs do not rely on a fixed tuning frequency and therefore exhibit broadband adaptability. Different NES configurations, such as bistable NESs [10], vibro-impact NESs [11,12], rotating NESs [13], and related optimization strategies [14], have been developed to extend their effective working ranges. NESs have also been applied to continuous structures such as beams, laminated plates, and sandwich plates [15–18]. However, their performance is strongly dependent on the input energy level. Under low-energy excitation, TET may be difficult to trigger effectively; under high-energy excitation, the NES may suffer from equivalent detuning, delayed energy capture, and even degradation of vibration reduction performance [4,6].

The ABH provides another passive vibration reduction mechanism based on flexural wave manipulation. By introducing a power-law thickness variation in beams or plates, the flexural wave speed gradually decreases and the vibration energy is concentrated near the ABH region, where it can be dissipated by an attached damping layer [7,19,20]. Traditional embedded ABHs require direct geometric modification of the host structure, which may be restricted by structural strength and manufacturing constraints. To overcome this limitation, add-on ABH absorbers have been developed, in which an independent ABH-featured unit is attached to the primary structure without altering the host geometry [21–23]. Such structures can use the multimodal response of the ABH region to receive and dissipate vibration energy and have shown good potential for mid- and high-frequency vibration suppression. Nevertheless, due to the truncation thickness and cut-on frequency, the low-frequency energy concentration and dissipation capability of standalone ABH absorbers remain limited [19,22].

To improve the limitations of NESs and ABHs, recent studies have explored structural modification and coupled designs. For ABH structures, geometric and contact nonlinearities have been introduced to promote energy transfer toward higher frequencies and improve vibration attenuation [24–26]. Other studies have combined ABHs with additional absorbers to enhance low-frequency performance. Li et al. [27] investigated the vibration reduction effect of attaching a tuned mass damper (TMD), NES, bi-stable NES (BNES), and vibro-impact ABH (VI-ABH) absorbers to an ABH beam, demonstrating that additional absorbers can reduce low-frequency resonance peaks. Wang et al. [28] proposed an ABH-BNES structure, in which the high-damping modes of the ABH were used to improve the performance of a BNES under high-energy inputs. He et al. [29] further studied an ABH cantilever beam with an attached NES, showing the potential of combining nonlinear and ABH effects. However, most existing studies are still limited to beam-type systems or add-on combinations in which the NES and ABH remain two separate absorbers. The use of a continuum ABH as the inertial component of an NES has received only limited attention, particularly for finite-energy transient vibration of plate-type host structures. A recent study by Zhang et al. [30] demonstrated that a structurally similar configuration, in which the lumped mass of the NES is replaced by an ABH, can significantly increase the flutter critical boundary of a supersonic panel by 27.8%. However, that problem is governed by aerodynamically sustained energy input and limit-cycle oscillation, which is essentially different from the impulsive vibration problem considered here. Under impulsive mechanical excitation, the initial energy is finite and the system undergoes free decay; therefore, the relative roles of the NES and ABH may evolve continuously as the energy level decreases. The resulting amplitude-dependent relay dissipation behavior and its implication for ABH structural design have not yet been systematically clarified.

To address this gap, this paper investigates an integrated NES-ABH absorber for plate vibration control, in which the lumped inertial mass of a conventional NES is replaced by a multimodal ABH continuum. Unlike conventional add-on configurations, the ABH is not retained as a separate absorber alongside the NES, but serves as the inertial component of the NES itself. Consequently, the internal modes of the ABH participate directly in the nonlinear energy-transfer process through the nonlinear spring-damper connection, forming a nonlinear-continuum coupled absorber rather than a combination of two independent absorbers. The structural topology is related to the mass-replacement concept reported in Ref. [30]; the present contribution therefore focuses on the transient energy-dependent mechanisms that arise under finite-energy impulsive excitation rather than claiming the mass-replacement topology itself as entirely unprecedented.

The main objective of this work is not only to evaluate the vibration reduction performance of the integrated NES-ABH absorber, but also to clarify the synergistic mechanism between nonlinear energy transfer and ABH multimodal dissipation in a freely decaying system. Three aspects are emphasized: (i) whether NES-induced nonlinear frequency broadening activates multiple ABH modes and converts otherwise detrimental broadband components into dissipative channels; (ii) whether the dominant dissipation pathway evolves from ABH-dominated dissipation at the early high-energy stage to NES-dominated targeted energy transfer at the later lower-energy stage; and (iii) how these mechanisms can be translated into a multimodal ABH design principle balancing low-order modal matching, mid-frequency modal density, and broadband high-order coverage.

The remainder of this paper is organized as follows. Section 2 establishes the continuum coupled dynamic model of a simply supported plate with the NES-ABH attachment based on classical thin plate theory, the mode superposition method, and the interfacial displacement compatibility condition. Section 3 compares the vibration suppression performance of the proposed configuration with standalone NES and standalone ABH schemes under different excitation amplitudes. Section 4 analyzes the synergistic dissipation mechanism through time-frequency and energy analyses, further refines the ABH parameters based on the revealed mechanism, and examines the sensitivity to pulse duration under a fixed peak force. Section 5 presents experimental verification. Section 6 summarizes the main conclusions.

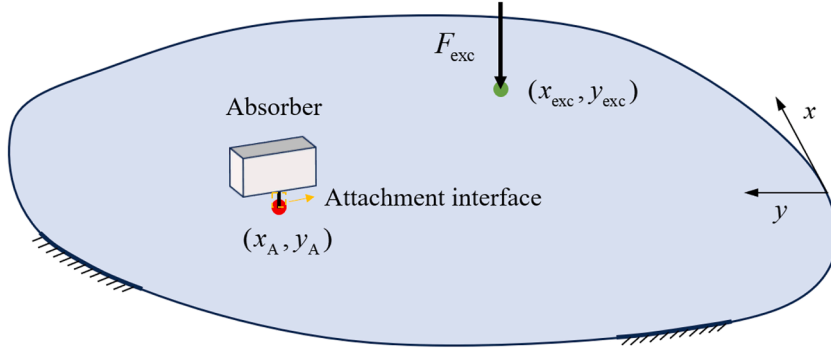


Fig. 1. Schematic of a generic host plate-absorber system.

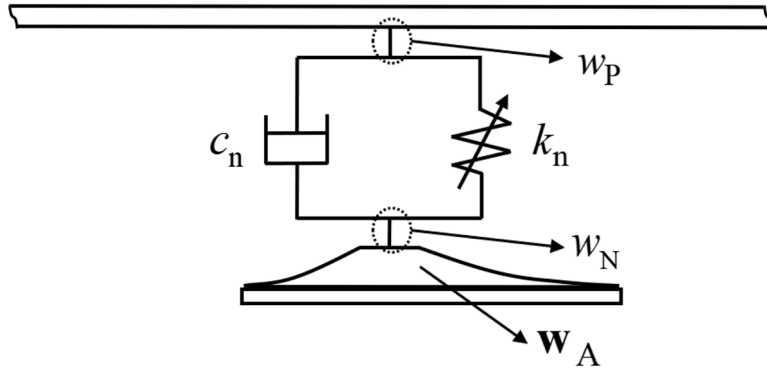


Fig. 2. Diagram of NES-ABH connection.

2. Model establishment

As shown in Fig. 1, a plate-like host structure is considered, with a generic absorber mounted at location (x_A, y_A) and an external excitation force F_{exc} applied at location (x_{exc}, y_{exc}) . The absorber is illustrated in a general form and may represent different configurations, such as a conventional NES, a directly attached ABH absorber, or an NES-ABH absorber. Depending on the specific configuration, the attachment to the host structure may be realized either through a nonlinear connecting element or through direct attachment. The NES-ABH integrated absorber is constructed by replacing the lumped mass in a conventional NES with an ABH structure, as shown in Fig. 2. Specifically, the ABH is connected to the host structure via a nonlinear spring with stiffness coefficient k_n and a viscous damper with coefficient c_n . The displacement at the interface between the host structure and the nonlinear spring/damper is denoted as w_p , and the displacement at the interface between the nonlinear spring/damper and the ABH is denoted as w_N .

2.1. Modal-space model of the system

For a general plate-like host structure subject to transverse excitation in Fig. 1, the governing equation is established based on classical thin plate theory [31]:

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = F_{exc} \delta(x - x_{exc}) \delta(y - y_{exc}) + F_{NES-ABH} \delta(x - x_A) \delta(y - y_A) \quad (1)$$

where w denotes the flexural deflection of the plate, $D = Eh^3/[12(1-\nu^2)]$ is the bending stiffness, and $\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial y^2} + \frac{\partial^4}{\partial y^4}$ is the biharmonic operator. F_{exc} represents the excitation force applied to the plate (x_{exc}, y_{exc}) , and $F_{NES-ABH}$ denotes the interaction force exerted by the NES-ABH attachment on the host plate at (x_A, y_A) .

The displacement response of the plate is expanded in terms of its natural mode shapes as:

$$w(x, y, t) = \sum_i \sum_j q_{ij}(t) \phi_{ij}(x, y) \quad (2)$$

where $q_{ij}(t)$ is the modal coordinate and $\phi_{ij}(x, y)$ is the mode shape function corresponding to the (i, j) -th mode, the specific form of which depends on the boundary conditions of the host structure and will be specified in Section 3 for the simply supported plate

considered in the numerical study.

Assuming small structural damping, a modal damping model is adopted by introducing a viscous damping term $2\zeta_{ij}\omega_{ij}\dot{q}_{ij}(t)$ for each mode after modal discretization, where ζ_{ij} and ω_{ij} are the modal damping ratio and natural angular frequency of the (i,j) -th mode. Substituting Eq. (2) into the governing Eq. (1) and applying the orthogonality of the mode shape functions yields the equations of motion in modal space [32,33]:

$$\ddot{q}_{ij}(t) + 2\zeta_{ij}\omega_{ij}\dot{q}_{ij}(t) + \omega_{ij}^2 q_{ij}(t) = \frac{F_{\text{exc}}(t)\phi_{ij}(x_{\text{exc}}, y_{\text{exc}}) + F_{\text{NES-ABH}}(t)\phi_{ij}(x_A, y_A)}{m_{ij}}. \quad (3)$$

The host plate is modeled within a geometrically linear classical thin-plate framework. Accordingly, geometric nonlinearity of the host plate is neglected, and the dominant nonlinearity of the coupled system is introduced through the NES element. This assumption is adopted to isolate the absorber-induced nonlinear dynamics. Host-plate geometric nonlinearity would need to be included when mid-plane stretching becomes significant, while thick-plate or substantially higher-frequency applications would require a formulation that accounts for transverse shear deformation and rotary inertia.

2.2. Analysis of interfacial coupling

The coupling between the NES-ABH absorber and the plate is governed by the interfacial displacement compatibility condition and the force equilibrium condition at the connection point, which together provide a rigorous and self-consistent framework for deriving the interaction forces without resorting to simplified impedance matching or assumed boundary stiffness.

The ABH is discretized in space, and considering only the motion along the z -direction its displacement vector at the nodes is denoted by $\mathbf{w}_A$. When the ABH is mounted on the plate via the nonlinear part, the displacement $\mathbf{w}_A$ consists of two components: the flexural displacement of the ABH and the displacement at the connection between the nonlinear part and the ABH, w_N . It can be expressed as [23]:

$$\mathbf{w}_A = \Phi_A \mathbf{q}_A + \mathbf{J} w_N \quad (4)$$

where Φ_A is the mode shape vector of the ABH normalized with respect to the mass matrix, $\mathbf{q}_A$ is the modal coordinate vector of the ABH, $\mathbf{J}$ is the modal vector for the translational mode, and w_N is the displacement at the connection between the nonlinear part and the ABH.

The modal dynamic equation of the ABH can be written as [23]:

$$\mathbf{I}\ddot{\mathbf{q}}_A + \mathbf{C}\dot{\mathbf{q}}_A + \mathbf{K}\mathbf{q}_A = \mathbf{F} \quad (5)$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{C}$ is a diagonal matrix with diagonal elements $2\omega_{Ai}\zeta_{Ai}$, $\mathbf{K}$ is a diagonal matrix with diagonal elements ω_{Ai}^2 , ω_{Ai} and ζ_{Ai} are the i -th natural angular frequency and modal damping ratio of the ABH, respectively. The physical damping of the ABH originates from the material loss of the damping layer attached to the ABH structure. During flexural vibration, deformation of the damping layer dissipates vibration energy through material hysteresis. After modal reduction, this distributed dissipation is represented by the mode-dependent damping ratio ζ_{Ai} , and the corresponding term $2\omega_{Ai}\zeta_{Ai}\dot{q}_{Ai}$ represents the equivalent viscous damping force of the i -th ABH mode. This equivalent viscous term is a reduced-order representation of the distributed hysteretic material loss and should not be interpreted as implying that the viscoelastic layer is physically governed by a viscous constitutive law.

The modal force $\mathbf{F}$ in Eq. (5) is induced by the base excitation transmitted through the nonlinear part, and can be expressed as [23]:

$$\mathbf{F} = -\Phi_A^T \mathbf{M}_A \mathbf{J} \ddot{w}_N \quad (6)$$

where $\mathbf{M}_A$ is the mass matrix of the ABH, and $\Phi_A^T \mathbf{M}_A \mathbf{J}$ represents the modal participation factor for different modes during the interaction between the plate and the ABH due to the translational motion.

The reaction force generated by the ABH and acting on the nonlinear part of the absorber is also induced by the inertial force, and can be determined from the modal displacement of the ABH. Its expression is [23]:

$$F_{\text{ABH}} = -\mathbf{J}^T \mathbf{M}_A \ddot{\mathbf{w}}_A = -\mathbf{J}^T \mathbf{M}_A \Phi_A \ddot{\mathbf{q}}_A - \mathbf{J}^T \mathbf{M}_A \mathbf{J} \ddot{w}_N \quad (7)$$

where $\mathbf{J}^T \mathbf{M}_A \mathbf{J}$ is the total mass of the ABH. Here, F_{ABH} denotes the reaction force generated by the ABH at the connection interface.

According to the force equilibrium of the nonlinear connecting element at the ABH interface, the following relation is obtained:

$$k_n(w_N - w_p)^3 + c_n(\dot{w}_N - \dot{w}_p) = -\mathbf{J}^T \mathbf{M}_A \Phi_A \ddot{\mathbf{q}}_A - \mathbf{J}^T \mathbf{M}_A \mathbf{J} \ddot{w}_N \quad (8)$$

The interaction force $F_{\text{NES-ABH}}$ acting on the host plate can then be expressed as

$$F_{\text{NES-ABH}} = k_n(w_N - w_p)^3 + c_n(\dot{w}_N - \dot{w}_p) \quad (9)$$

The positive directions of w_p , w_N , F_{ABH} , and $F_{\text{NES-ABH}}$ are all defined along the positive transverse direction of the host plate.

With this convention, the action-reaction forces at the plate-attachment and nonlinear-element-ABH interfaces are defined consistently throughout the analytical derivation and the numerical implementation.

Table 1
Structural material parameters.

Structure	Density (kg /m ³)	Young's modulus (Pa)	Poisson's ratio	Loss factor
Plate	2700	7×10^{10}	0.35	0.001
ABH	2700	9×10^9	0.35	0.001
Damping	1800	2×10^8	0.45	0.3

Table 2
First four modal frequencies of the plate.

Mode order	1	2	3	4
Frequency (Hz)	29.6	73.9	73.9	118.3

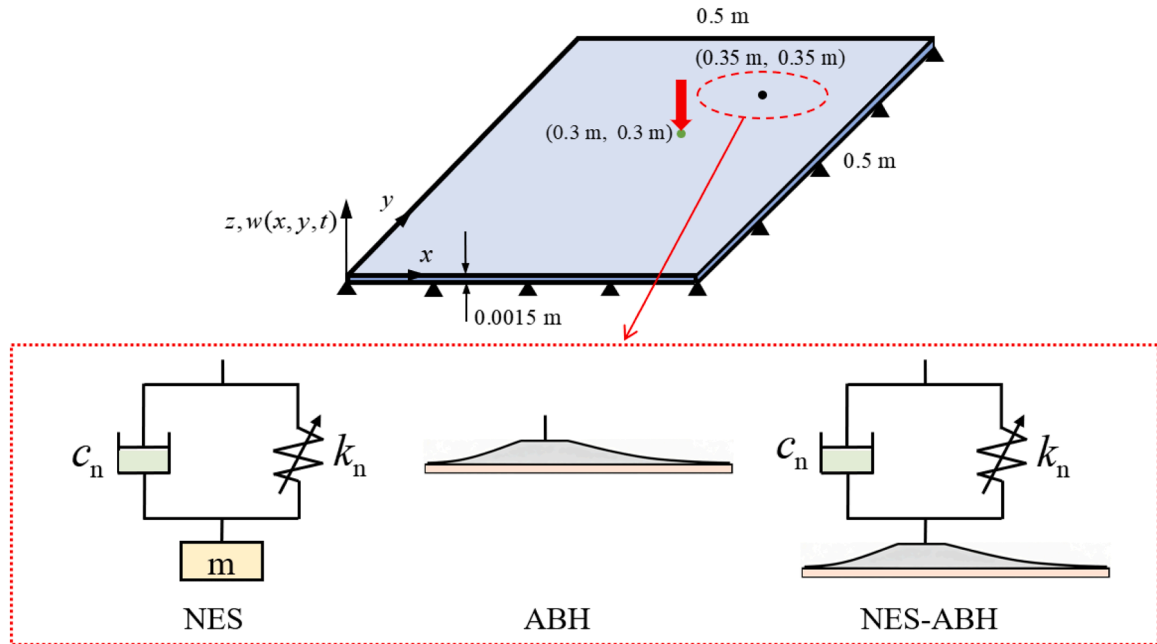


Fig. 3. Schematic diagram of different attachment schemes.

2.3. Solution of the coupled system

Based on the modal governing equations of the thin plate discretized in Section 2.1 and the interfacial displacement compatibility and force equilibrium relationships established in Section 2.2, a time-domain solution framework for the coupled system of the thin plate with the NES-ABH attachment absorber is constructed. To facilitate numerical integration, the governing equations of the system are rewritten into a first-order state-space form in this paper, and the fourth-order Runge-Kutta method (RK4) is employed for the step-by-step time-domain integration [23].

During the solution process, within each integration time step, the real-time displacement and velocity responses of the thin plate at the NES-ABH coupling point are first reconstructed based on the system state variables at the current moment. Combined Eq. (9), the nonlinear coupling force between the NES-ABH and the thin plate is calculated. Subsequently, this coupling force is projected onto the truncated modes of the thin plate and the ABH, respectively. The modal acceleration responses of the thin plate and the ABH are then solved by Eqs. (3) and (5), while the acceleration response of the NES part is obtained based on the force equilibrium relationship Eq. (8) at the coupling interface. After simultaneously updating the derivatives of the state variables for the three parts—the thin plate, ABH, and NES part—the system state variables at the end of the current time step are calculated using the RK4 method. Once the state update is completed, the calculation proceeds to the next integration time step, repeating the above process until the dynamic response solution for the entire time domain is completed.

3. Performance verification

3.1. Benchmark model and absorber configurations

To evaluate the vibration suppression performance of the proposed NES-ABH absorber, a simply supported rectangular plate is used as a benchmark host structure. Although the formulation in Section 2 is written in a general modal form, this benchmark is selected because its analytical modes are available and its low-order modal response can be clearly interpreted. The dimensions of the plate are $500\text{mm} \times 500\text{mm} \times 1.5\text{mm}$, corresponding to a thickness-to-span ratio of 0.003. This slender geometry is consistent with the classical thin-plate assumption adopted in Section 2. For a simply supported rectangular thin plate, the mode function can be expressed as [32]:

$$\phi_{ij}(x, y) = \sin\left(\frac{i\pi x}{a}\right) \sin\left(\frac{j\pi y}{b}\right), (i, j = 1, 2, \dots) \quad (10)$$

The (i, j) -th natural circular frequency is $\omega_{ij} = \pi^2 \sqrt{\frac{D}{\rho h}} \cdot \left[\left(\frac{i}{a}\right)^2 + \left(\frac{j}{b}\right)^2 \right]$, and the (i, j) -th modal mass is $m_{ij} = \frac{\rho h a b}{4}$.

The material parameters of the plate are listed in Table 1. This paper uses an impulsive excitation as the external load input. To determine the modal basis required to represent the dominant host-plate response under the present impulsive excitation, a modal-truncation sensitivity analysis was performed for the bare plate. A 20-mode plate model shows that approximately 98.94% of the squared displacement-response spectral content is contained below 130 Hz. Since the fourth and fifth natural frequencies are 118.3 and 147.9 Hz, respectively, the first four modes include all plate natural frequencies within this dominant response range. Within the same low-frequency range, adding the fifth mode changes the response by only 0.80%, while the tested higher-order enrichments produce variations below 1%. Therefore, the first four plate modes are adopted to represent the dominant low-frequency host-plate response in the subsequent reduced-order calculations. The corresponding frequencies of the first four plate modes are listed in Table 2.

The four-mode truncation is therefore used specifically to represent the dominant low-frequency response of the present host plate under the considered impulsive loading and should not be generalized to cases in which higher-order host-plate modes are strongly excited. In contrast, the ABH continuum retains the first 20 modes in the coupled calculation so that the low- and mid-frequency modal channels involved in the nonlinear frequency-broadening mechanism are preserved.

The excitation and absorber locations are shown in Fig. 3. The external excitation is applied at (0.3 m, 0.3 m), and the absorber is attached at (0.35 m, 0.35 m). The excitation point is selected away from the nodal lines of the dominant modes to ensure sufficient modal participation, while the absorber is placed in a region with relatively large low-order modal amplitudes to enhance the coupling between the host plate and the absorber.

Three absorber configurations are considered for comparison: a standalone NES, a standalone ABH absorber, and the proposed integrated NES-ABH absorber, as shown in Fig. 3. To ensure a fair comparison, the total mass ratio of the attached absorber to the host plate is kept identical for all configurations, at 8.5%. The nonlinear stiffness and damping coefficients of the NES element are also kept the same in the standalone NES and NES-ABH configurations, with $k_n = 5 \times 10^{10} \text{N/m}^3$ and $c_n = 1 \text{N} \cdot \text{s/m}$. Thus, the differences in vibration suppression performance mainly reflect the intrinsic dissipation mechanisms of the attached absorbers rather than the trivial effect of added mass.

For completeness, the standalone NES and standalone ABH models are briefly derived within the same modeling framework. For the standalone NES, the ABH continuum is replaced by a lumped mass m_{NES} , while the nonlinear spring-damper connection is retained. Its equation of motion is

$$m_{\text{NES}} \ddot{w}_N = c_n (\dot{w}_p - \dot{w}_N) + k_n (w_p - w_N)^3 \quad (11)$$

The force exerted by the NES on the host plate is equal in magnitude and opposite in direction to the force on the right-hand side of Eq. (11), and is introduced into the host-plate equation accordingly.

For the standalone ABH, the nonlinear connection is removed and the ABH is directly attached to the host plate. Its displacement is therefore written as

$$\mathbf{w}_A = \Phi_A \mathbf{q}_A + \mathbf{J} \mathbf{w}_p \quad (12)$$

and the modal equation becomes

$$\mathbf{I} \ddot{\mathbf{q}}_A + \mathbf{C} \dot{\mathbf{q}}_A + \mathbf{K} \mathbf{q}_A = -\Phi_A^T \mathbf{M}_A \mathbf{J} \ddot{\mathbf{w}}_p \quad (13)$$

The corresponding reaction force exerted by the standalone ABH on the host plate is obtained from Eq. (7) by replacing w_N with w_p . The external excitation is modeled as a rectangular impulse,

$$F_{\text{exc}} = \begin{cases} F_p, & t_0 \leq t < t_0 + \tau \\ 0, & t \geq t_0 + \tau \end{cases} \quad (14)$$

where F_p is the excitation amplitude, $t_0 = 0.01\text{s}$ is the trigger time, and $\tau = 0.001\text{s}$ is the pulse duration. Three excitation amplitudes, $F_p = 7\text{N}$, 30N , and 120N , are adopted to represent different input energy levels. Unless otherwise specified, the pulse duration is fixed at $\tau = 0.001\text{s}$, and all host-structure parameters are kept unchanged.

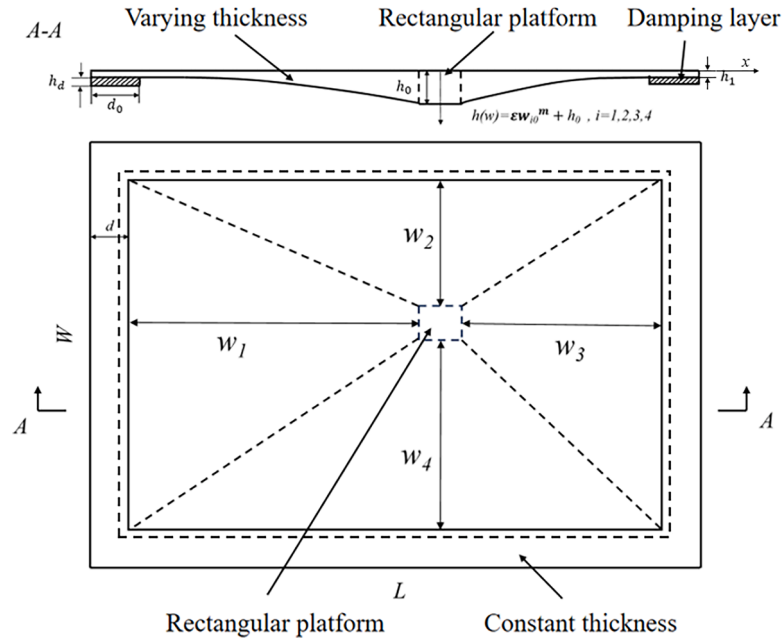


Fig. 4. Schematic diagram of the ABH structure.

Table 3
ABH geometric parameters.

$L(\text{mm})$	280	$w_1(\text{mm})$	125	$w_4(\text{mm})$	65
$W(\text{mm})$	160	$w_2(\text{mm})$	40	$d(\text{mm})$	20
$h_0(\text{mm})$	3	$w_3(\text{mm})$	99	m	2.5
$h_1(\text{mm})$	0.3	$h_d(\text{mm})$	1	$d_0(\text{mm})$	10

Table 4
Modal parameters of the first 10 modes of the ABH structure.

Order	Frequency (Hz)	Damping ratio	Effective modal mass
1	14.271	0.0386	0.011589
2	17.439	0.0573	0.000378
3	18.267	0.0424	0.010055
4	21.558	0.0543	0.001802
5	32.508	0.0650	0.003940
6	36.266	0.0788	0.001028
7	43.242	0.0750	0.001166
8	51.498	0.0982	0.000096
9	56.234	0.0696	0.008836
10	63.006	0.0809	0.000003

3.2. ABH configuration and parameters

The ABH used in the present study is a two-dimensional plate-type structure with a power-law thickness profile. As shown in Fig. 4, the ABH structure mainly consists of a rectangular uniform connecting platform with dimensions of $16\text{mm} \times 15\text{mm} \times h_0$, four variable-thickness regions, and a uniform layer. Among them, the variable-thickness regions are wrapped around the outer surface of the connecting post, extending from the four outer surfaces of the central connecting post away from it according to a specific thickness variation law, with extension lengths of w_1, w_2, w_3 and w_4 . Surrounding the four variable-thickness regions is a uniform layer with a width of d and a thickness of h_1 , which together form the main ABH structure with a length of L and a width of W . A damping layer with a width of d_0 and a thickness of h_d is bonded onto the surrounding uniform layer. The dimensional parameters of the ABH are provided in Table 3. The thickness variation in the ABH region follows the expression:

$$h(x) = \frac{h_1 - h_0}{w_i^m} x^m + h_0, i = 1, 2, 3, 4 \quad (15)$$

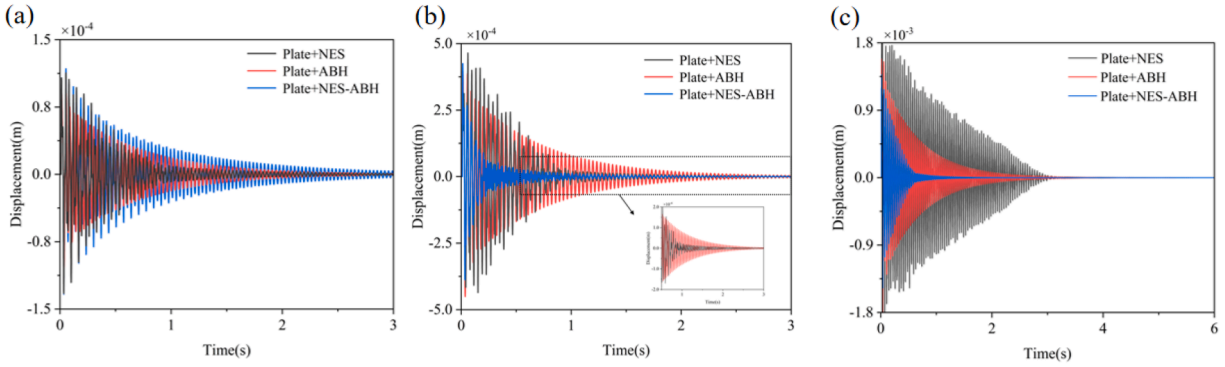


Fig. 5. Response at the plate coupling point: (a) $F_p = 7\text{N}$; (b) $F_p = 30\text{N}$; (c) $F_p = 120\text{N}$.

For the designed ABH structure, ABAQUS is used to perform finite element discretization and modal analysis. The C3D20R element is adopted, and the finite element model contains 2548 elements and 17775 nodes. The first 20 modes of the ABH are retained in the subsequent coupled dynamic calculation. The natural frequencies, modal damping ratios, and effective modal masses of the first 10 modes are listed in Table 4.

The modal frequencies of the ABH are distributed over a relatively broad frequency range. The first 10 natural frequencies range from 14.271 Hz to 63.006 Hz, covering the low-frequency band of the host plate. In particular, the 5th ABH mode at 32.508 Hz is close to the first natural frequency of the host plate, 29.6 Hz, indicating that this mode can provide an important low-frequency energy reception channel.

The effective modal mass reflects the contribution of each ABH mode to the translational energy exchange at the coupling interface. As shown in Table 4, the 1st, 3rd, 5th, and 9th modes have relatively large effective modal masses, suggesting that these modes are more strongly involved in the energy transfer between the host plate and the ABH continuum. These modes therefore play important roles in the subsequent multimodal dissipation process of the NES-ABH system.

In the coupled dynamic solution, modal truncation is applied to the ABH continuum. Retaining the first 20 modes provides a balance between computational efficiency and accuracy, while preserving the main low- and mid-frequency energy reception channels required for the synergistic dissipation mechanism.

3.3. Vibration response under different excitation levels

The performance verification in this study is organized around different impulsive excitation amplitudes because the NES-ABH absorber is essentially energy dependent. Its dissipation behavior is governed by how the nonlinear NES and the multimodal ABH respond as the input energy level changes. Therefore, three excitation amplitudes are considered to examine the effectiveness and robustness of the proposed configuration over different energy levels.

The coupled equations are solved using the fourth-order Runge-Kutta time-domain integration scheme described in Section 2.3. The time step is set to $\Delta t = 5 \times 10^{-4}\text{s}$, which provides a balance between numerical accuracy and computational efficiency while avoiding instability induced by the nonlinear coupling terms. Fig. 5 compares the displacement response at the coupling point of the plate for the three absorber configurations under different excitation amplitudes.

Under the low excitation level of $F_p = 7\text{N}$, the standalone NES exhibits the fastest response attenuation. In this case, the input energy is suitable for triggering targeted energy transfer, and the lumped mass of the NES can efficiently receive energy from the host plate. The NES-ABH configuration also reduces the vibration response, but its attenuation is slightly weaker than that of the standalone NES. This indicates that, at low energy levels, part of the energy is distributed into the elastic modes of the ABH continuum, which slightly weakens the direct NES-type energy transfer.

When the excitation amplitude increases to $F_p = 30\text{N}$, the NES-ABH configuration shows the best vibration suppression performance. The response decays more rapidly than that of both the standalone NES and the standalone ABH. By contrast, the standalone NES begins to exhibit a delay in response attenuation, implying that its energy transfer process becomes less stable as the input energy increases.

Under the large excitation level of $F_p = 120\text{N}$, the degradation of the standalone NES becomes more evident. Its response attenuation is delayed, suggesting that the nonlinear oscillator experiences energy-dependent detuning. In contrast, the NES-ABH configuration maintains a rapid and stable decay process. This result demonstrates that replacing the lumped NES mass with the ABH continuum improves the robustness of the absorber under large-amplitude excitation.

Overall, Fig. 5 indicates that the standalone NES is highly effective only within a suitable energy range, while the standalone ABH provides smoother but relatively limited attenuation. The proposed NES-ABH absorber combines the nonlinear energy transfer capability of the NES with the multimodal dissipation capability of the ABH, leading to more robust performance over a wider range of excitation levels.

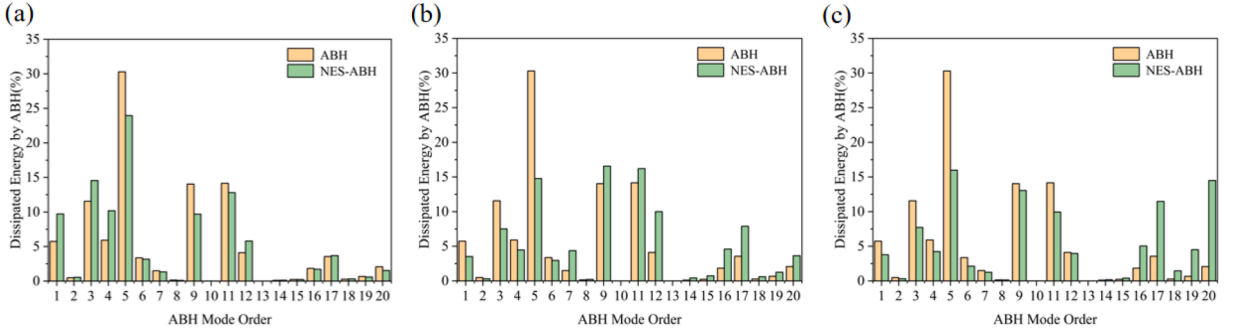


Fig. 6. Percentage of energy dissipated by each ABH mode relative to the total ABH energy dissipation: (a) $F_p = 7\text{N}$; (b) $F_p = 30\text{N}$; (c) $F_p = 120\text{N}$.

3.4. Modal energy dissipation of the ABH continuum

To clarify how the NES affects the energy dissipation pattern of the ABH continuum, the energy dissipated by each ABH mode is calculated. The modal dissipated energy is evaluated as:

$$E_{ABH} = \int_0^{t_d} \dot{\mathbf{q}}_A^T \mathbf{C} \dot{\mathbf{q}}_A dt = \sum_{i=1}^{20} 2\omega_{Ai} \zeta_{Ai} \int_0^{t_d} \dot{q}_{Ai}^2 dt \quad (16)$$

where t_d is chosen such that the vibration energy has been essentially dissipated. In the present calculation, $t_d = 3\text{s}$ for the 7 N and 30 N excitations, and $t_d = 6\text{s}$ for the 120 N excitation. The percentage of energy dissipated by each ABH mode relative to the total ABH dissipation is shown in Fig. 6 for both the standalone ABH and NES-ABH configurations.

For the standalone ABH, the modal energy dissipation distribution is almost unchanged as the excitation amplitude increases. Under all three excitation levels, the dissipated energy remains concentrated in a few modes close to the dominant frequency of the host plate. In particular, the 5th ABH mode contributes more than 30% of the total ABH dissipation and acts as the dominant energy-dissipating mode, whereas the modes above the 13th order contribute only weakly. This indicates a clear narrow-band dissipation feature. Since the standalone ABH is a linear continuum absorber, its response is governed mainly by modal superposition, and its energy dissipation pattern is primarily determined by frequency matching and effective modal mass rather than excitation amplitude.

In contrast, the NES-ABH configuration exhibits an excitation-dependent redistribution of modal energy dissipation. Under the 7 N excitation, the nonlinear effect of the NES is not yet fully activated, and the ABH dissipation is still dominated by low-order modes, especially the 5th mode and its neighboring modes. Under the 30 N excitation, the low-order modes remain active, but the energy originally concentrated in a few narrow-band modes begins to spread to more ABH modes. This indicates that the introduction of the NES broadens the range of participating ABH modes. When the excitation amplitude increases to 120 N, the dominant energy-dissipating modes further shift toward higher orders, and the modes above the 16th order become significantly activated.

These results show that the NES does not merely serve as a nonlinear connector between the plate and the ABH. Instead, it changes how the ABH receives and dissipates energy. As the excitation level increases, the nonlinear dynamics of the NES broaden the frequency content of the response, allowing the ABH continuum to receive and dissipate energy through multiple modes. The ABH therefore evolves from a narrow-band linear absorber in the standalone case to a multimodal energy receiver in the NES-ABH system. This provides the first evidence for the synergistic mechanism discussed in Section 4.

4. Synergistic dissipation mechanism and design implication

4.1. Energy dissipation evaluation and frequency broadening

To quantitatively evaluate the dissipation capability of different absorber configurations, the energy dissipation measure (EDM) is introduced. For a freely decaying system subjected to an impulsive excitation, the initial energy is finite and is assumed to be mainly stored in the primary structure immediately after the impulse. The EDM is defined as

$$\text{EDM} = E_d/E_0 \quad (17)$$

where E_d is the total energy dissipated by the absorber, and E_0 is the initial input energy of the system. In the present study, E_0 is evaluated as the work done by the external excitation on the primary structure, namely

$$E_0 = \int_0^{t_d} F_{\text{exc}} \dot{w}_{\text{exc}} dt \quad (18)$$

where $\dot{w}_{\text{exc}}$ is the transverse velocity of the primary structure at the excitation point.

For the NES-ABH configuration, the total dissipated energy consists of the energy dissipated by the NES damper and that dissipated

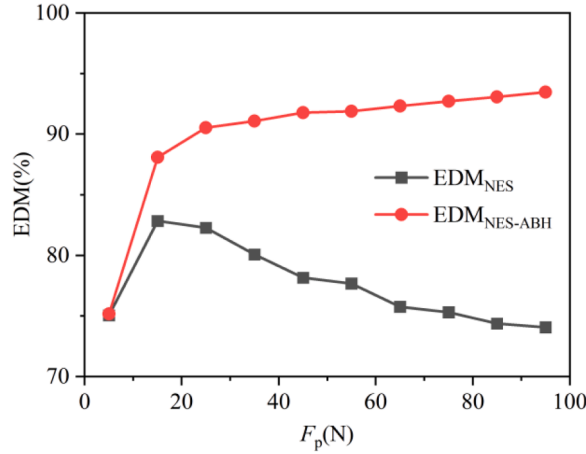


Fig. 7. Vibration suppression efficiency of the standalone NES scheme and NES-ABH scheme.

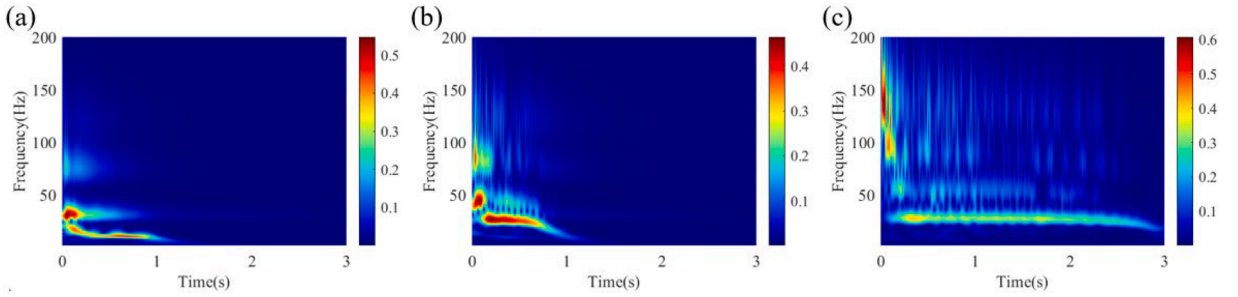


Fig. 8. Time-frequency diagram of NES relative displacement: (a) $F_p = 7$ N; (b) $F_p = 30$ N; (c) $F_p = 120$ N.

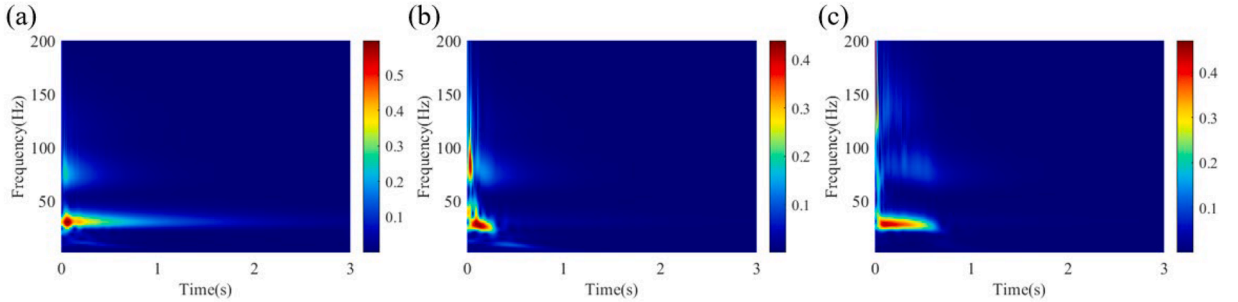


Fig. 9. Time-frequency diagram of NES-ABH relative displacement: (a) $F_p = 7$ N; (b) $F_p = 30$ N; (c) $F_p = 120$ N.

by the damping layer of the ABH continuum. The energy dissipated by the NES damper is calculated as

$$E_{NES} = c_n \int_0^{t_d} (\dot{w}_p - \dot{w}_N)^2 dt \quad (19)$$

where c_n is the NES damping coefficient and $\dot{w}_p - \dot{w}_N$ denotes the relative velocity between the host plate and the ABH connection point. The energy dissipated by the ABH is obtained by summing the modal damping dissipation of the retained ABH modes, as defined in Eq. (16).

Fig. 7 compares the EDM of the standalone NES and the NES-ABH configurations under different excitation amplitudes. When the excitation amplitude is small, the two configurations exhibit comparable dissipation efficiency, indicating that the input energy lies within the effective range for targeted energy transfer. As the excitation amplitude increases, the EDM of the standalone NES decreases after reaching a limited peak, which reflects the degradation of the conventional NES under large-amplitude excitation. This degradation is associated with the energy-dependent detuning of the nonlinear oscillator and the delayed establishment of stable resonance

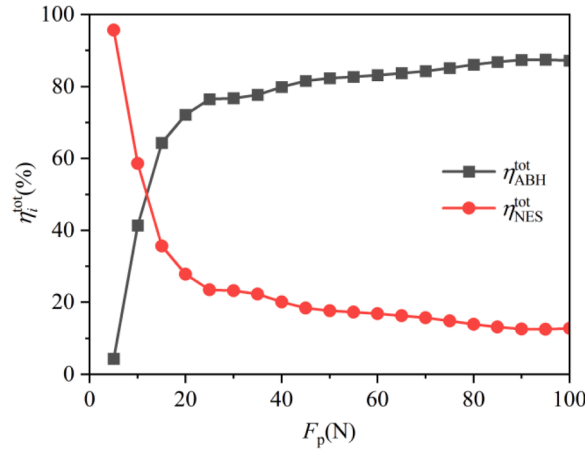


Fig. 10. Cumulative energy partition of NES and ABH under different F_p .

capture. In contrast, the NES-ABH configuration maintains a relatively high EDM over a wider excitation range, indicating that the ABH continuum provides additional energy reception and dissipation channels when the standalone NES becomes less effective.

To reveal the source of this difference, a wavelet transform is used to analyze the time-frequency evolution of the relative displacement between the primary structure and the attached absorber. Fig. 8 shows the time-frequency diagrams of the standalone NES. Under the 7 N excitation, the response energy is mainly concentrated in the low-frequency band of 0–50 Hz, close to the dominant frequency of the primary structure. When the excitation amplitude increases to 30 N, additional components appear in the 50–200 Hz range, and the frequency distribution becomes broader. Under the 120 N excitation, the energy spreads more evidently over the 0–200 Hz range, and the high-frequency components become denser and last longer. These results indicate that the nonlinear stiffness of the NES induces an amplitude-dependent frequency broadening effect.

For the standalone NES, however, the broadened frequency components are not necessarily beneficial. Once the nonlinear oscillator moves away from the effective resonance-capture condition, the broadband components cannot be efficiently dissipated by the NES damping alone, and the establishment of stable TET is delayed. This explains why the standalone NES performs well at relatively low energy levels but suffers from performance degradation under large excitation.

Fig. 9 shows the corresponding time-frequency response of the NES-ABH configuration. Under low excitation, the energy distribution is still mainly concentrated in the low-frequency band, and the ABH receives energy primarily through its dominant low-order modes. As the excitation amplitude increases, the nonlinear frequency broadening induced by the NES becomes activated. Unlike the standalone NES case, the broadened components do not persist for a long time in the NES-ABH system. Instead, the mid- and high-frequency components are received and dissipated by the multiple modes of the ABH continuum. Under large excitation, the broadband energy generated by the NES can be synchronously absorbed by the multi-order ABH modes, preventing long-term accumulation of high-frequency components.

Therefore, the nonlinear frequency broadening generated by the NES acquires a new role in the integrated NES-ABH system. It converts part of the narrow-band low-frequency energy of the primary structure into broader frequency components, while the ABH provides multiple modal channels to receive and dissipate these components. This frequency-domain complementarity is the first key mechanism underlying the improved robustness of the proposed absorber.

4.2. Energy partition and relay-type dissipation

To clarify the respective contributions of the NES and ABH in the integrated system, two types of energy partition are considered. The first one is the cumulative energy partition over the whole decay process, which is used to evaluate the overall contribution of each component under a given excitation amplitude. The second one is the cycle-based energy partition, which is used to describe how the dominant dissipation component evolves with time during the free-decay process.

For the cumulative energy partition, the total dissipated energies of the NES damper and ABH damping layer over the entire response duration are denoted as E_{NES} and E_{ABH} , respectively. Their cumulative contribution ratios are defined as

$$\eta_{NES}^{tot} = \frac{E_{NES}}{E_{NES} + E_{ABH}}, \quad \eta_{ABH}^{tot} = \frac{E_{ABH}}{E_{NES} + E_{ABH}} \quad (20)$$

These ratios describe the overall energy-sharing relationship between the NES and ABH parts after the vibration energy has been essentially dissipated.

Fig. 10 shows the variation of η_{NES}^{tot} and η_{ABH}^{tot} with the excitation amplitude from 5 N to 95 N. At low excitation levels, η_{NES}^{tot} is much larger than η_{ABH}^{tot} , reaching a maximum of about 96%, whereas the ABH contribution is nearly negligible. This indicates that when the input energy lies in the effective TET range, the NES is the dominant dissipation component. In this case, the energy transfer path is

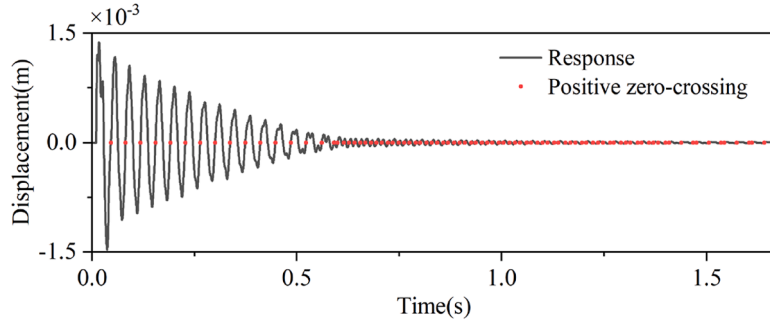


Fig. 11. Response of the primary structure and zero-crossing selection.

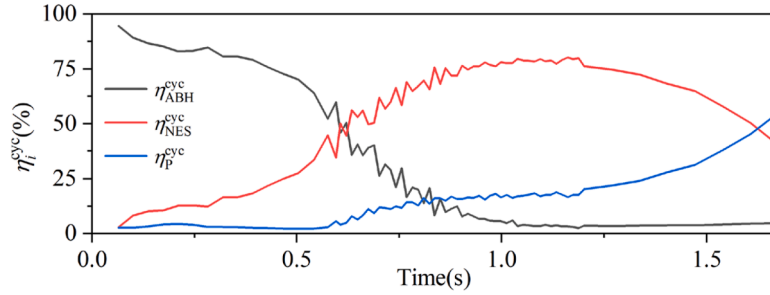


Fig. 12. Cycle-based dissipation proportions of the primary structure, NES, and ABH under $F_p = 120\text{N}$.

mainly governed by the nonlinear spring–damper element, and the ABH behaves more like an elastic receiving body with limited modal participation.

As the excitation amplitude increases, $\eta_{\text{NES}}^{\text{tot}}$ decreases continuously, while $\eta_{\text{ABH}}^{\text{tot}}$ increases significantly. This means that the dominant dissipation path gradually shifts from the NES damper to the ABH damping layer. Physically, the large-amplitude response activates the nonlinear frequency-broadening effect of the NES, which transfers part of the low-frequency energy into a wider frequency range. These broadened components are then better matched with the multiple modal channels of the ABH. Therefore, the ABH becomes increasingly involved in the energy dissipation process as the input energy increases. The synergistic effect of the two keeps the total energy dissipation efficiency of the system consistently around 90%, confirming their complementary advantages in the high and low energy ranges and the strong robustness of the system under wide energy input. This energy-dependent redistribution also explains why the total dissipation efficiency of the NES-ABH system remains around a high level over a wide excitation range, despite the degradation of the standalone NES.

However, the cumulative ratios in Fig. 10 do not reveal when each component dominates during the decay process. Therefore, a cycle-based energy partition is further introduced to capture the time evolution of the dissipation path. Since the instantaneous dissipated power oscillates strongly within each vibration cycle, the energy contribution is evaluated over one vibration period rather than at a single time instant.

Fig. 11 illustrates how the vibration cycles are selected. The positive zero-crossings of the primary-structure response are used as the boundaries of each cycle. If two adjacent positive zero-crossing instants are denoted by t_k and t_{k+1} , the corresponding local vibration period is

$$T_k = t_{k+1} - t_k \quad (21)$$

The energy dissipated by each component during this cycle is calculated over the interval $[t_k, t_{k+1}]$.

The cycle-based dissipation proportion is then defined as

$$\eta_i^{\text{cyc}}(t_k) = \frac{E_i(t_k, t_{k+1})}{E_P(t_k, t_{k+1}) + E_{\text{NES}}(t_k, t_{k+1}) + E_{\text{ABH}}(t_k, t_{k+1})} \quad (22)$$

where $i = P, \text{NES}, \text{ABH}$. Here, $E_P(t_k, t_{k+1})$, $E_{\text{NES}}(t_k, t_{k+1})$, and $E_{\text{ABH}}(t_k, t_{k+1})$ are the energies dissipated by the primary-structure damping, NES damper, and ABH damping layer during the k -th vibration cycle, respectively. These are defined as

Table 5
ABH-2 geometric parameters.

$L(\text{mm})$	160	$w_1(\text{mm})$	80	$w_4(\text{mm})$	51
$W(\text{mm})$	105	$w_2(\text{mm})$	17	$d(\text{mm})$	15
$h_0(\text{mm})$	3	$w_3(\text{mm})$	43	m	2.5
$h_1(\text{mm})$	0.3	$h_d(\text{mm})$	1.5	$d_0(\text{mm})$	15

$$\begin{cases} E_P(t_k, t_{k+1}) = \sum_{i=1}^2 \sum_{j=1}^2 2\zeta_{ij} m_{ij} \omega_{ij} \int_{t_k}^{t_{k+1}} \dot{q}_{ij}^2 dt \\ E_{\text{NES}}(t_k, t_{k+1}) = c_n \int_{t_k}^{t_{k+1}} (\dot{w}_P - \dot{w}_N)^2 dt \\ E_{\text{ABH}}(t_k, t_{k+1}) = \int_{t_k}^{t_{k+1}} \dot{\mathbf{q}}_A^T \mathbf{C} \dot{\mathbf{q}}_A dt = \sum_{i=1}^{20} 2\omega_{Ai} \zeta_{Ai} \int_{t_k}^{t_{k+1}} \dot{q}_{Ai}^2 dt \end{cases} \quad (23)$$

Fig. 12 shows the cycle-based dissipation proportions under large excitation $F_p = 120\text{N}$. In the early stage, approximately from 0 to 0.6 s, the ABH contribution rises rapidly and remains dominant, with its proportion exceeding 90% during part of this stage. This indicates that, immediately after the large impulse, the ABH first receives and dissipates the broadband energy components generated by the nonlinear response. During this period, the NES has not yet entered its most effective TET regime.

Around $t \approx 0.6\text{s}$, the ABH and NES dissipation proportions intersect. This time is defined as the relay transition time t_r ,

$$\eta_{\text{ABH}}^{\text{cyc}}(t_r) = \eta_{\text{NES}}^{\text{cyc}}(t_r) \quad (24)$$

Before t_r , the dissipation process is mainly ABH-dominated; after t_r , the NES gradually becomes the dominant dissipation component. After approximately 1 s, the ABH contribution decreases to a low level, while the NES contribution increases and reaches its peak around 1.1 s. This indicates that, after the ABH dissipates a large portion of the initial high-energy broadband response, the remaining energy level becomes more suitable for resonance capture and TET of the NES.

The contribution of the primary-structure damping increases slightly in the later stage, but remains lower than that of the attached structures during the main attenuation process. This confirms that the vibration energy is mainly transferred to and dissipated by the NES-ABH attachment rather than by the inherent damping of the host plate.

Therefore, the NES-ABH system exhibits a relay-type dissipation process. The cumulative energy partition in Fig. 10 shows that the dominant dissipation component depends on the input energy level, while the cycle-based partition in Fig. 12 further shows that, under large excitation, the dominant dissipation path changes with time from the ABH to the NES. Physically, t_r is not an intrinsic time constant of the absorber, but marks the point at which the decaying system passes from an ABH-favorable regime to an NES-favorable regime. At high response levels, strong nonlinear frequency broadening produces a wide frequency content that can be received by multiple ABH modes, while stable NES resonance capture is less favorable. As energy is dissipated and the response amplitude decreases, the broadened frequency content weakens and the system gradually approaches an energy range more favorable for NES resonance capture and TET. Therefore, t_r depends on the energy-decay trajectory. In principle, t_r could be predicted using an averaged energy model if the energy-dependent cycle-averaged dissipation rates of the ABH and NES were available. A crossover state could then be identified when the two dissipation rates become equal, and the corresponding transition time could be estimated from the energy-decay trajectory. In the present multimodal nonlinear system, however, the frequency content, ABH modal participation, and NES capture condition evolve simultaneously, so a simple closed-form expression is not available. In the present study, the transition time is therefore identified numerically from the intersection of the cycle-based ABH and NES dissipation contributions. This time-varying energy redistribution forms the basis of the energy-gating effect of the integrated NES-ABH absorber.

The clear sequential relay behavior identified here is specifically associated with finite-energy free decay following impulsive excitation. Under sustained harmonic, random, or continuously varying loads, the system energy is continuously replenished and need not decrease monotonically; therefore, a unique one-way ABH-to-NES relay transition or a single transition time should not be expected in general. Instead, the ABH- and NES-dominated regimes may coexist or alternate as the instantaneous response level changes. The more general conclusion is the energy-dependent complementarity of the two components: multimodal ABH dissipation is favored at relatively high response levels, whereas NES-type resonance capture and targeted energy transfer become more favorable when the system enters an appropriate energy range. The detailed manifestation of this complementarity under sustained and stochastic excitations remains to be investigated.

Taken together, the modal-dissipation distribution in Fig. 6, the cumulative energy partition in Fig. 10, and the cycle-based partition in Fig. 12 provide complementary views of the same mechanism: excitation-dependent broadening activates progressively more ABH modes, while the dominant dissipation pathway changes in time as the system energy decays.

4.3. Influence of the NES-ABH parameter

The mechanism discussed above indicates that the ABH in the NES-ABH system plays two different roles. First, it provides a low-frequency energy reception channel that can directly couple with the dominant mode of the host plate. Second, it provides multiple mid- and high-order modal channels to receive and dissipate the broadened frequency components generated by the nonlinear NES. Therefore, the ABH should not be designed only by maximizing the performance of a single mode. Instead, its modal distribution

Table 6

The first 10 modal parameters of ABH-2.

Order	Frequency (Hz)	Damping ratio	Effective modal mass
1	25.643	0.0833	0.010260
2	35.319	0.0819	0.000346
3	40.463	0.1112	0.009010
4	60.122	0.1202	0.000005
5	72.707	0.1299	0.005793
6	93.806	0.1752	0.000261
7	110.47	0.1425	0.000014
8	142.07	0.1344	0.001958
9	152.07	0.1704	0.003042
10	183.75	0.1498	0.000048

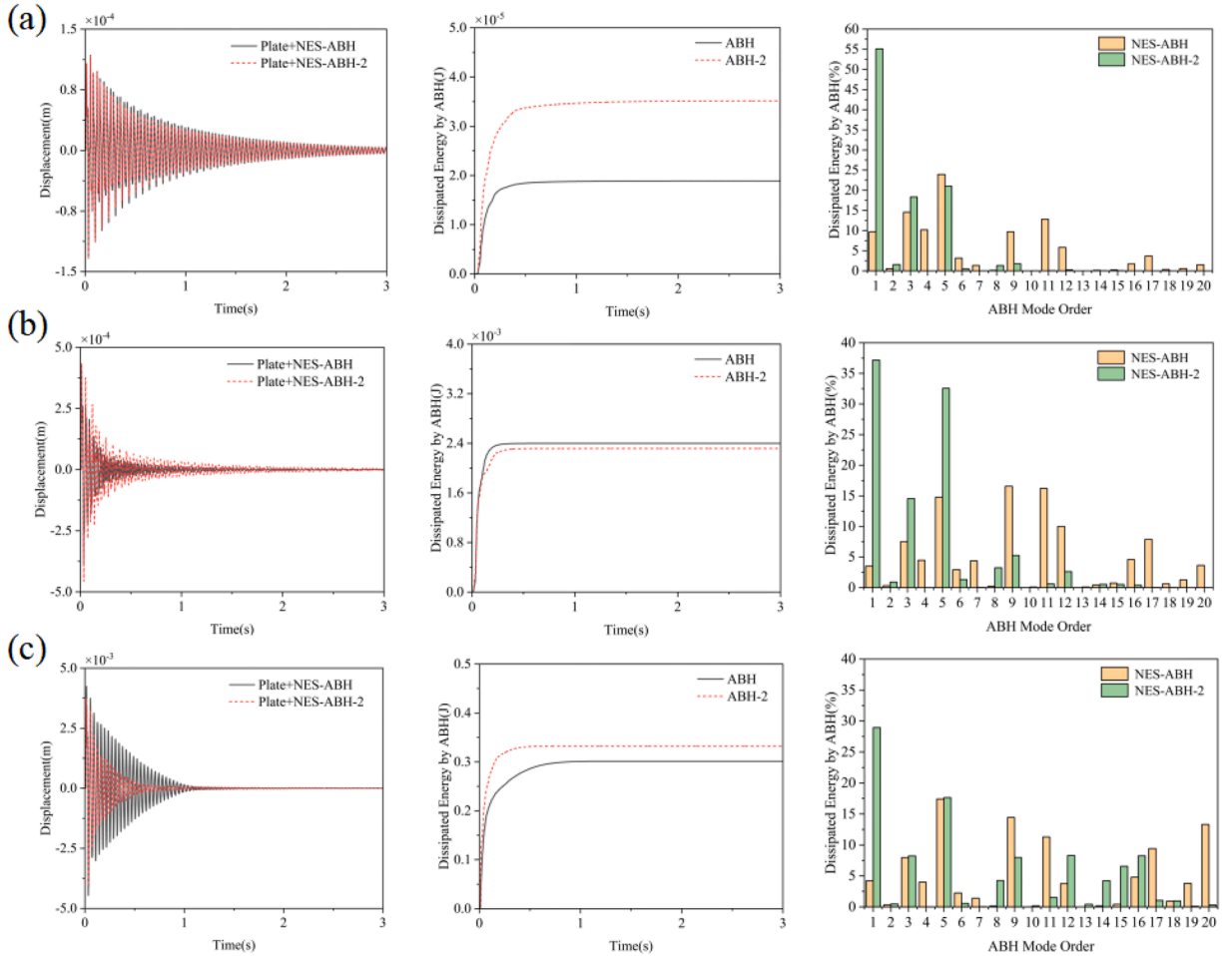


Fig. 13. Response at the plate coupling point, cumulative dissipated energy of ABH, and ABH modal energy dissipation proportion: (a) $F_p = 7\text{N}$; (b) $F_p = 30\text{N}$; (c) $F_p = 300\text{N}$.

should be tailored to balance three requirements: low-order modal matching, mid-frequency modal density, and high-order broadband coverage.

Based on this consideration, an optimized ABH structure, denoted as ABH-2, is introduced. Its geometric parameters are listed in Table 5, and the first ten modal parameters are given in Table 6. Compared with the original ABH, ABH-2 shifts its first mode to 25.643 Hz, which is closer to the fundamental frequency of the host plate at 29.6 Hz. Meanwhile, several mid- and high-order modes of ABH-2 show relatively high damping ratios. This design is intended to strengthen the low-frequency direct energy transfer channel while retaining broadband energy reception capability for the frequency components generated by NES nonlinear broadening.

To evaluate the influence of this parameter refinement, the original NES-ABH and the optimized NES-ABH-2 are compared under different excitation amplitudes. The response at the plate coupling point, the cumulative energy dissipated by the ABH, and the ABH

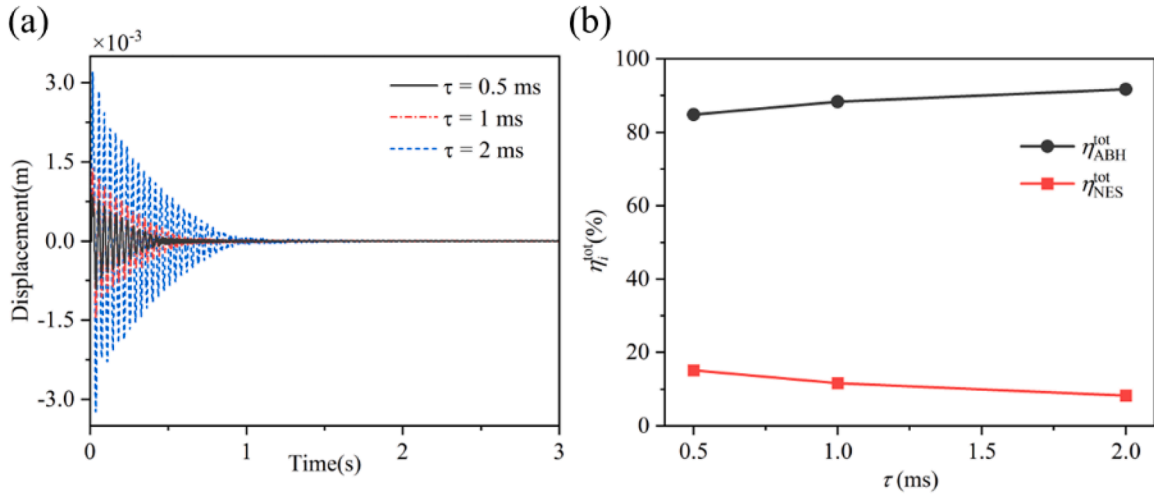


Fig. 14. Sensitivity to pulse duration of the NES-ABH system under fixed peak force at $F_p = 120\text{N}$: (a) displacement responses at the plate coupling point; (b) cumulative dissipation ratios of the NES and ABH.

modal energy dissipation distribution are shown in Fig. 13.

Under the low excitation level ($F_p = 7\text{N}$), the responses of the two configurations decay at similar rates. However, the cumulative energy dissipated by ABH-2 reaches a higher final value and exhibits a slightly larger initial slope. This indicates that ABH-2 receives more energy through the coupling interface even when the NES nonlinearity is weak. The reason is that its first mode is better matched with the fundamental mode of the host plate, and its effective modal mass is relatively large. Therefore, in the low-energy linear-dominant stage, ABH-2 mainly improves the direct low-order coupling channel rather than relying on nonlinear frequency broadening.

Under the moderate excitation level ($F_p = 30\text{N}$), the cumulative energy dissipated by the original ABH and ABH-2 becomes comparable, but the original ABH shows a faster energy dissipation rate during the early attenuation stage. Correspondingly, the response of the host plate decays slightly faster with the original NES-ABH configuration. This suggests that, under moderate excitation, the key factor is not only the final amount of energy dissipated by the ABH, but also the rate at which the broadened energy can be received. At this stage, the NES nonlinearity has been activated, but the broadened energy mainly extends to the low- and mid-frequency ranges. The original ABH possesses a denser modal distribution in this range, which forms a more continuous reception network for the mid-frequency energy components. In contrast, ABH-2 has relatively sparse modal coverage in this band, resulting in a lower early-stage energy dissipation rate.

When the excitation amplitude is further increased to the large-energy level ($F_p = 300\text{N}$), ABH-2 shows clear advantages. Its cumulative energy dissipation rises more rapidly and reaches a higher final value. The response of the host plate also decays faster with NES-ABH-2. This improvement comes from two aspects. On the one hand, the low-order mode of ABH-2 is better matched with the fundamental mode of the host plate, which strengthens the initial energy transfer channel. On the other hand, under strong nonlinear excitation, the NES broadens the vibration energy into a wider frequency range, and the mid- and high-order modes of ABH-2 can effectively receive and dissipate these broadband components. As a result, ABH-2 provides both faster initial extraction and larger total energy dissipation under large excitation.

These results show that the preferred ABH modal distribution depends on the target energy range rather than on a single tuning objective. Better low-order matching is advantageous when the response is dominated by direct low-frequency coupling, but it is not necessarily optimal under moderate excitation if the low- and mid-frequency modal density is insufficient. As the excitation level increases and nonlinear frequency broadening becomes stronger, broadband high-order coverage and modal damping become increasingly important. The design objective is therefore not to maximize one isolated ABH mode, but to provide a hierarchy of modal reception and dissipation channels that remains effective as the response spectrum evolves with energy level.

Therefore, the ABH design in the integrated NES-ABH system should follow a multimodal design principle rather than a single-mode tuning principle. Specifically:

- 1) The low-order ABH mode should be close to the dominant frequency of the host structure to ensure efficient early-stage energy transfer.
- 2) Sufficient modal density should be maintained in the low- and mid-frequency ranges to receive moderately broadened energy components.
- 3) Mid- and high-order modes with adequate damping should be preserved to dissipate broadband components generated under large-amplitude excitation.

This design implication also highlights the irreplaceable role of the ABH continuum in the NES-ABH system. If the lumped mass of the NES were replaced by a single-degree-of-freedom linear absorber, the energy reception would remain limited to a narrow tuning

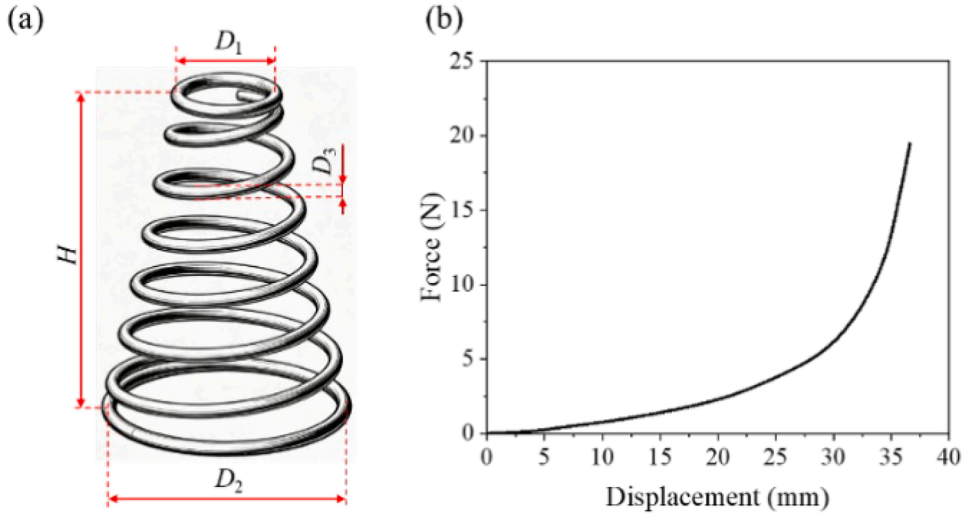


Fig. 15. Conical spring and its characteristic curve: (a) Schematic of the conical spring; (b) Force-displacement characteristic curve of the structure.

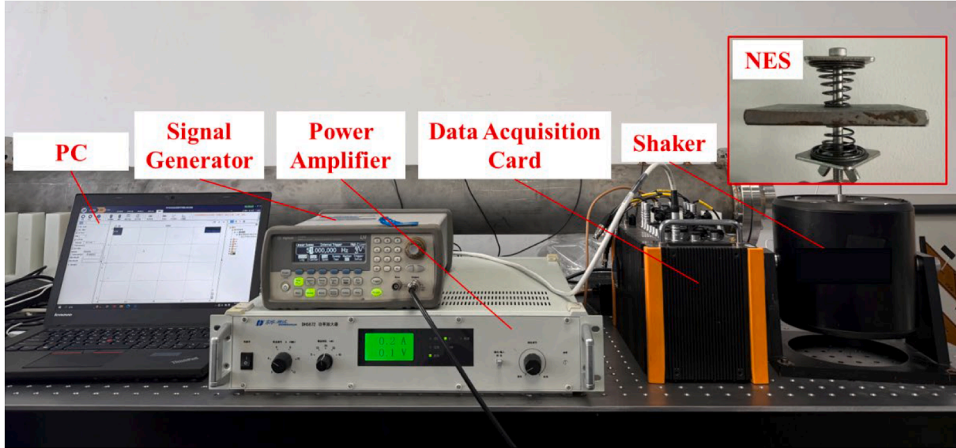


Fig. 16. Schematic of the experimental testing and acquisition system.

frequency. Once the nonlinear frequency broadening shifts energy away from that frequency, the absorber would lose effectiveness. In contrast, the ABH continuum provides multiple modal channels over a broader frequency range, allowing it to receive and dissipate energy even when the frequency content changes with excitation amplitude.

4.4. Sensitivity to pulse duration under fixed peak force

To further examine the sensitivity of the energy-dependent dissipation behavior to the impulsive loading condition, the effect of pulse duration was investigated at $F_p = 120\text{ N}$. Fig. 14 compares the displacement responses and the cumulative dissipation ratios of the NES and ABH for $\tau = 0.5\text{ ms}$, 1.0 , and 2.0 ms . As shown in Fig. 14(a), increasing τ leads to a stronger transient response. Meanwhile, $\eta_{\text{ABH}}^{\text{tot}}$ the cumulative dissipation contribution of the ABH increases from 84.83% to 91.71%, whereas $\eta_{\text{NES}}^{\text{tot}}$ decreases from 15.17% to 8.29%, as shown in Fig. 14(b). These results indicate that the energy-sharing behavior of the NES-ABH system is sensitive to the pulse duration. Under the fixed-peak-force condition considered here, increasing the pulse duration increases the overall energy input and progressively shifts the dominant dissipation contribution toward the ABH. This behavior is consistent with the energy-dependent mechanism identified in Section 4.2: as the excitation energy increases, nonlinear frequency broadening becomes more pronounced, activating a wider range of ABH modes and thereby enhancing the contribution of multimodal ABH dissipation.

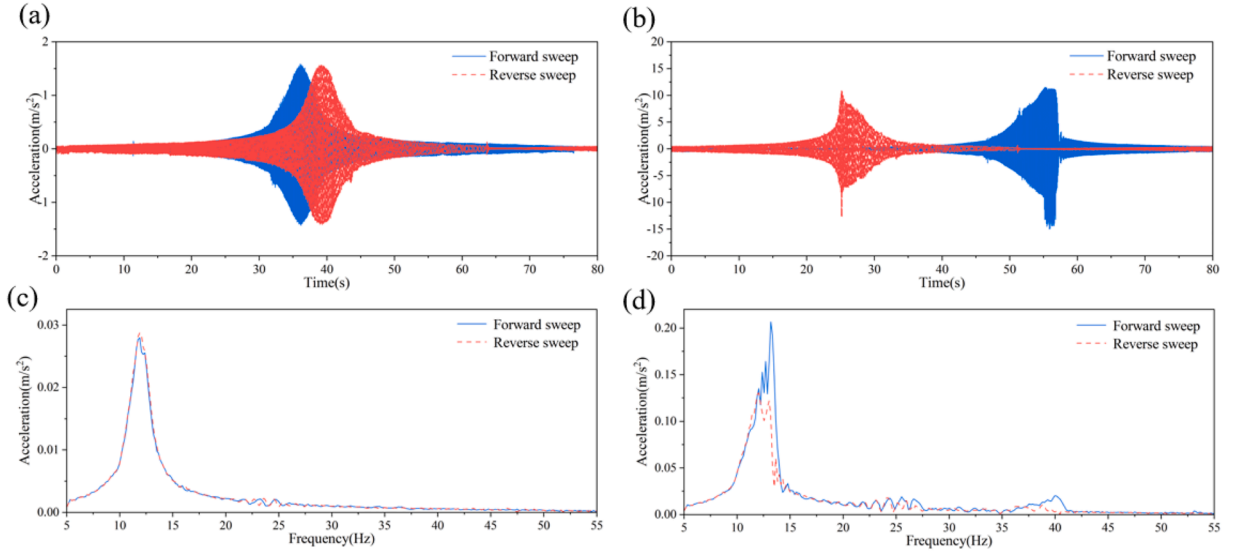


Fig. 17. Comparison of time-domain and frequency-domain results from forward and reverse sweeps under different excitations (0.2 V and 1 V): (a) Time domain at small amplitude; (b) Time domain at large amplitude; (c) Frequency domain at small amplitude; (d) Frequency domain at large amplitude.

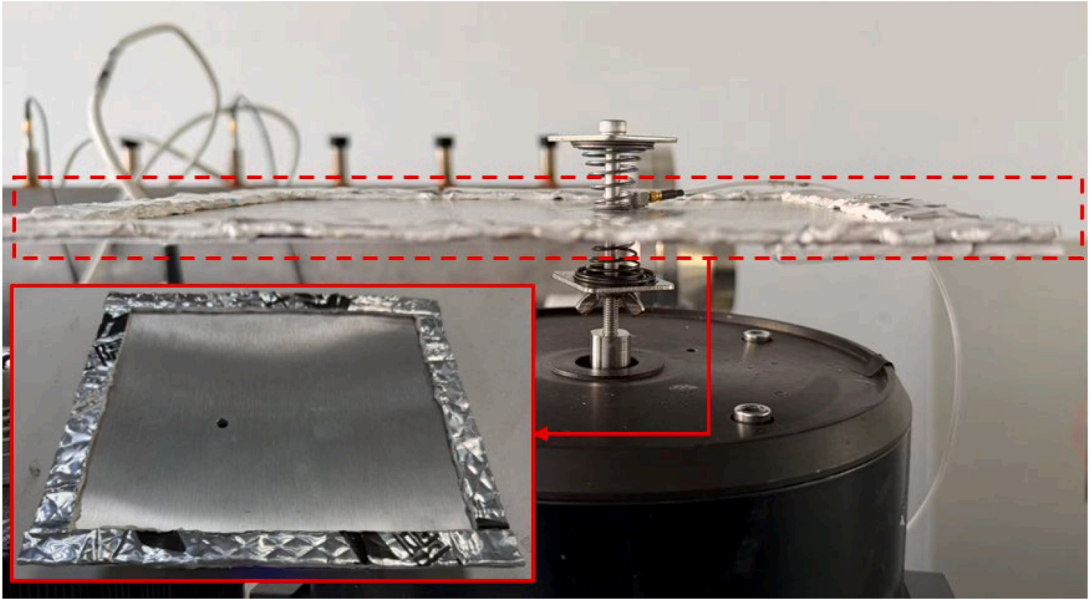


Fig. 18. Schematic of the NES-ABH structure.

5. Experimental verification

5.1. Nonlinear characteristic of the NES element

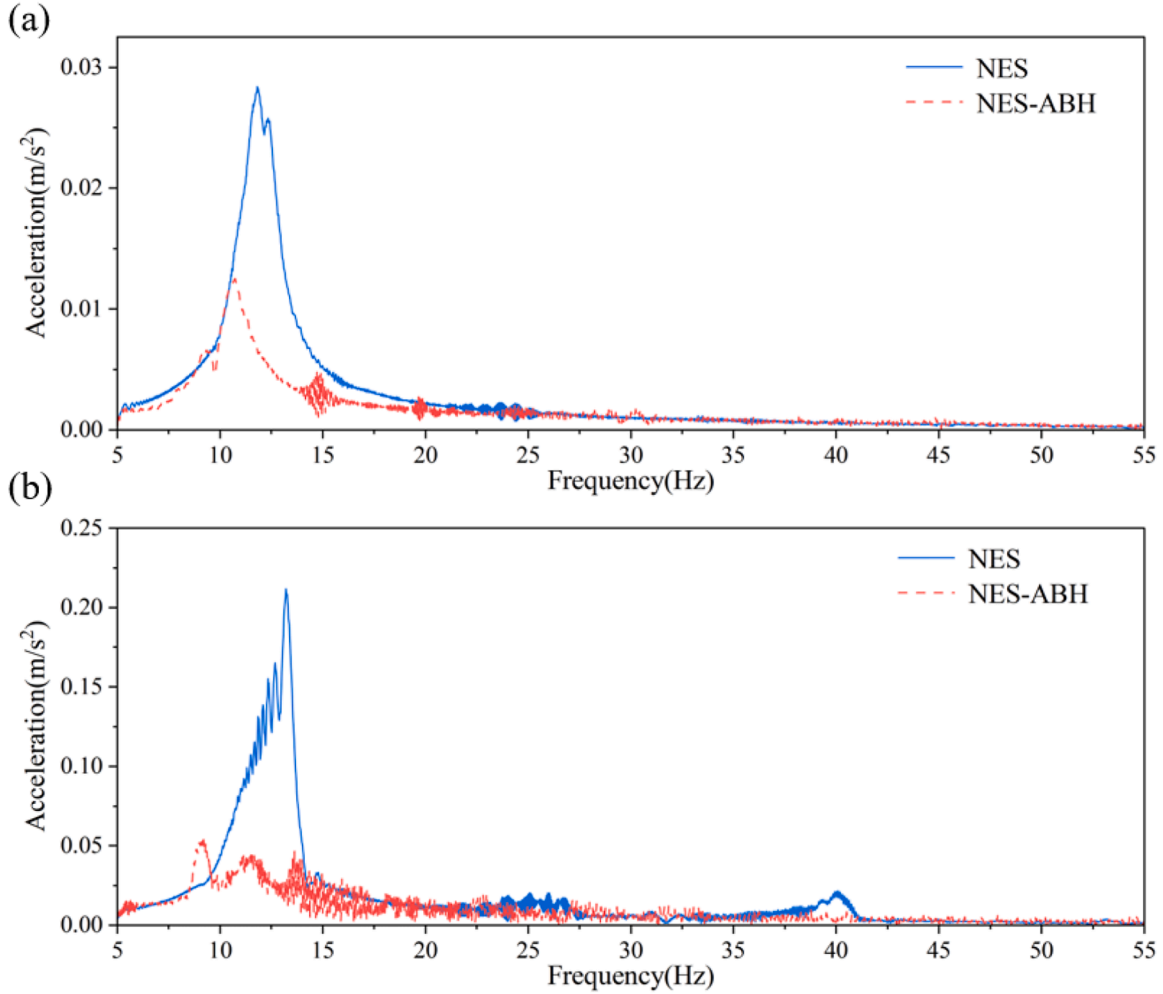
To experimentally verify the feasibility of the proposed NES-ABH configuration, a nonlinear spring element was first designed using a pair of conical springs, as shown in Fig. 15(a). The geometric parameters of the conical spring are $D_1 = 12$ mm, $D_2 = 26$ mm, $D_3 = 1$ mm and $H = 39$ mm, with seven coils. The measured force–displacement curve is shown in Fig. 15(b). The result indicates that the restoring force exhibits obvious nonlinear characteristics as the compression displacement increases, which provides the required nonlinear stiffness for the NES.

It should be noted that the experimental setup is not intended to reproduce the numerical benchmark model quantitatively. In the simulations, the host plate, nonlinear stiffness, damping coefficients, and ABH modal parameters can be prescribed independently,

Table 7

Geometric parameters of the ABH experimental specimen.

$L(\text{mm})$	223	$w_1(\text{mm})$	110.5	$w_4(\text{mm})$	77
$W(\text{mm})$	186	$w_2(\text{mm})$	49	$d(\text{mm})$	20
$h_0(\text{mm})$	5	$w_3(\text{mm})$	52.5	m	3
$h_1(\text{mm})$	0.3	$h_d(\text{mm})$	1.5	$d_0(\text{mm})$	15

**Fig. 19.** Comparison of frequency response results of NES-ABH and NES.

whereas in the experiment the nonlinear stiffness of the conical springs, assembly preload, friction, boundary conditions, and ABH fabrication details are difficult to control precisely. Therefore, the experimental validation focuses on verifying the key physical trends and vibration suppression capability of the NES-ABH concept, rather than providing a one-to-one comparison with the optimized numerical model. In particular, the optimized ABH-2 discussed in Section 4 is used to illustrate the design implication of the revealed mechanism, while the experimental prototype is designed to demonstrate the feasibility and robustness of replacing the NES lumped mass with an ABH continuum.

Based on the conical spring, a standalone NES prototype was assembled, as shown in Fig. 16. The structure consists of two conical springs, a guide rod, and an attached mass block. The mass block is a uniformly drilled thin iron sheet with a mass of 155.3 g, capable of vibrating axially along the smooth rod. The bottom nut can adjust the pre-compression of the springs. Forward and backward swept-sine tests were conducted to identify its nonlinear dynamic behavior. The sweep frequency range was 5–85 Hz, with a sweep duration of 80 s and a sampling frequency of 2000 Hz.

Fig. 17 compares the time-domain and frequency-domain responses under small- and large-amplitude excitations. Under small-amplitude excitation, the forward and backward responses nearly overlap, and no obvious jump or hysteresis is observed, indicating that the system mainly behaves linearly. Under large-amplitude excitation, however, the response curves become asymmetric

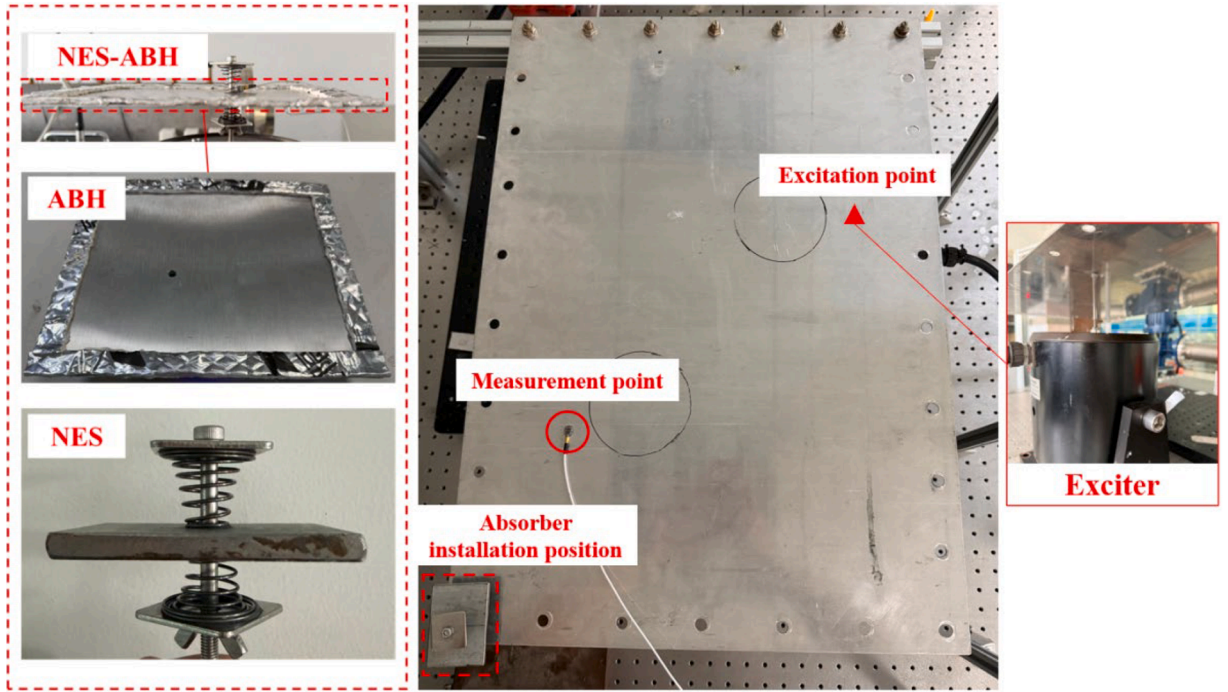


Fig. 20. Overall schematic of the control experiment.

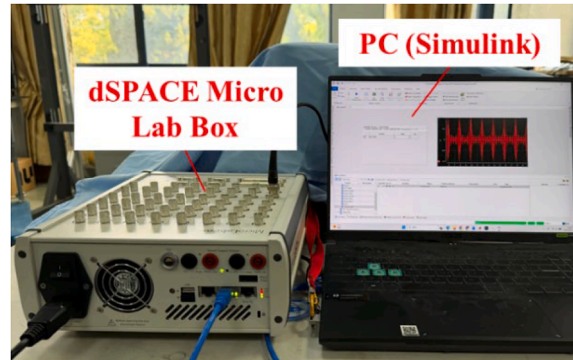


Fig. 21. dSPACE MicroLabBox real-time simulation system.

and a clear hysteresis loop appears in the frequency response. A pronounced capture peak is observed around 13.2 Hz, confirming that the designed NES can exhibit nonlinear energy capture when the excitation amplitude is sufficiently large.

5.2. Experimental comparison between NES and NES-ABH

To construct the NES-ABH prototype, the lumped mass of the standalone NES was replaced by an ABH structure with the same mass, as shown in Fig. 18. The ABH was fabricated from aluminum alloy, and a constrained damping layer made of butyl rubber with aluminum film was attached to the surface. The geometric parameters of the experimental ABH specimen are listed in Table 7. This mass-equivalent design ensures that the comparison between the NES and NES-ABH reflects the effect of replacing the lumped mass with a multimodal continuum absorber, rather than a simple mass change.

The frequency response characteristics of the standalone NES and NES-ABH were compared under the same swept-frequency excitation, as shown in Fig. 19. Under small-amplitude excitation, both systems mainly exhibit linear resonance behavior. However, the resonance peak of the NES-ABH is significantly lower than that of the standalone NES, indicating that the high-damping ABH continuum can provide additional dissipation even before strong nonlinear behavior is fully activated.

Under large-amplitude excitation, the standalone NES shows a sharp resonance peak associated with nonlinear energy capture. In contrast, the NES-ABH response becomes more distributed and its peak amplitude is clearly reduced. This phenomenon indicates that,

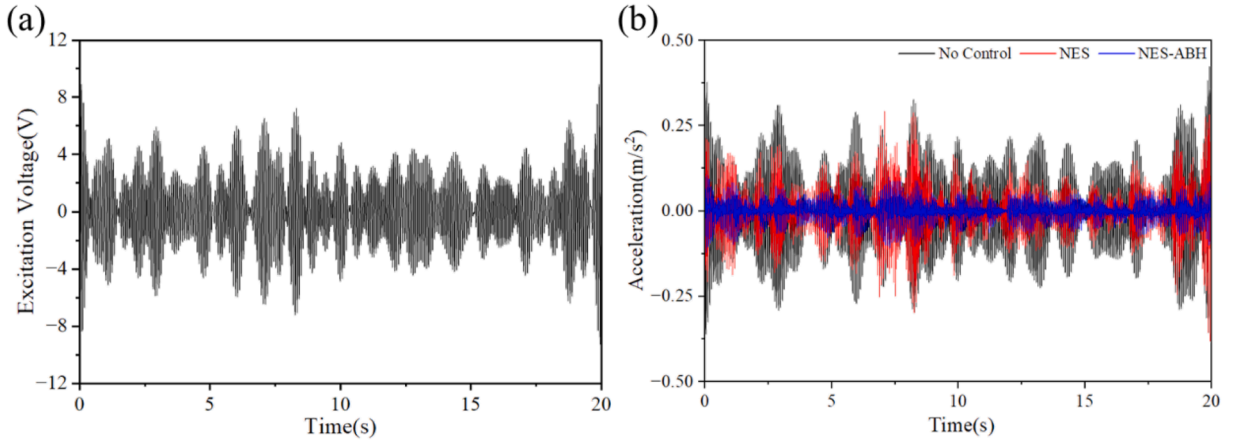


Fig. 22. Excitation signal and vibration test results under 12–14 Hz bandwidth: (a) Excitation signal; (b) Vibration test results.

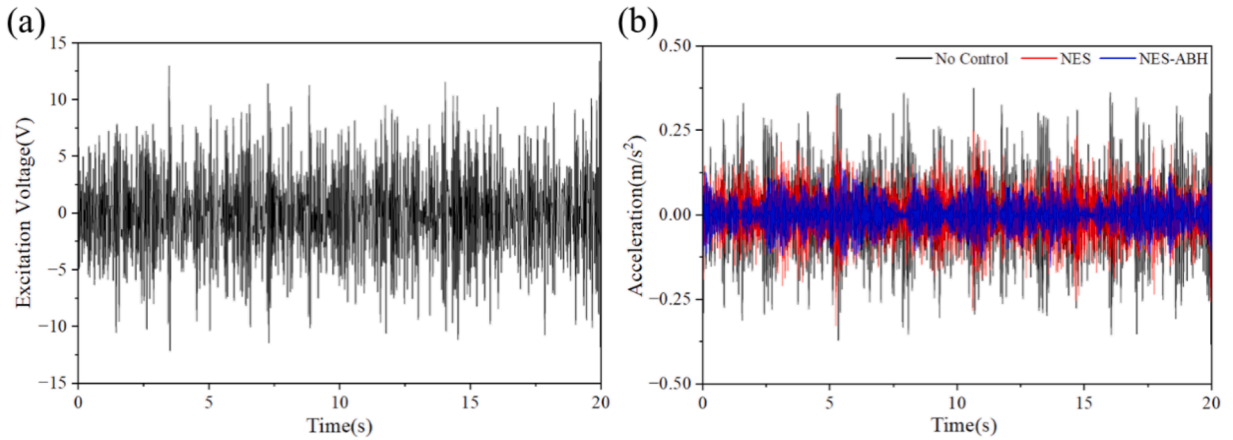


Fig. 23. Excitation signal and vibration test results under 8–20 Hz bandwidth: (a) Excitation signal; (b) Vibration test results.

after the nonlinear spring captures energy, the energy is transmitted into the ABH continuum and redistributed among multiple flexural modes. The damping layer of the ABH then dissipates the distributed energy, leading to a broader and more stable attenuation effect. This result is consistent with the numerical mechanism revealed in Sections 3 and 4: the ABH transforms the concentrated NES response into a multimodal dissipation process.

5.3. Vibration control test on a cantilever plate

A cantilever plate was further used as the controlled structure to compare the vibration suppression performance of different absorbers, as shown in Fig. 20. The plate was clamped along one edge and free along the other three edges. The excitation point, measurement point, and absorber installation position are indicated in Fig. 20. The standalone NES, standalone ABH, and NES-ABH were installed at the same location to ensure consistency among the tests.

First, a narrow-band excitation was applied to activate the nonlinear energy capture range of the NES. Based on the nonlinear response identified in Fig. 17(d), the excitation was constructed by superimposing sinusoidal components with random frequencies and phases in the 12–14 Hz band. The signal was generated in Simulink and output through the dSPACE MicroLabBox system to drive the shaker, as shown in Fig. 21.

The excitation signal and vibration responses are shown in Fig. 22. Under the 12–14 Hz excitation, both the standalone NES and the NES-ABH exhibit vibration reduction effects. However, the NES-ABH achieves a lower response amplitude than the standalone NES. This indicates that, within the effective nonlinear working range of the NES, replacing the lumped mass with the ABH continuum further enhances the dissipation capacity through multimodal reception and damping.

To evaluate broadband robustness, the excitation bandwidth was then extended to 8–20 Hz. The broadband excitation was generated by superimposing 30 sinusoidal components with random frequencies and phases. The test results are shown in Fig. 23. Compared with the narrow-band case, the vibration reduction effect of the standalone NES decreases significantly because the

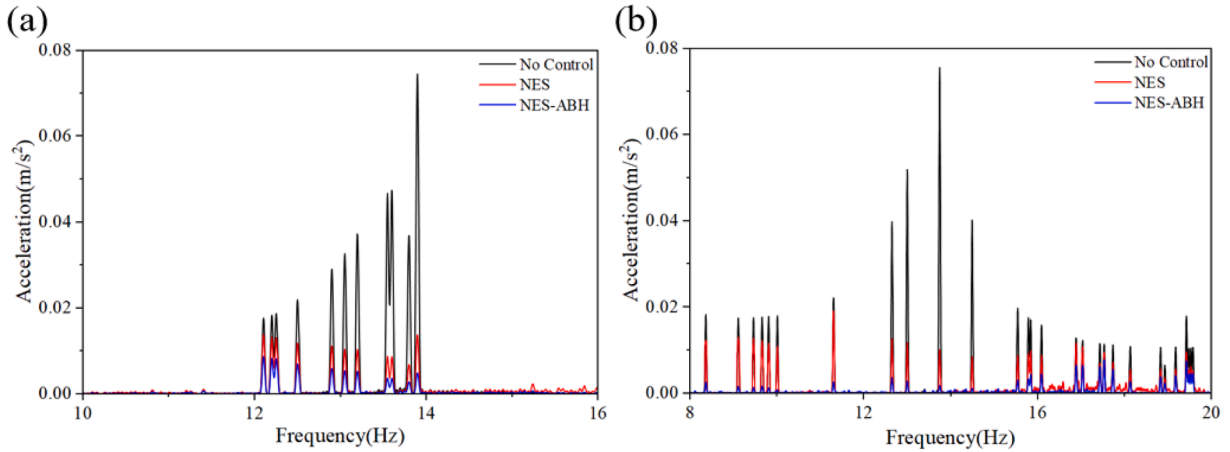


Fig. 24. Vibration spectra: (a) 12–14 Hz bandwidth excitation; (b) 8–20 Hz bandwidth excitation.

excitation band extends beyond its most effective capture range. In contrast, the NES-ABH maintains a stable suppression effect over the broader frequency range. This demonstrates that the ABH continuum helps compensate for the limited effective bandwidth of the standalone NES.

The response spectra under the two excitation conditions are further compared in Fig. 24. In the 12–14 Hz band, the standalone NES works effectively because its nonlinear energy capture mechanism is activated. The NES-ABH achieves additional suppression due to the high damping and multimodal characteristics of the ABH. Outside this narrow band, the standalone NES becomes less effective, whereas the NES-ABH still maintains lower spectral amplitudes over a wider frequency range. This result experimentally supports the proposed mechanism that the ABH provides additional modal dissipation channels when the excitation frequency content broadens.

Overall, the experimental results confirm three points. First, the designed nonlinear spring can provide the required amplitude-dependent NES behavior. Second, replacing the NES lumped mass with an ABH continuum reduces the resonance peak and redistributes the response energy. Third, the NES-ABH structure exhibits better robustness than the standalone NES under both narrow-band and broadband excitations. These observations are consistent with the numerical results and support the proposed nonlinear-continuum synergistic dissipation mechanism.

6. Conclusions

This paper proposes a nonlinear-continuum coupled NES-ABH absorber by replacing the lumped mass of a conventional NES with a two-dimensional ABH continuum. Beyond demonstrating vibration reduction performance, the work reveals how nonlinear energy transfer and ABH multimodal dissipation interact in a freely decaying plate system. The main findings and contributions are summarized as follows:

- 1) An integrated NES-ABH configuration is developed, in which the ABH is not an additional absorber but acts as the continuum mass of the NES. This configuration preserves the nonlinear energy transfer capability of the NES while introducing multimodal energy reception and dissipation through the ABH.
- 2) An amplitude-dependent relay-type dissipation mechanism is identified. Under large-amplitude excitation, the NES first induces frequency broadening, and the ABH preferentially dissipates the broadened energy components through multiple modes. As the system energy decreases, the NES gradually becomes dominant and targeted energy transfer is established more effectively.
- 3) The study shows that the NES fundamentally changes the modal dissipation pattern of the ABH. Compared with a standalone ABH, the NES-ABH system activates more mid- and high-order ABH modes under large excitation, enabling the ABH to evolve from a narrow-band absorber into a broadband multimodal energy receiver.
- 4) A multimodal ABH design principle is proposed. Effective NES-ABH design should balance low-order modal matching with the host structure, mid-frequency modal density, and high-order broadband coverage, rather than pursuing single-mode tuning.
- 5) Experimental tests under swept-frequency, narrow-band, and broadband random excitations confirm the feasibility and robustness of the proposed concept. The results support the conclusion that replacing the NES lumped mass with an ABH continuum can improve vibration suppression over a broader excitation range.

CRediT authorship contribution statement

Jinhao Qiu: Supervision, Methodology, Funding acquisition, Conceptualization. **Ruisi Hu:** Writing – original draft, Software, Formal analysis, Data curation. **Zhuogeng Zhang:** Visualization, Validation, Investigation. **Hongli Ji:** Writing – review & editing, Supervision, Project administration. **Li Cheng:** Supervision, Resources.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at [doi:10.1016/j.jsv.2026.120162](https://doi.org/10.1016/j.jsv.2026.120162).

Data availability

Data will be made available on request.

References

- [1] T.T. Soong, B.F. Spencer Jr., Supplemental energy dissipation: state-of-the-art and state-of-the-practice, *Eng. Struct.* 24 (2002) 243–259, [https://doi.org/10.1016/S0141-0296\(01\)00092-X](https://doi.org/10.1016/S0141-0296(01)00092-X).
- [2] B.F. Spencer Jr., S. Nagarajaiah, State of the art of structural control, *J. Struct. Eng.* 129 (2003) 845–856, [https://doi.org/10.1061/\(ASCE\)0733-9445\(2003\)129:7\(845\)](https://doi.org/10.1061/(ASCE)0733-9445(2003)129:7(845)).
- [3] Z. Lu, Z.X. Wang, Y. Zhou, et al., Nonlinear dissipative devices in structural vibration control: a review, *J. Sound Vib.* 423 (2018) 18–49, <https://doi.org/10.1016/j.jsv.2018.02.052>.
- [4] Y.S. Lee, A.F. Vakakis, L.A. Bergman, et al., Passive non-linear targeted energy transfer and its applications to vibration absorption: a review, *Proc. Inst. Mech. Eng. Part K J. Multi. Body Dyn.* 222 (2008) 77–134, <https://doi.org/10.1243/14644193JMBD118>.
- [5] A.F. Vakakis, O.V. Gendelman, L. Bergman, et al., Nonlinear targeted energy transfer: state of the art and new perspectives, *Nonlinear Dyn.* 108 (2022) 711–741, <https://doi.org/10.1007/s11071-022-07216-w>.
- [6] A.S. Saeed, R. Abdul Nasar, M.A. Al-Shudeifat, A review on nonlinear energy sinks: designs, analysis and applications of impact and rotary types, *Nonlinear Dyn.* 111 (2023) 1–37, <https://doi.org/10.1007/s11071-022-08094-y>.
- [7] V.V. Krylov, F.J.B.S. Tilman, Acoustic “black holes” for flexural waves as effective vibration dampers, *J. Sound Vib.* 274 (2004) 605–619, <https://doi.org/10.1016/j.jsv.2003.05.010>.
- [8] O. Gendelman, L.I. Manevitch, A.F. Vakakis, et al., Energy pumping in nonlinear mechanical oscillators: part I—dynamics of the underlying Hamiltonian systems, *J. Appl. Mech.* 68 (2001) 34–41, <https://doi.org/10.1115/1.1345524>.
- [9] A.F. Vakakis, O. Gendelman, Energy pumping in nonlinear mechanical oscillators: part II—resonance capture, *J. Appl. Mech.* 68 (2001) 42–48, <https://doi.org/10.1115/1.1345525>.
- [10] D. Qiu, T. Li, S. Seguy, et al., Efficient targeted energy transfer of bistable nonlinear energy sink: application to optimal design, *Nonlinear Dyn.* 92 (2018) 443–461, <https://doi.org/10.1007/s11071-018-4067-7>.
- [11] T. Li, S. Seguy, A. Berlioz, Dynamics of cubic and vibro-impact nonlinear energy sink: analytical, numerical, and experimental analysis, *J. Vib. Acoust.* 138 (2016) 031010, <https://doi.org/10.1115/1.4032725>.
- [12] T. Li, S. Seguy, A. Berlioz, On the dynamics around targeted energy transfer for vibro-impact nonlinear energy sink, *Nonlin. Dyn.* 87 (2017) 1453–1466, <https://doi.org/10.1007/s11071-016-3127-0>.
- [13] G. Sigalov, O.V. Gendelman, M.A. Al-Shudeifat, et al., Resonance captures and targeted energy transfers in an inertially-coupled rotational nonlinear energy sink, *Nonlinear Dyn.* 69 (2012) 1693–1704, <https://doi.org/10.1007/s11071-012-0379-1>.
- [14] R. Vigié, G. Kerschen, Nonlinear vibration absorber coupled to a nonlinear primary system: a tuning methodology, *J. Sound Vib.* 326 (2009) 780–793, <https://doi.org/10.1016/j.jsv.2009.05.023>.
- [15] Z.N. Ahmadabadi, S.E. Khadem, Nonlinear vibration control of a cantilever beam by a nonlinear energy sink, *Mech. Mach. Theor.* 50 (2012) 134–149, <https://doi.org/10.1016/j.mechmachtheory.2011.11.007>.
- [16] M. Kani, S.E. Khadem, M.H. Pashaei, et al., Vibration control of a nonlinear beam with a nonlinear energy sink, *Nonlinear. Dyn.* 83 (2016) 1–22, <https://doi.org/10.1007/s11071-015-2304-x>.
- [17] Y.W. Zhang, H. Zhang, S. Hou, et al., Vibration suppression of composite laminated plate with nonlinear energy sink, *Acta Astronaut.* 123 (2016) 109–115, <https://doi.org/10.1016/j.actaastro.2016.02.021>.
- [18] J.E. Chen, W. Zhang, M.H. Yao, et al., Vibration reduction in truss core sandwich plate with internal nonlinear energy sink, *Compos. Struct.* 193 (2018) 180–188, <https://doi.org/10.1016/j.compstruct.2018.03.048>.
- [19] A. Pelat, F. Gautier, S.C. Conlon, et al., The acoustic black hole: a review of theory and applications, *J. Sound Vib.* 476 (2020) 115316, <https://doi.org/10.1016/j.jsv.2020.115316>.
- [20] L. Tang, L. Cheng, H. Ji, et al., Characterization of acoustic black hole effect using a one-dimensional fully-coupled and wavelet-decomposed semi-analytical model, *J. Sound Vib.* 374 (2016) 172–184, <https://doi.org/10.1016/j.jsv.2016.03.031>.
- [21] C.A. McCormick, M.R.F. Shepherd, Design optimization and performance comparison of three styles of one-dimensional acoustic black hole vibration absorbers, *J. Sound Vib.* 470 (2020) 115164, <https://doi.org/10.1016/j.jsv.2019.115164>.
- [22] H. Ji, N. Wang, C. Zhang, et al., A vibration absorber based on two-dimensional acoustic black holes, *J. Sound Vib.* 500 (2021) 116024, <https://doi.org/10.1016/j.jsv.2021.116024>.
- [23] Z.G. Zhang, H.L. Ji, J.H. Qiu, et al., Coupled aeroelastic analysis of a panel in supersonic flow with add-on acoustic black hole, *Chin. J. Aeronaut.* 38 (2025) 103390, <https://doi.org/10.1016/j.cja.2024.103390>.
- [24] H. Li, C. Touzé, A. Pelat, et al., A vibro-impact acoustic black hole for passive damping of flexural beam vibrations, *J. Sound Vib.* 450 (2019) 28–46, <https://doi.org/10.1016/j.jsv.2019.03.004>.
- [25] V. Denis, A. Pelat, C. Touzé, et al., Improvement of the acoustic black hole effect by using energy transfer due to geometric nonlinearity, *Int. J. Non-Linear Mech.* 94 (2017) 134–145, <https://doi.org/10.1016/j.ijnonlinmec.2016.11.012>.

- [26] H. Li, C. Touzé, F. Gautier, et al., Linear and nonlinear dynamics of a plate with acoustic black hole, geometric and contact nonlinearity for vibration mitigation, *J. Sound Vib.* 508 (2021) 116206, <https://doi.org/10.1016/j.jsv.2021.116206>.
- [27] H. Li, C. Touzé, A. Pelat, et al., Combining nonlinear vibration absorbers and the acoustic black hole for passive broadband flexural vibration mitigation, *Int. J. Non-Linear Mech.* 129 (2021) 103558, <https://doi.org/10.1016/j.ijnonlinmec.2020.103558>.
- [28] T. Wang, Y. Tang, T. Yang, et al., Bistable enhanced passive absorber based on integration of nonlinear energy sink with acoustic black hole beam, *J. Sound Vib.* 544 (2023) 117409, <https://doi.org/10.1016/j.jsv.2022.117409>.
- [29] M.X. He, Y. Tang, Q. Ding, Dynamic analysis and optimization of a cantilevered beam with both the acoustic black hole and the nonlinear energy sink, *J. Intell. Mater. Syst. Struct.* 33 (2022) 70–83, <https://doi.org/10.1177/1045389x211011679>.
- [30] Z. Zhang, H. Ji, H. Qian, L. Cheng, Panel flutter suppression via combined-nonlinear acoustic black hole, *J. Sound Vib.* 629 (2026) 119694, <https://doi.org/10.1016/j.jsv.2026.119694>.
- [31] S.P. Timoshenko, S. Woinowsky-Krieger, *Theory of Plates and Shells*, second ed., McGraw-Hill, New York, 1959.
- [32] A.W. Leissa, *Vibration of Plates*, NASA SP-160, National Aeronautics and Space Administration, Washington, D.C., 1969.
- [33] D. Jia, C. Gao, Y. Yang, et al., A semi-analytical method for dynamic analysis of a rectangular plate with general boundary conditions based on FSDT, *Rev. Adv. Mater. Sci.* 61 (2022) 477–492, <https://doi.org/10.1515/rams-2022-0033>.