



Full Length Article

Semi-analytical study on low-frequency bandgap of lightweight corrugated-core sandwich *meta*-plate using spectral element methodDingkang Chen^{a,b} , Xunyu Li^{a,b} , Xiaobin Li^{a,b}, Li Cheng^c, Kian Meng Lim^d , Yinggang Li^{a,b,*}^a Key Laboratory of High Performance Ship Technology (Wuhan University of Technology), Ministry of Education, Wuhan 430063, PR China^b Department of Naval Architecture, Ocean and Structural Engineering, School of Naval Architecture, Ocean and Energy Power Engineering, Wuhan University of Technology, Wuhan 430063, PR China^c Department of Mechanical Engineering, The Hong Kong Polytechnic University, Hong Kong 999077, PR China^d Department of Mechanical Engineering, National University of Singapore, 117575, Singapore

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ABSTRACT

In this paper, the elastic wave propagation and low-frequency bandgap characteristics of a lightweight high-strength corrugated-core sandwich *meta*-plate (CSMP) with local resonators under simply supported boundary conditions is studied. Based on the spectral element method, a non-equivalent high-precision theoretical wave dynamic model of CSMP is established to calculate the band structure and transmission characteristics of the structure. The low-frequency bandgap generation mechanism is revealed and the comprehensive performance of the proposed CSMP is compared with that of the typical sandwich metastructures. Numerical calculation and experimental tests are conducted to verify the proposed theoretical model. Results show that the proposed CSMP with only 1.77% mass ratio local resonators can exhibit a low-frequency bandgap, demonstrating significant flexural waves attenuation and excellent vibration isolation performance. The starting and cutoff frequencies of the low-frequency band gap are mainly attributed to the vibration coupling of the local resonator with the connecting plate and corrugated-core sandwich plate respectively. The prediction results of the theoretical model are consistent with those of numerical simulation and experimental test, which verifies the correctness of the proposed theoretical model. The flexural wave bandgap can be effectively adjusted by reasonably changing the design parameters of the CSMP.

1. Introduction

As transportation engineering in aerospace, automotive, and marine engineering evolves toward green lightweighting [1–5], demands for structural mechanical properties continue to increase. Sandwich structures combine lightweight properties with high strength, high specific stiffness, and impact resistance [6–10], demonstrating exceptional potential for engineering applications. However, compared with conventional monolithic plates, lightweight sandwich structures with thin faces and a thick low-density core

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Nomenclature

a	lattice constant of the CSMP
b	width of the CSMP
c_t	shear wave velocity
$\mathbf{d}_b$	spectral nodal displacement vector associated with the two free edges of the plate element
$\mathbf{d}_{s\text{-CSP}}$	nodal displacement vector of the CSP unit cell
D	bending stiffness of the plate
D_0	dynamic stiffness of the local resonator
E	Young's modulus
f_1	natural frequency of the local resonator
f_2	natural frequency of the host sandwich structure
$\mathbf{f}_b$	spectral nodal force vector associated with the two free edges of the plate element
f_s	normalized bandgap starting frequency
$\mathbf{f}_{s\text{-CSP}}$	nodal force vector of the unit cell of CSP
F_a	magnitude of the applied concentrated force
F_c	modal force coefficient of the applied concentrated force
h	thickness of the plate element
H	height of the CSMP
k_L	wavenumber of the in-plane longitudinal wave
k_r	the spring stiffness of the local resonator
k_S	wavenumber of the in-plane shear wave
k_{ym}	wavenumber of the m -th retained mode in the y -direction
K_1	the equivalent stiffness of the connecting plate
K_2	the equivalent stiffness of CSP
m_0	lumped mass attached to the upper facesheet
m_r	local resonator mass
M	number of retained modes in the y -direction
M_1	the mass of the connecting plate
M_2	the mass of CSP
N	number of frequency components
P_E	excitation point
$P_{LR,j}$	response position on the j -th local resonator
q	Bloch wave vector
R_b	relative bandwidth
$\mathbf{S}_b$	spectral stiffness matrix of plate element
$\mathbf{S}_b^g$	spectral stiffness matrix of plate element in the global coordinate system
$\mathbf{S}_{in}$	spectral stiffness matrix of in-plane vibration
S_{in}^{ij}	(i, j) th element of the in-plane spectral stiffness matrix
$\mathbf{S}_{out}$	spectral stiffness matrix of out-of-plane vibration
S_{out}^{ij}	(i, j) th element of the out-of-plane spectral stiffness matrix
$\mathbf{S}_{s\text{-CSP}}$	spectral stiffness matrix of the CSP unit cell in the global coordinate system
u	displacement in the x direction
U_{1nm}	displacement of node 1 in the x -direction in the frequency and wavenumber domain
U_{2nm}	displacement of node 2 in the x -direction in the frequency and wavenumber domain
ν	Poisson's ratio
v	displacement in the y direction
V_{1nm}	displacement of node 1 in the y -direction in the frequency and wavenumber domain
V_{2nm}	displacement of node 2 in the y -direction in the frequency and wavenumber domain
$w(x, y, t)$	displacement in the z -direction
$W_{c\text{-in}}$	vertical displacement at the excitation point
$W_{c\text{-out}}$	vertical displacement at the response point
W_{1nm}	displacement of node 1 in the z -direction in the frequency and wavenumber domain
W_{2nm}	displacement of node 2 in the z -direction in the frequency and wavenumber domain
$w(y_0)$	displacement of the plate at the local resonator attachment position y_0
y_s	position of the excitation point on the free edge in the y -direction
y_r	position of the response point on the free edge in the y -direction
μ	mass ratio of the local resonator to CSP
η	material loss factor

η_r	loss factor of the local resonator
λ	Bloch factor
φ	corrugation angle
ρ	density of the plate element
ω_n	n -th circular frequency
Θ_{1nm}	rotation of node 1 in the frequency and wavenumber domain
Θ_{2nm}	rotation of node 2 in the frequency and wavenumber domain

are mainly optimized for a high bending stiffness-to-weight ratio. As a result, their low-frequency vibration and noise reduction performance is still unsatisfactory [11–15], which motivates the development of more innovative design approaches.

Acoustic metamaterials/metastructures exhibit extraordinary elastic mechanical properties and physical characteristics [16–24] such as elastic wave bandgaps, in which wave propagation can be prohibited due to the Bragg scattering effect of periodic structure or local resonance effects [25–33]. The unique bandgap characteristics of acoustic metamaterials/metastructures provide a new and promising avenue for the vibration control of sandwich metastructures, as evidenced by the increasing attention received. For example, Song et al. [34] numerically studied the wave propagation and vibration characteristics of a honeycomb sandwich metastructure plate with an add-on resonator using finite element method (FEM). The results showed that the vibration of the honeycomb sandwich metastructure plate was significantly reduced in a low and wide frequency range. An et al. [35] proposed a Kagome lattice sandwich metastructure with added resonators and numerically investigated the low-frequency vibration reduction performance and bandgap generation mechanism of the structure. Chen et al. [36] studied the low-frequency vibration isolation characteristics of sandwich metastructure and experimentally validated the results. It was shown that grille sandwich metastructures exhibited significant low-frequency sub-wavelength flexural wave bandgaps conducive to vibration isolation and low-frequency vibration control of the ship power systems. Existing works have mainly focused on the effects of different topological configurations (such as honeycomb and Kagome lattices) and local resonators on bandgap characteristics and vibration attenuation performance, and the effectiveness of sandwich metastructures in low-frequency isolation has been verified through finite element simulations and experiments. However, these studies have not sufficiently addressed the formation mechanism of low-frequency bandgaps. In particular, the effects of geometric parameters on the starting and cutoff frequencies of low-frequency bandgap remain to be systematically clarified.

Moreover, above studies were carried out by using numerical approaches and experimental methods. Therefore, in recent years, there has been an increasing effort in developing theoretical tools to study the vibration characteristics of sandwich metastructures more quickly and efficiently. For example, Jin et al. [37] proposed a multifunctional metastructure combining a honeycomb sandwich structure and a locally resonant metastructure. The sandwich metastructure was equated to a homogeneous plate with periodic resonators, allowing for the establishment of an analytical model to study the vibration characteristics based on Hamilton's principle and energy averaging technique. An et al. [38] simplifies the *meta*-orthogrid sandwich panel with spring-mass resonators into a thin plate with spring-mass resonators based on the equivalent principle and thin plate small deflection bending theory, and predicts its bandgap. Zhang et al. [39] combines dynamic equivalent theory and spectral element method to establish an effective bandgap prediction and vibration transmission analysis model for honeycomb sandwich metastructures. Nevertheless, above studies on theoretical modeling usually adopt equivalent treatments for sandwich structures, in which they are often simplified as homogeneous beam or plate elements. Such simplifications neglect not only the internal interactions within the sandwich structure but also the coupling effects between local resonators and the connecting plates. Therefore, in order to accurately study the low-frequency bandgap characteristics of sandwich metastructures, it is urgently necessary to establish a non-equivalent high-precision theoretical wave dynamics model.

The motivation for the present work, which shows its novelty in multiple aspects, is as follows: 1) to establish a non-equivalent high-precision theoretical wave dynamic model for the corrugated-core sandwich *meta*-plate (CSMP) with simply supported boundary conditions; and 2) to reveal the low-frequency formation mechanism of the CSMP and guide the regulation and optimization design of sandwich metastructures. More specifically, the elastic wave propagation and low-frequency bandgap characteristics of a light-weight high-strength CSMP with local resonators under simply supported boundary conditions is studied. Based on the spectral element method (SEM), a non-equivalent high-precision theoretical wave dynamic model of CSMP is established to calculate the band

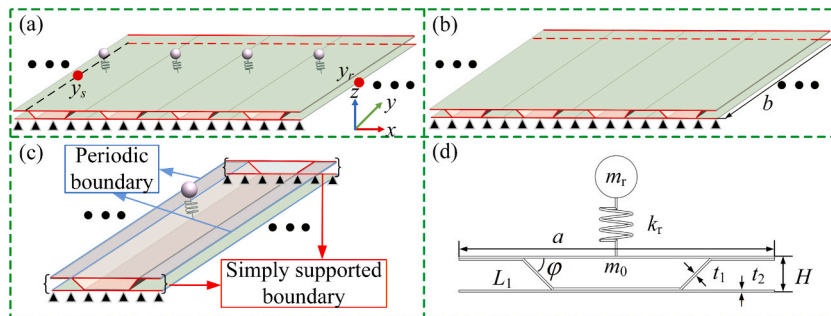


Fig. 1. (a) A periodic CSMP; (b) A periodic CSP; (c) The unit cell of the CSMP; (d) Schematic front view and geometric dimensions of the unit cell.

structure and transmission characteristics of the structure. The low-frequency bandgap generation mechanism is revealed and the comprehensive performance of the proposed CSMP is compared with that of the typical sandwich metastructures. Numerical calculation and experimental tests are conducted to verify the proposed theoretical model.

2. Model and theoretical formulation

2.1. Physical model of CSMP

Combining the concepts of acoustic metamaterials and sandwich structures, a periodic CSMP is proposed as shown in Fig. 1(a). Simply supported boundary conditions are applied to both sides of the CSMP along the x -axis. The CSMP consists of a periodic corrugated-core sandwich plate (CSP) as shown in Fig. 1(b) and local resonators attached to the upper facesheet at discrete points. The CSP is composed of upper and lower facesheets and a corrugated core. Fig. 1(c) and (d) shows the unit cell of the CSMP and its schematic front view in the x - z plane, respectively. The lattice constant, the width and height of CSMP are denoted by a , b and H . The thickness of the facesheet and the corrugated core are t_1 and t_2 , respectively. The angle of the corrugated core is φ . Lumped mass attached to the upper facesheet, resonator mass and spring stiffness are denoted by the letters m_0 , m_r and k_r , respectively. E , ρ , and ν represent the Young's modulus, density and Poisson's ratio of CSMP, respectively.

2.2. Theoretical formulation

In this subsection, the theoretical wave dynamic model of the CSMP with simply supported boundary is established by using the spectral element method. As shown in Fig. 2, the unit cell of the CSMP is divided into nine plate elements and a local resonator. 'X' and 'X°' represent the node number and plate number respectively. Therefore, based on the equations of motion, the spectral stiffness matrices of the plate elements and local resonators in the local coordinate system are first derived. Then, the stiffness matrix of the plate elements is transformed from the local coordinate system to the global coordinate system and assembled to obtain the stiffness matrix of the CSP. Furthermore, the Kevorkian-Pascal projection method is used to map the projected coordinates of the local resonators into the global degree of freedom space, thereby achieving coupling between the CSP and the local resonators. Finally, by combining Bloch's theorem and boundary conditions, the calculation methods of band structure and transmission characteristics of the CSMP are proposed.

2.2.1. The spectral stiffness matrix for the plate element in local coordinate system

In this study, the in-plane and out-of-plane vibrations of the plate element are decoupled, and the spectral element method is employed to formulate the corresponding spectral stiffness matrices based on the plane-stress elastodynamic model and the classical Kirchhoff plate model. It is worth noting that the authors [25] have previously described in detail the construction process of the in-plane and out-of-plane spectral stiffness matrices of the plate element. Therefore, this paper only introduces the key derivation steps. For more information, please refer to the relevant literature.

Fig. 3 shows a Kirchhoff plate element with simply supported opposite edges whose length is L , width is b , and thickness is h .

The equations of the in-plane vibration of the Kirchhoff plate in the time domain are given by:

$$\frac{\partial^2 u}{\partial x^2} + \frac{(1-\nu)}{2} \frac{\partial^2 u}{\partial y^2} + \frac{(1+\nu)}{2} \frac{\partial^2 v}{\partial x \partial y} - \frac{(1-\nu^2)\rho}{E} \frac{\partial^2 u}{\partial t^2} = 0 \quad (1)$$

$$\frac{\partial^2 v}{\partial y^2} + \frac{(1-\nu)}{2} \frac{\partial^2 v}{\partial x^2} + \frac{(1+\nu)}{2} \frac{\partial^2 u}{\partial x \partial y} - \frac{(1-\nu^2)\rho}{E} \frac{\partial^2 v}{\partial t^2} = 0 \quad (2)$$

For the in-plane vibration, the SS1-type simply supported boundary conditions, $u = 0$ and $N_y = 0$, are imposed at $y = 0$ and $y = b$, where N_y denotes the in-plane normal force resultant in the y direction. Accordingly, u and v can be expressed as [40]:

$$u(x, y, t) = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m=1}^M U_{nm}(x; k_{ynm}, \omega_n) \sin(k_{ynm}y) e^{i\omega_n t} \quad (3)$$

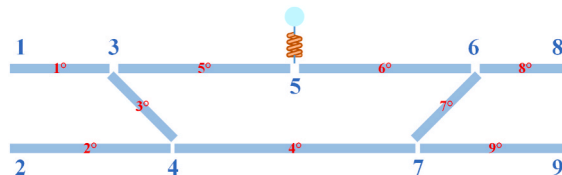


Fig. 2. The elemental division of the unit cell of the CSMP.

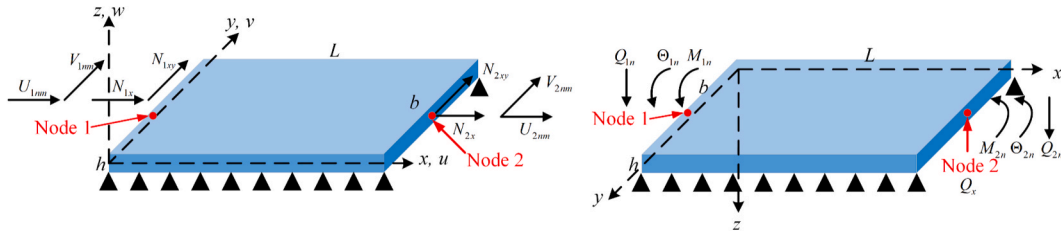


Fig. 3. Displacement and force of the Kirchhoff plate element with simply supported boundary.

$$v(x, y, t) = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m=1}^M V_{nm}(x; k_{ynm}, \omega_n) \cos(k_{ynm}y) e^{i\omega_n t} \quad (4)$$

where ω_n represents the n -th circular frequency, N is the number of frequency components, whereas M is the number of retained modes in the y direction, and U_{nm} , V_{nm} are the in-plane displacements in the frequency and k_{ynm} wavenumber domain.

$$k_{ynm} = \frac{m\pi}{b}, \quad (m = 1, 2, 3, \dots) \quad (5)$$

The specific form of the displacement of the plate element with simply supported boundary conditions can be expressed as:

$$\mathbf{d}_{in}(k_{ynm}, \omega_n) = \begin{Bmatrix} U_{1nm}(k_{ynm}, \omega_n) \\ V_{1nm}(k_{ynm}, \omega_n) \\ U_{2nm}(k_{ynm}, \omega_n) \\ V_{2nm}(k_{ynm}, \omega_n) \end{Bmatrix} = \begin{Bmatrix} U_{nm}(0; k_{ynm}, \omega_n) \\ V_{nm}(0; k_{ynm}, \omega_n) \\ U_{nm}(L; k_{ynm}, \omega_n) \\ V_{nm}(L; k_{ynm}, \omega_n) \end{Bmatrix} \quad (6)$$

where U_{1nm} , V_{1nm} , U_{2nm} and V_{2nm} denote the in-plane displacements of the nodes 1 and 2 in the frequency and k_{ynm} wavenumber domain, respectively, as shown in Fig. 3.

Equations (3) and (4) are substituted into Equations (1) and (2) and U_{nm} and V_{nm} are solved. Substituting U_{nm} and V_{nm} into Equation (6) leads to:

$$\mathbf{d}_{in}(k_y, \omega_n) = \Phi_{in}(k_y, \omega_n) \mathbf{A} \quad (7)$$

where $\mathbf{A}$ is the vector of four unknown wave-amplitude coefficients, and Φ_{in} is the corresponding coefficient matrix, whose expression is provided in the Appendix.

Based on the relation between in-plane force and displacement, the in-plane nodal force expression can be expressed as:

$$\mathbf{f}_{in}(k_{ynm}, \omega_n) = \begin{Bmatrix} N_{1xnm}(k_{ynm}, \omega_n) \\ N_{1xym}(k_{ynm}, \omega_n) \\ N_{2xnm}(k_{ynm}, \omega_n) \\ N_{2xym}(k_{ynm}, \omega_n) \end{Bmatrix} = \begin{Bmatrix} -N_{xnm}(0; k_{ynm}, \omega_n) \\ -N_{xym}(0; k_{ynm}, \omega_n) \\ N_{xnm}(L; k_{ynm}, \omega_n) \\ N_{xym}(L; k_{ynm}, \omega_n) \end{Bmatrix} \quad (8)$$

Transforming Equation (8) into matrix multiplication form, we can obtain:

$$\mathbf{f}_{in}(k_{ynm}, \omega_n) = \mathbf{G}_{in}(k_{ynm}, \omega_n) \mathbf{A} \quad (9)$$

where the expression of $\mathbf{G}_{in}$ is shown in the appendix.

Combining Equation (7) and Equation (9), we can obtain:

$$\mathbf{S}_{in}(k_{ynm}, \omega_n) \mathbf{d}_{in} = \mathbf{f}_{in} \quad (10)$$

where $\mathbf{S}_{in}$ is the spectral stiffness matrix of in-plane vibration, and its form can be expressed as:

$$\mathbf{S}_{in}(k_{ynm}, \omega_n) = \mathbf{G}_{in}(k_{ynm}, \omega_n) \Phi_{in}^{-1}(k_{ynm}, \omega_n) \quad (11)$$

The equation of the out-of-plane vibration of the Kirchhoff plate in the time domain is given by:

$$D \nabla^4 w(x, y, t) + \rho h \frac{\partial^2 w(x, y, t)}{\partial t^2} = 0 \quad (12)$$

where $w(x, y, t)$ is the displacement in the z -direction; D represents the bending stiffness of the plate, which is calculated as follows: $D = Eh^3/[12(1-\nu^2)]$.

For the out-of-plane vibration, the simply supported boundary conditions $w = 0$ and $M_y = 0$ are imposed at $y = 0$ and $y = b$, where M_y denotes the bending moment resultant in the y direction. Accordingly, the out-of-plane displacement can be expressed as:

$$w(x, y, t) = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m=1}^M W_{nm}(x; k_{ynm}, \omega_n) \sin(k_{ynm}y) e^{i\omega_n t} \quad (13)$$

Following the same derivation procedure as that for the in-plane vibration, the relationship between the nodal force and displacement vectors for the out-of-plane vibration can be obtained as:

$$\begin{aligned} S_{out}(k_{ynm}, \omega_n) \mathbf{d}_{out} &= \mathbf{f}_{out} \\ \mathbf{d}_{out} &= [W_{1nm}, \Theta_{1nm}, W_{2nm}, \Theta_{2nm}]^T \\ \mathbf{f}_{out} &= [Q_{1nm}, M_{1nm}, Q_{2nm}, M_{2nm}]^T \end{aligned} \quad (14)$$

where S_{out} is the spectral stiffness matrix of out-of-plane vibration. Its form can be written as:

$$S_{out}(k_{ynm}, \omega_n) = \mathbf{G}_{out}(k_{ynm}, \omega_n) \Phi_{out}^{-1}(k_{ynm}, \omega_n) \quad (15)$$

The in-plane and out-of-plane spectral stiffness matrices are combined to obtain the spectral stiffness matrix of the plate element.

$$S_b = \begin{bmatrix} S_{in11} & S_{in12} & 0 & 0 & S_{in13} & S_{in14} & 0 & 0 \\ S_{in21} & S_{in22} & 0 & 0 & S_{in23} & S_{in24} & 0 & 0 \\ 0 & 0 & S_{out11} & S_{out12} & 0 & 0 & S_{out13} & S_{out14} \\ 0 & 0 & S_{out21} & S_{out22} & 0 & 0 & S_{out23} & S_{out24} \\ S_{in31} & S_{in32} & 0 & 0 & S_{in33} & S_{in34} & 0 & 0 \\ S_{in41} & S_{in42} & 0 & 0 & S_{in43} & S_{in44} & 0 & 0 \\ 0 & 0 & S_{out31} & S_{out32} & 0 & 0 & S_{out33} & S_{out34} \\ 0 & 0 & S_{out41} & S_{out42} & 0 & 0 & S_{out43} & S_{out44} \end{bmatrix} \quad (16)$$

where S_{inij} and S_{outij} represent the (i, j) elements of the in-plane and out-plane spectral stiffness matrix, respectively.

The relationship between nodal force and nodal displacement of spectral plate element can be written as:

$$\begin{aligned} S_b(k_{ynm}, \omega_n) \mathbf{d}_b &= \mathbf{f}_b \\ \mathbf{d}_b &= [U_{1nm}, V_{1nm}, W_{1nm}, \Theta_{1nm}, U_{2nm}, V_{2nm}, W_{2nm}, \Theta_{2nm}]^T \\ \mathbf{f}_b &= [N_{1xnm}, N_{1xynm}, Q_{1nm}, M_{1nm}, N_{2xnm}, N_{2xynm}, Q_{2nm}, M_{2nm}]^T \end{aligned} \quad (17)$$

where $\mathbf{d}_b$ and $\mathbf{f}_b$ are the spectral nodal displacement and force vectors associated with the two free edges of the plate element, respectively.

Since the analytical field functions employed in the present SEM formulation depend on the prescribed boundary conditions, the present formulation is directly applicable to plate elements with two opposite simply supported edges. For other boundary conditions in the y direction, the general SEM framework can be extended when the corresponding analytical field functions are available by reformulating the y -direction displacement expansions and associated element matrices.

2.2.2. The dynamic stiffness matrix for the local resonator in local coordinate system

The equations of motion of the local resonator under simple harmonic oscillation conditions can be expressed as [41]:

$$\left(\begin{bmatrix} k_r & -k_r \\ -k_r & k_r \end{bmatrix} - \omega^2 \begin{bmatrix} m_0 & 0 \\ 0 & m_r \end{bmatrix} \right) \begin{Bmatrix} w_0 \\ w_r \end{Bmatrix} = \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix} \quad (18)$$

By solving Equation (18) and introducing the resonator damping by replacing k_r with the complex stiffness $k_r(1 + i\eta_r)$, the dynamic stiffness D_0 of the local resonator can be obtained:

$$D_0 = \frac{F_0}{w_0} = -\omega^2 m_0 + \frac{-\omega^2 m_r}{1 - \omega^2 / [\omega_r^2(1 + i\eta_r)]} \quad (19)$$

where η_r is the loss factor of the local resonator.

$$\omega_r = 2\pi f_r = (k_r/m_r)^{1/2} \quad (20)$$

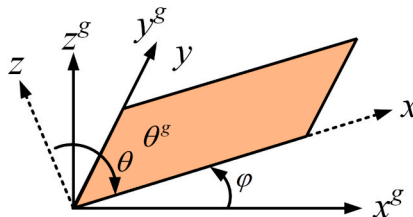


Fig. 4. Transformation from the local coordinate system to the global coordinate system.

2.2.3. Assembly and coupling of stiffness matrices in global coordinate system

In this section, in order to connect the local resonator and the plate element, the transformation matrix of the local resonator is constructed based on the Kevorkian-Pascal projection method, and the spectral stiffness matrix of the CSMP in the global coordinate system is assembled.

The plate elements of the CSMP are defined in different local coordinate systems, as illustrated in Fig. 2. Therefore, the spectral stiffness matrices of all plate elements need to be transformed into a unified global coordinate system. As shown in Fig. 4, the nodal displacement vectors in the local and global coordinate systems are related by $\mathbf{d}_b = \mathbf{T}_r \mathbf{d}_b^e$. Accordingly, the spectral stiffness matrix in the global coordinate system can be written as:

$$\mathbf{S}_b^g = \mathbf{T}_r^T \mathbf{S}_b \mathbf{T}_r \quad (21)$$

where $\mathbf{S}_b^g$ is the spectral stiffness matrix of plate element in the global coordinate system, and $\mathbf{T}_r$ is the coordinate transformation matrix.

$$\mathbf{T}_r = \begin{bmatrix} \cos\varphi & 0 & \sin\varphi & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\sin\varphi & 0 & \cos\varphi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\varphi & 0 & \sin\varphi & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sin\varphi & 0 & \cos\varphi & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (22)$$

The spectral stiffness matrix of the nine spectral plate elements of the CSP can be assembled as shown in Fig. 5.

The whole spectral stiffness matrix for the unit cell of the CSP can be expressed as:

$$\mathbf{S}_{s\text{-CSP}}(\mathbf{k}_y, \omega_n) \mathbf{d}_{s\text{-CSP}} = \mathbf{f}_{s\text{-CSP}} \quad (23)$$

where $\mathbf{d}_{s\text{-CSP}}$ and $\mathbf{f}_{s\text{-CSP}}$ correspond to the nodal displacement and force vector of the unit cell of the CSP, respectively.

Next, the plate element and the local resonator are assembled to obtain the whole spectral stiffness matrix for the unit cell of the CSMP. It can be seen from Fig. 2 that, based on whole spectral stiffness matrix for the unit cell of the CSP, the stiffness matrix of the CSMP can be obtained by assembling the stiffness matrix of the local resonator with the plate element 5° . Nevertheless, sandwich plates are two-dimensional continua, whose boundary displacement fields require continuous description, while local resonators are essentially discrete point elements (one-dimensional degrees of freedom), resulting in a fundamental dimensional disparity between the two. This dimensional disparity between the “continuous boundary displacement field” and the “discrete point element” makes it impossible to directly assemble the stiffness matrix of the local resonator and the plate element.

Therefore, in this paper, the projection method is utilized to construct the transformation matrix, mapping the sub-structure projection coordinates to the global degree-of-freedom space, thereby achieving coupling between the corrugated sandwich structure (two-dimensional continuum) and the local resonator (one-dimensional discrete point). The specific process is as follows:

First, according to Equation (19) and Dirac delta functions, forces acting on the plate boundary can be written as:

$$F_0 = D_0 w(y_0) \delta(y - y_0) \quad (24)$$

where the displacement of the plate at the local resonator attachment position y_0 is represented as $w(y_0)$.

Secondly, the equilibrium equation along the plate boundary is given as:

$$Q_{\alpha n} + F_0 = 0 \quad (25)$$

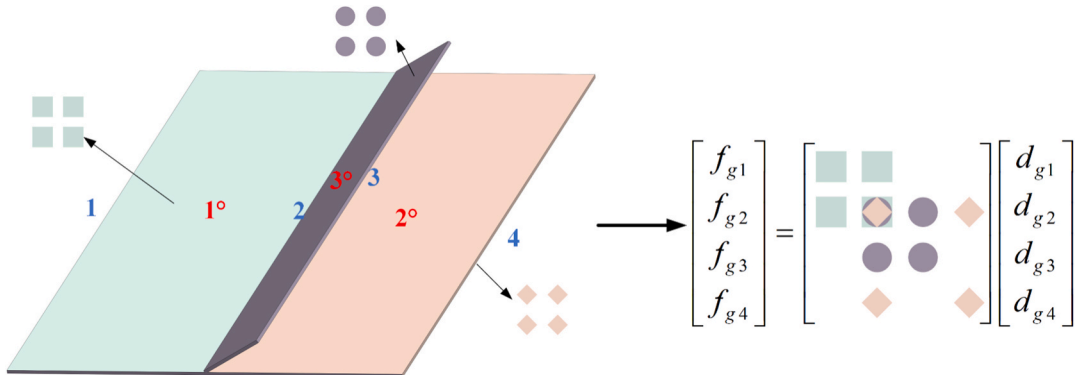


Fig. 5. Assembly of spectral stiffness matrix of plate element.

Forces F_0 can be projected onto a set of corresponding Kevorkian-Pascal projection functions[42]:

$$h_m(y) = \sin(k_{ym}y) \quad (26)$$

The projection of the force vector onto a set of orthogonal spatial modal functions can be obtained:

$$\tilde{F}_{0m} = \langle F_0, h_m(y) \rangle = \frac{2}{b} \int_0^b F_0 \delta\left(y - \frac{b}{2}\right) h_m(y) dy = \frac{2}{b} (-1)^m F_0 \quad (27)$$

Finally, substituting Equation (24) into Equation (27), the relations between the force and displacement are obtained:

$$\tilde{F}_{0m} = \frac{2}{b} \sin \frac{m\pi y_0}{b} D_0 w(y_0) = \frac{2}{b} \sin \frac{m\pi y_0}{b} D_0 \sum_{n=1}^M w_n(y_0) \sin \frac{n\pi y_0}{b} \quad (28)$$

$$\tilde{F}_{0mn} = \frac{2}{b} \sin \frac{m\pi y_0}{b} D_0 w_n(y_0) \sin \frac{n\pi y_0}{b} \quad (29)$$

$$D_{mn} = \frac{\tilde{F}_{0mn}}{w_n(y_0)} = \frac{2}{b} \sin \frac{m\pi y_0}{b} D_0 \sin \frac{n\pi y_0}{b} \quad (30)$$

It is noted that each node of the plate element has three degrees of freedom, but the local resonator is only coupled with the lateral displacement degrees in it. Therefore, it is necessary to transform the plate element spectral stiffness matrix to facilitate the coupled procedure. Before and after the transformation, the displacements and forces vectors are arranged as follows:

$$\begin{aligned} f_{bef} &= [N_{1xn1} \quad N_{1xyn1} \quad Q_{1n1} \quad M_{1n1} \quad N_{2xn1} \quad N_{2xyn1} \quad Q_{2n1} \quad M_{2n1} \\ &\quad \dots \quad N_{1xnM} \quad N_{1xynM} \quad Q_{1nM} \quad M_{1nM} \quad N_{2xnM} \quad N_{2xynM} \quad Q_{2nM} \quad M_{2nM}] \\ d_{bef} &= [U_{1n1} \quad V_{1n1} \quad W_{1n1} \quad \Theta_{1n1} \quad U_{2n1} \quad V_{2n1} \quad W_{2n1} \quad \Theta_{2n1} \\ &\quad \dots \quad U_{1nM} \quad V_{1nM} \quad W_{1nM} \quad \Theta_{1nM} \quad U_{2nM} \quad V_{2nM} \quad W_{2nM} \quad \Theta_{2nM}] \\ f_{aft} &= [N_{1xn1} \quad \dots \quad N_{1xnM} \quad N_{1xyn1} \quad \dots \quad N_{1xynM} \quad Q_{1n1} \quad \dots \quad Q_{1nM} \quad M_{1n1} \quad \dots \quad M_{1nM} \\ &\quad N_{2xn1} \quad \dots \quad N_{2xnM} \quad N_{2xyn1} \quad \dots \quad N_{2xynM} \quad Q_{2n1} \quad \dots \quad Q_{2nM} \quad M_{2n1} \quad \dots \quad M_{2nM}] \\ d_{aft} &= [U_{1n1} \quad \dots \quad U_{1nM} \quad V_{1n1} \quad \dots \quad V_{1nM} \quad W_{1n1} \quad \dots \quad W_{1nM} \quad \Theta_{1n1} \quad \dots \quad \Theta_{1nM} \\ &\quad U_{2n1} \quad \dots \quad U_{2nM} \quad V_{2n1} \quad \dots \quad V_{2nM} \quad W_{2n1} \quad \dots \quad W_{2nM} \quad M_{2n1} \quad \dots \quad M_{2nM}] \end{aligned} \quad (31)$$

By assembling the dynamic stiffness matrix of the local resonator with the spectral stiffness matrix of the unit cell of the CSP after the transformation, the total stiffness matrix of the unit cell of the CSMP can be obtained.

2.2.4. The calculation methods of band structure and transmission characteristics

The calculation methods of the band structure and transmission characteristics are proposed in this section. The left node, the inner node and the right node in the unit cell are distinguished as shown in Fig. 2, and the whole spectral stiffness matrix of the unit cell of the CSMP can be written as follows:

$$\begin{bmatrix} \bar{\mathbf{S}}_{LL} & \bar{\mathbf{S}}_{LM} & \bar{\mathbf{S}}_{LR} \\ \bar{\mathbf{S}}_{ML} & \bar{\mathbf{S}}_{MM} & \bar{\mathbf{S}}_{MR} \\ \bar{\mathbf{S}}_{RL} & \bar{\mathbf{S}}_{RM} & \bar{\mathbf{S}}_{RR} \end{bmatrix} \begin{Bmatrix} \bar{\mathbf{d}}_L \\ \bar{\mathbf{d}}_M \\ \bar{\mathbf{d}}_R \end{Bmatrix} = \begin{Bmatrix} \bar{\mathbf{f}}_L \\ \bar{\mathbf{f}}_M \\ \bar{\mathbf{f}}_R \end{Bmatrix} \quad (32)$$

The internal nodes of the unit cell are not subject to external forces, so Equation (32) can be simplified as follows [43]:

$$\begin{bmatrix} \hat{\mathbf{S}}_{LL} & \hat{\mathbf{S}}_{LR} \\ \hat{\mathbf{S}}_{RL} & \hat{\mathbf{S}}_{RR} \end{bmatrix} \begin{Bmatrix} \bar{\mathbf{u}}_L \\ \bar{\mathbf{u}}_R \end{Bmatrix} = \begin{Bmatrix} \bar{\mathbf{f}}_L \\ \bar{\mathbf{f}}_R \end{Bmatrix} \quad (33)$$

where

$$\begin{aligned} \hat{\mathbf{S}}_{LL} &= \bar{\mathbf{S}}_{LL} - \bar{\mathbf{S}}_{LM} \bar{\mathbf{S}}_{MM}^{-1} \bar{\mathbf{S}}_{ML}, \quad \hat{\mathbf{S}}_{LR} = -\bar{\mathbf{S}}_{LM} \bar{\mathbf{S}}_{MM}^{-1} \bar{\mathbf{S}}_{MR} \\ \hat{\mathbf{S}}_{RL} &= -\bar{\mathbf{S}}_{RM} \bar{\mathbf{S}}_{MM}^{-1} \bar{\mathbf{S}}_{ML}, \quad \hat{\mathbf{S}}_{RR} = \bar{\mathbf{S}}_{RR} - \bar{\mathbf{S}}_{RM} \bar{\mathbf{S}}_{MM}^{-1} \bar{\mathbf{S}}_{MR} \end{aligned} \quad (34)$$

According to Bloch theorem, the boundary displacement and force of the unit cell are expressed as follows:

$$\bar{\mathbf{u}}_R = e^{-iqa} \bar{\mathbf{u}}_L, \quad \bar{\mathbf{f}}_R = e^{-iqa} \bar{\mathbf{f}}_L \quad (35)$$

where q stands for Bloch wave vector.

The following equation can be obtained by substituting Equation (35) into Equation (33).

$$[\hat{\mathbf{S}}_{RL} + (\hat{\mathbf{S}}_{LL} + \hat{\mathbf{S}}_{RR})e^{-iqa} + \hat{\mathbf{S}}_{LR}e^{-2iqa}] \bar{\mathbf{u}}_L = \mathbf{0} \quad (36)$$

To solve Equation (36), the Bloch factor $\lambda = e^{-iqa}$ is introduced, and Equation (36) is rewritten as the following quadratic polynomial eigenvalue problem:

$$(\mathbf{A}_0 + \lambda \mathbf{A}_1 + \lambda^2 \mathbf{A}_2) \bar{\mathbf{u}}_L = \mathbf{0} \quad (37)$$

$$\mathbf{A}_0 = \hat{\mathbf{S}}_{RL}, \mathbf{A}_1 = \hat{\mathbf{S}}_{LL} + \hat{\mathbf{S}}_{RR}, \mathbf{A}_2 = \hat{\mathbf{S}}_{LR}$$

For each prescribed circular frequency, the coefficient matrices in Equation (37) are stored in sparse form, and the quadratic polynomial eigenvalue problem is solved using the polynomial eigenvalue solver `polyeig` to obtain the Bloch factors λ . The corresponding Bloch wave vectors are then recovered from $q = i \ln(\lambda)/a$. This approach avoids the explicit evaluation of the high-dimensional determinant and the associated scalar root-searching procedure. By repeating this procedure over the prescribed frequency range, the band structure of the CSMP with two opposite edges simply supported is obtained.

The spectral stiffness matrix of the CSMP with simply supported boundary is assembled according to the period number of the structure.

$$\mathbf{S}_{\text{CSMP}} \mathbf{d}_{\text{CSMP}} = \mathbf{f}_{\text{CSMP}} \quad (38)$$

Next, the application of the force and the extraction of the displacement are considered. The vertical concentrated force in the z direction can be expressed as:

$$F_c = \frac{2F_a}{b} \sin(k_{ym} y_s) \quad (39)$$

where F_a represents the magnitude of the force, F_c is the corresponding modal force coefficient and y_s is the position of the excitation point on the free edge in the y -direction.

The vertical displacements at the excitation and response points can be reconstructed as:

$$W_{c-\text{in}} = \sum_{m=1}^M W_{m-\text{in}} \sin(k_{ym} y_s), \quad W_{c-\text{out}} = \sum_{m=1}^M W_{m-\text{out}} \sin(k_{ym} y_r) \quad (40)$$

where $W_{c-\text{in}}$ and $W_{c-\text{out}}$ denote the vertical displacements at the excitation point and the response points, respectively, and y_r denotes the y -coordinate of the response point on the free edge. The vibration transmissibility can then be calculated as follows:

$$\text{Trans} = 20 \log_{10} \frac{|W_{c-\text{out}}|}{|W_{c-\text{in}}|} \quad (41)$$

3. Results and discussion

3.1. Low-frequency bandgap and vibration isolation characteristics

In this section, the band structure is calculated using the SEM to investigate the elastic wave propagation and low-frequency bandgap characteristics of the proposed CSMP with simply supported boundary conditions shown in Fig. 1. Each unit cell contains one local resonator attached to the geometric center of the connecting plate. The resonator is modeled as an equivalent mass-spring-mass system characterized by m_0 , k_r , and m_r . These parameters are determined according to the target isolation frequency range and practical fabrication constraints. The geometric and resonator parameters and the material parameters of the CSMP are listed in Tables 1 and 2.

To determine the number of modes retained in the y direction, a convergence study is conducted using $M = 1, 5, 9, 13, 17, 21$, and 35. The lower and upper boundaries of the two bandgaps obtained for each value of M are summarized in Table 3. As M increases, the bandgap boundaries gradually stabilize. The four bandgap boundaries remain unchanged when M is further increased from 17 to 21 and 35, indicating that convergence is achieved at $M = 17$. Accordingly, $M = 17$ is adopted in the subsequent band structure calculations.

In the band structure shown in Fig. 6(a), the left vertical axis represents the frequency, whereas the right vertical axis represents the normalized frequency. The normalized frequency is defined as fa/c_t , where f denotes the wave frequency and c_t is the shear wave velocity of mild steel which is equal to 3140 m/s. The horizontal axis represents the reduced wave vector along the boundary of the irreducible Brillouin zone. As can be seen from Fig. 6(a), in the range of 0–600 Hz, the CSMP exhibits two flexural wave bandgaps (pink and blue shaded area) ranging from 105 Hz to 122 Hz and from 352 Hz to 528 Hz, respectively. In attempt to verify the band structure of the CSMP and study its vibration isolation characteristics, the vibration transmission characteristics of the 4×1 unit cells periodic CSMP are calculated using the SEM. The excitation and response points are located on the lower face sheet at the left and right ends of

Table 1
Geometric and resonator parameters of the CSMP.

Parameter	a/mm	b/mm	H/mm	t_1/mm	t_2/mm	φ	m_0/kg	m_r/kg	$k_r/(\text{N/m})$
Dimension	300	1000	30	1.37	1.37	$\pi/4$	0.026	0.176	1.6×10^5

Table 2
Material parameters of the CSMP.

Parameter	E/Pa	$\rho/(\text{kg/m}^3)$	ν
Dimension	2.1×10^{11}	7850	0.3

Table 3
Convergence of the SEM bandgap boundaries with respect to the number M of modes retained in the y direction.

M	Lower boundary of Bandgap 1 (Hz)	Upper boundary of Bandgap 1 (Hz)	Lower boundary of Bandgap 2 (Hz)	Upper boundary of Bandgap 2 (Hz)
1	125	143	349	528
5	112	128	349	528
9	107	124	353	529
13	106	123	352	528
17	105	122	352	528
21	105	122	352	528
35	105	122	352	528

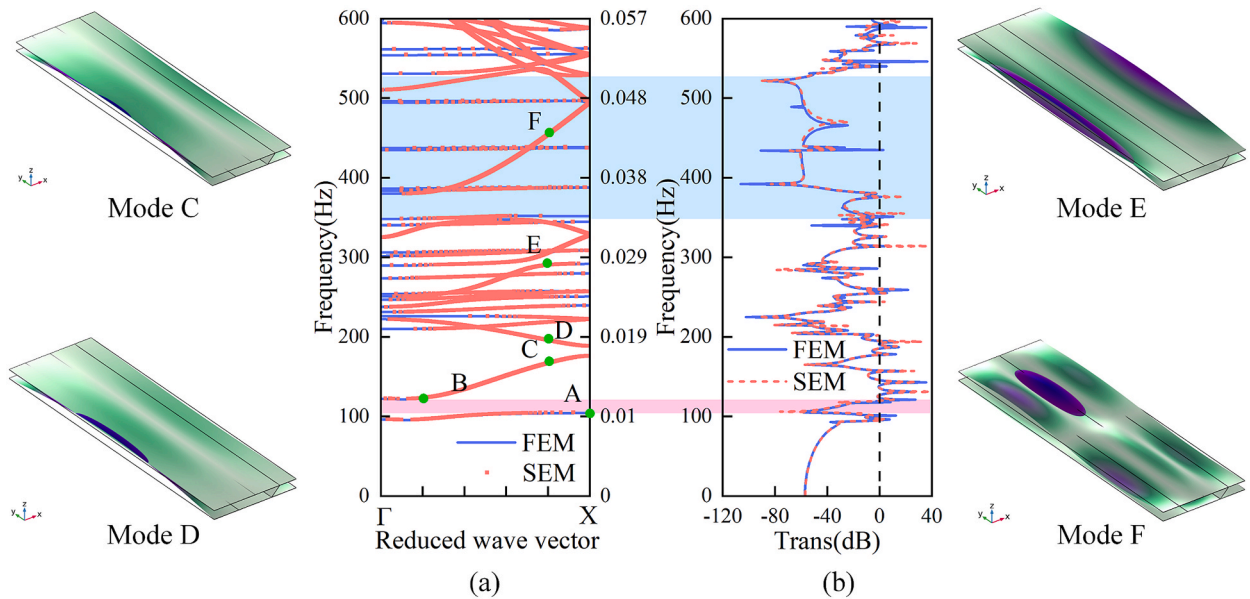


Fig. 6. (a) Band structure and (b) Transmission characteristics of the CSMP. Results from FEM (blue line) overlaps with SEM (red dots) and zones in pink and blue denote the two bandgaps. Eigenmodes at specific frequencies (C, D, E, F) are shown. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the CSMP, respectively, with $y_s = y_r = b/2 = 500$ mm. As shown in Fig. 6(b), two obvious vibration attenuation regions are observed in the frequency range of 0–600 Hz, and their positions and widths are consistent with the bandgaps in the band structure.

In order to compare and verify the advantages of the CSMP in low-frequency bandgap, the bandgap characteristics of the CSP are studied. The band structure and vibration transmission characteristics of the CSP are calculated by using the SEM, as shown in Fig. 7. It can be observed that in the range of 0–600 Hz, the CSP exhibits only a flexural wave bandgap (blue shaded area) ranging from 352 Hz to 528 Hz, which also corresponds to a vibration attenuation region in the transmission characteristics. So, it can be inferred that the first bandgap is attributed to the local resonators, while the second bandgap is the Bragg bandgap formed by the destructive interference of the forward wave and the reverse wave [44,45].

The band structures and transmission characteristics of the CSMP and CSP are calculated by using the FEM via COMSOL Multiphysics to verify the correctness of the established theoretical wave dynamic model. In the finite element calculation model, the corrugated sandwich structure is modeled using shell elements, and the local resonator is modeled using the Lumped Mechanical System. The shell elements are meshed using free quadrilateral meshes. It can be seen from Figs. 6 and 7 that the SEM and FEM calculation results show excellent agreement, confirming the accuracy and reliability of the theoretical wave dynamic model. In addition, it can be found that there is a band in the second bandgap of the CSMP. In order to explain this phenomenon, the displacement fields of eigenmodes at points C, D, E, and F marked in Fig. 6(a) are conducted. It can be clearly observed that the flexural waves in Modes C–E primarily propagate along the x -direction, while those in Mode F primarily propagate along the y -direction. This

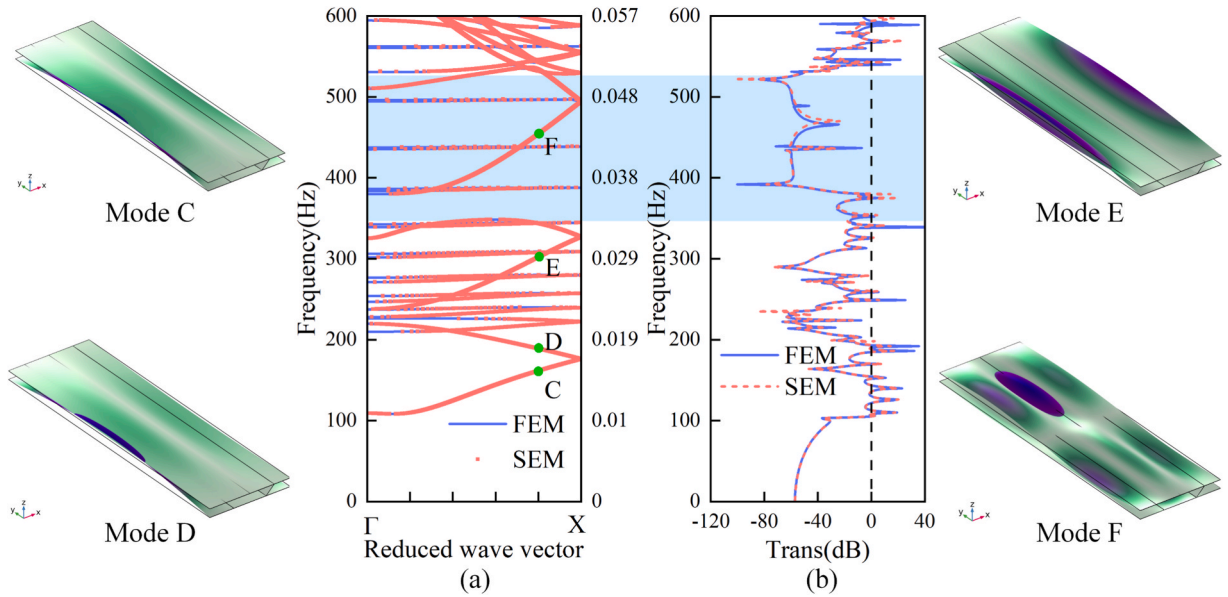


Fig. 7. (a) Band structure and (b) Transmission characteristics of the CSP. Results from FEM (blue line) overlaps with SEM (red dots) and zone in pink denotes the bandgap. Eigenmodes at specific frequencies (C, D, E, F) are shown. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

leads to the formation of the vibration attenuation region in the transmission characteristics, which is considered a flexural wave bandgap. Similarly, the displacement fields of eigenmodes at points C, D, E, and F marked in Fig. 7(a) are conducted. Modes C, D, E, and F of the CSP are basically consistent with the eigenmodes of CSP of the CSMP, indicating that the local resonators have little effect on the high-frequency band structure.

To demonstrate the computational efficiency of the SEM, a comparison is conducted between SEM and the FEM in terms of degrees of freedom (DOF) and computational time for calculating the band structure of the CSMP, as shown in Table 4. In the SEM calculation, $M = 17$ modes are retained in the y direction, whereas the FEM model is discretized using a mesh size of $10 \text{ mm} \times 10 \text{ mm}$. Both methods are executed on the same computer equipped with an NVIDIA GeForce RTX 4070 Ti GPU and an Intel Core i7-13700 K CPU. The results show that the SEM requires substantially fewer degrees of freedom and significantly less computational time than the FEM, underscoring its superiority in efficient computation.

In addition, to intuitively illustrate the vibration isolation characteristics of the CSP and CSMP, the frequency responses inside and outside the flexural wave bandgap are computed using the FEM. Fig. 8(a) and (b) shows the vibration response outside the bandgap (at 106 Hz and 326 Hz), while Fig. 8(c) shows the vibration response inside the bandgap (at 392 Hz). The flexural wave outside the bandgap is smoothly transmitted from the vibration excitation point to the other side of the CSP. In contrast, the flexural wave inside the bandgap is suppressed, and the other side of the CSP has almost no displacement. This phenomenon occurs because destructive interference between the forward and reflection waves occurs when the elastic wave propagates in the periodic CSP. For CSMP, similarly, the vibration responses outside and inside the bandgap are output separately, as shown in Fig. 8(d)-(f). The wave propagation inside and outside the bandgap of the CSMP exhibits the same characteristics as that of the CSP. Furthermore, only the first local resonator has a large displacement inside the first bandgap. The flexural wave propagates only up to the first local resonator, and the subsequent propagation is suppressed. Compared to the CSP, the CSMP can suppress the propagation of the flexural wave well in the first bandgap (low-frequency) range. It not only explains the local resonance mechanism but also demonstrates the good vibration isolation effect of the CSMP.

To further illustrate the attenuation of flexural waves by the local resonators, the displacement responses at the excitation point and the four local resonators of the finite 4×1 CSMP are calculated using the theoretical model. Fig. 9(a) shows the excitation point P_E and the response positions $P_{LR,j}$ ($j = 1, 2, 3, 4$) on the four local resonators. Fig. 9(b)-(d) compare the displacement responses at P_E and $P_{LR,1}$ with those at $P_{LR,2}$, $P_{LR,3}$, and $P_{LR,4}$, respectively. Across the frequency range of 0–600 Hz, all four local resonators generally exhibit substantially lower displacement amplitudes than the excitation point. Within the two shaded bandgap ranges, the responses at $P_{LR,2}$,

Table 4

Comparison of the SEM and FEM in DOFs and computational time for CSMP band structure.

Method	DOFs	Time/s
SEM	612	34.0
FEM	173667	849

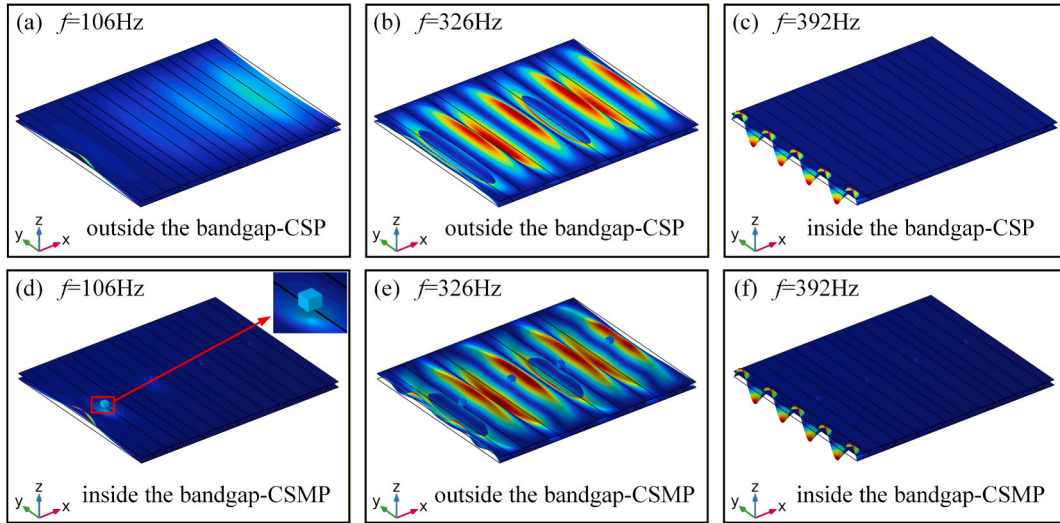


Fig. 8. Comparison of vibration frequency responses inside and outside the bandgap. Flexural waves outside the bandgap can propagate smoothly, but flexural waves inside the bandgap will be suppressed.

$P_{LR,3}$, and $P_{LR,4}$ are further attenuated relative to that at $P_{LR,1}$. At the representative frequency shown in Fig. 8(d), the motion of the first local resonator is also larger than those of the second, third, and fourth local resonators. This observation is consistent with the vibration frequency responses plots, further illustrating the critical role of local resonators.

3.2. Low-frequency bandgap mechanism

In order to study the generation mechanism of the low-frequency bandgap of the CSMP, the eigenmodes corresponding to the starting and cutoff frequencies of the bandgap are output using the FEM. Modes A and B correspond to points A and B in the band structure diagram of the Fig. 6(a), respectively. In Mode A, the vibration along the z -direction is primarily concentrated in the local resonator and the connecting plate, while the vibration of the rest of CSMP is almost completely suppressed, as shown in Fig. 10(a). This phenomenon can be explained as the mutual coupling of the vertical vibration of the local resonator and the flexural vibration of the connecting plate in the z -direction, which is the key to the generation of the low-frequency band gap of the CSMP. This mode can first be approximated by the simplified model shown in Fig. 10(b). From a physical perspective, energy is primarily concentrated in the local resonator and the connecting plate, which can be equivalent to a spring-mass system respectively, ultimately forming the two degrees of freedom spring-mass system illustrated in Fig. 10(c). Here, K_1 , M_1 represent the equivalent stiffness and mass of the connecting plate, respectively. Therefore, the starting frequency of the low-frequency bandgap of the CSMP is influenced not only by the local resonator but also by the equivalent stiffness and mass of the connecting plate.

In mode B, the entire CSMP exhibits vibration along the z -direction, as shown in Fig. 10(d). This mode can be explained as the coupling between a local resonant mode and a flexural wave mode propagating along the x -direction in CSP. Similarly, Mode B can be approximated by the simplified model in Fig. 10(e), and then the local resonator and CSP can be equivalent to a spring-mass system respectively, also resulting in the two degrees of freedom spring-mass system shown in Fig. 10(f). Here, K_2 and M_2 represent the equivalent stiffness and mass of CSP, respectively. Therefore, the cutoff frequency of the low-frequency bandgap of the CSMP is primarily governed by the equivalent stiffness and mass of the local resonator and CSP.

Obviously, the explanation of the low-frequency bandgap formation mechanism of the CSMP in this work differs significantly from that in the Refs. [37–39]. In previous theoretical studies, the host sandwich structure is usually equivalent to a homogeneous beam or plate, neglecting the coupling effects between the constituent plates as well as between the connector plate and the local resonators. This incomplete understanding of the low-frequency bandgap mechanism has hindered the design and performance optimization of sandwich metastructures. Therefore, the investigation of the low-frequency bandgap formation mechanism in this work provides valuable guidance for the optimized design of sandwich metastructures.

Furthermore, the normalized comparison of comprehensive performance between the proposed CSMP based on the low-frequency bandgap mechanism and the typical sandwich metastructures in Refs. [34–38,46–48] is shown in Fig. 11. The three axes represent the normalized bandgap starting frequency (f_s), the mass ratio of the local resonator to CSP (μ), and relative bandwidth (normalized bandgap width to normalized bandgap center frequency ratio, R_b), respectively. It should be noted that in the radar chart of Fig. 11, to uniformly represent performance levels, the coordinates of f_s and μ are taken as their reciprocals, and the reciprocal of f_s is taken as its common logarithm. It can be observed that the proposed CSMP exhibits not only excellent normalized bandgap starting frequencies and relative bandwidths, but also a significant mass advantage over typical sandwich metastructures, with mass ratios of only 1.77%. Consequently, the proposed CSMP can be utilized to realize simultaneous lightweight and high-strength performance and low-frequency vibration isolation ability.

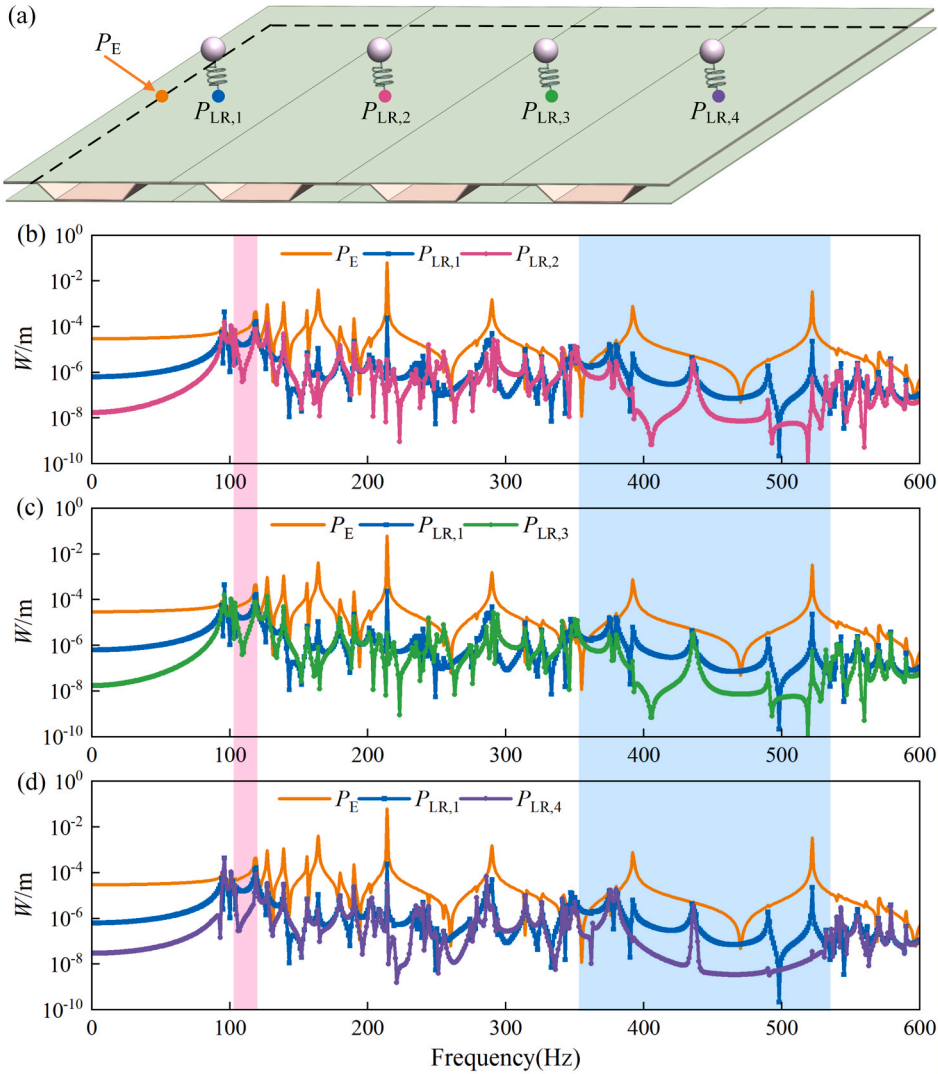


Fig. 9. Displacement responses at the excitation point and local-resonator positions of the finite 4×1 CSMP: (a) locations of the excitation point P_E and the response positions $P_{LR,j}$ ($j = 1, 2, 3, 4$); comparisons of the responses at (b) P_E , $P_{LR,1}$, and $P_{LR,2}$; (c) P_E , $P_{LR,1}$, and $P_{LR,3}$; and (d) P_E , $P_{LR,1}$, and $P_{LR,4}$.

3.3. Experimental validations

In an attempt to verify the reliability of the theoretical wave dynamic model, the vibration transmission characteristics of the CSP and CSMP are measured as shown in Fig. 12. The experimental measurement system and measurement principle are shown in Fig. 12 (a) and (b), respectively. The experimental system consists of an M + P test system, a vibration exciter, a power amplifier, three accelerometers, and a computer for control and analysis. The vibration exciter is mounted on one side of the specimen to generate excitations of varying amplitudes and frequencies. The power amplifier amplifies the excitation generated by the vibration exciter. The accelerometers measure the acceleration signals, which are processed by the M + P test system. All three single-axis accelerometers are oriented in the z -direction to measure the transverse acceleration responses. Finally, the equation to calculate the vibration transmission characteristic of the specimen can be written as:

$$\text{Trans} = 20 \log_{10} \frac{|a_{c-\text{out}}|}{|a_{c-\text{in}}|} \quad (42)$$

where $a_{c-\text{in}}$ and $a_{c-\text{out}}$ denote the vertical displacements at the excitation point and extraction point of the specimen, respectively.

To simulate the simply supported boundary conditions in the theoretical model and numerical simulation, the experimental model shown in Fig. 12(c) is designed. The two edges at $y = 0$ and $y = b$ are inserted into C-shaped steel channels A, which extend continuously along the supported edges and only enclose the upper and lower face sheets without adhesive bonding or additional

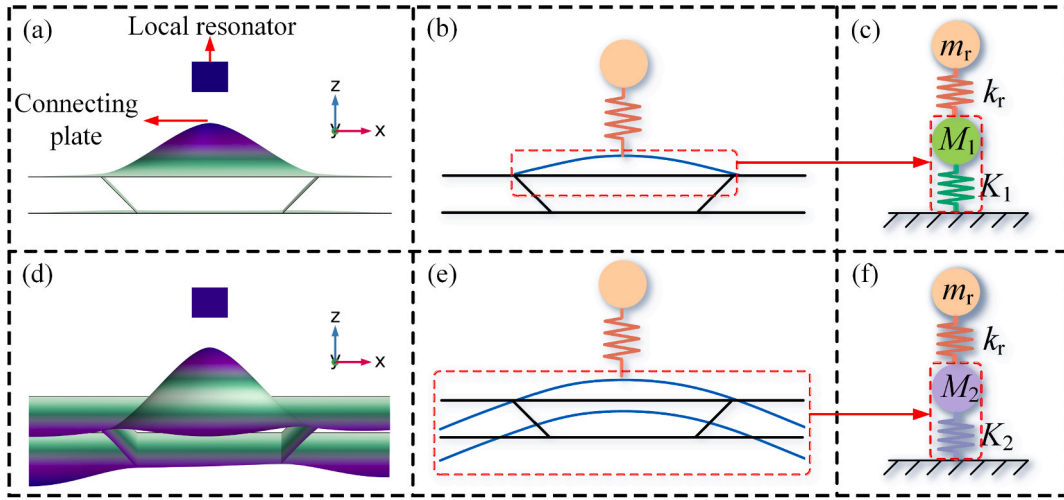


Fig. 10. Schematic of modal simplification and equivalent two degrees of freedom modeling for low-frequency bandgap edges.

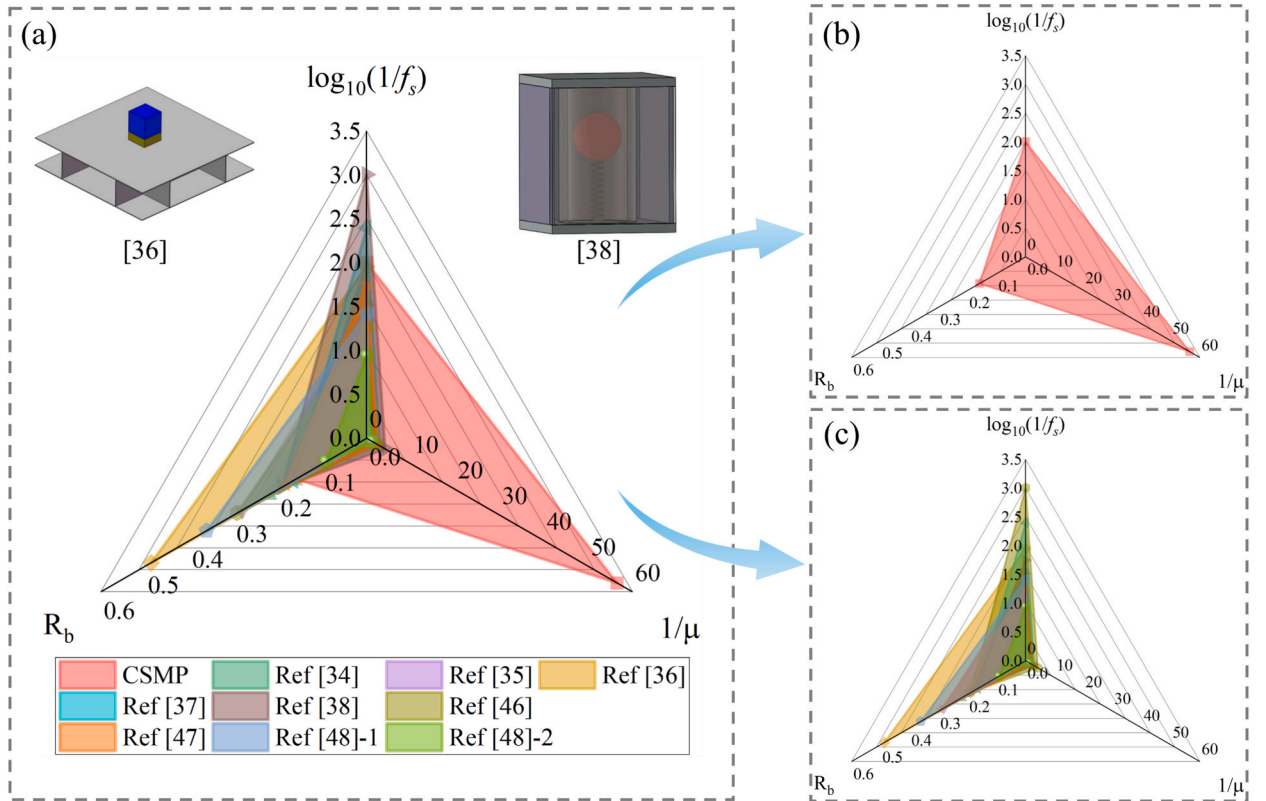


Fig. 11. (a) Comparisons between the proposed CSMP and typical sandwich metastructures; (b) Performance of the proposed CSMP; (c) Performance of typical sandwich metastructures. The proposed CSMP based on the low-frequency bandgap mechanism exhibits a significant mass advantage over typical sandwich metastructures, with mass ratios of only 1.77%.

clamping. A semi-cylindrical member B is welded to the bottom of each channel and seated in a continuous grooved support C fixed to the rigid test bench. The contact between B and C restrains the transverse displacement of the supported edges, whereas their relative rotation allows rotation about the support lines. The other two edges remain free. The self-weight of the mild-steel specimen maintains stable contact between B and C during the test.

As shown in Fig. 12(d)-(f), three specimens, namely CSP, CSMP-1, and CSMP-2, are fabricated and tested. The only difference between CSMP-1 and CSMP-2 lies in their local resonators, whose parameters are listed in Table 5. Each spring is installed vertically,

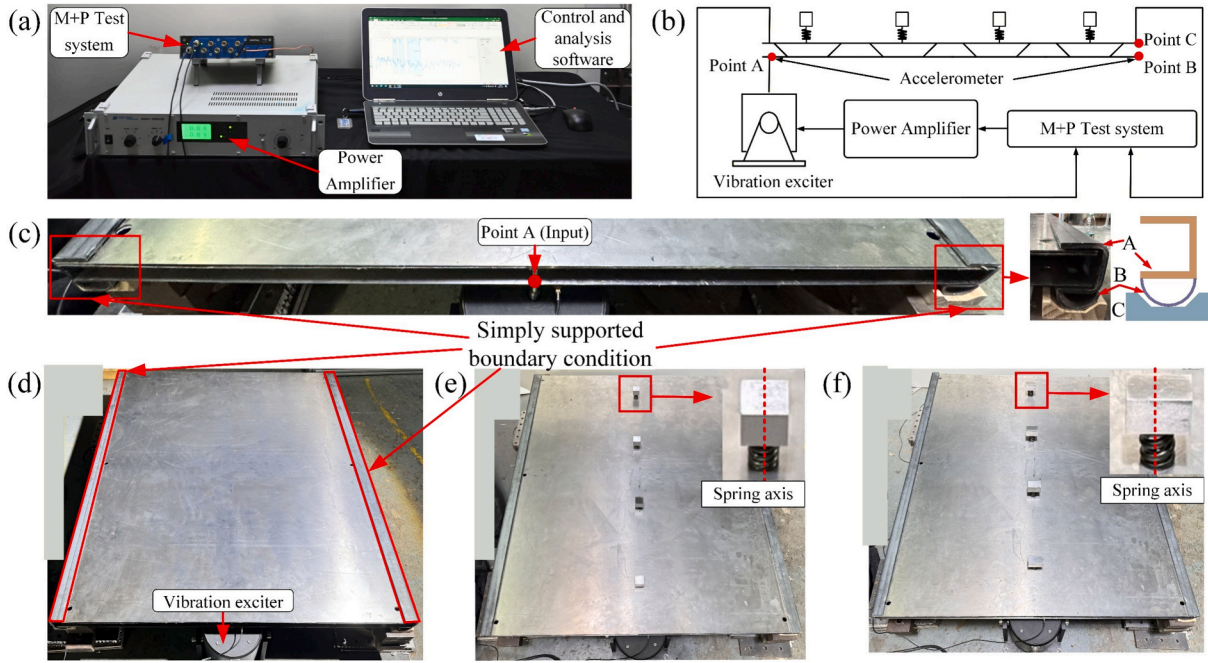


Fig. 12. (a) Experimental measurement system; (b) Measurement principle; (c) Schematic diagram of simply supported boundary; The specimens of (d) the CSP, (e) the CSMP-1 and (f) the CSMP-2.

Table 5

The local resonator parameters of the two CSMP models.

Model	m_0/kg	m_r/kg	$k_r/(\text{N/m})$	μ
CSMP-1	0.026	0.176	1.6×10^5	1.77%
CSMP-2	0.0808	0.35	1.017×10^6	3.52%

with its lower and upper ends adhesively bonded to the upper face sheet and the center of the resonator mass, respectively, and the resonator mass is coaxially aligned with the spring. The remaining material and geometric parameters of the specimen are consistent with those in the theoretical model calculations.

The excitation point is located at the midpoint of the lower face sheet (Point A) on the left side of the specimen. The input acceleration is measured at Point A, while the output accelerations are measured at the midpoints of the lower face sheet (Point B) and the upper face sheet (Point C) on the right side. The vibration transmission characteristics of the CSP, CSMP-1, and CSMP-2 are obtained using the SEM and FEM and through experimental measurements, as shown in Figs. 13–15. Experiment-Point B indicates that the accelerations at Points A and B obtained from the experimental tests are substituted into Equation (42) to calculate the transmission characteristics of the structure, whereas Experiment-Point C is calculated using the accelerations measured at Points A and C. The experimental attenuation regions observed at Points B and C generally correspond to the bandgap frequency ranges predicted by the SEM and FEM. The CSP exhibits one bandgap, whereas CSMP-1 and CSMP-2 each exhibit two bandgaps. This correspondence supports the capability of the theoretical wave dynamics model to predict the bandgap frequency ranges.

In addition, the attenuation of the experimental vibration transmission characteristics is lower than the theoretical predicted attenuation, which can be attributed to material damping. To account for this effect, material damping is incorporated into the calculations of the finite structure transmission characteristics through the complex Young's modulus $E^* = E(1 + i\eta)$, where η denotes the material loss factor. The vibration transmission characteristics of the three structures with damping are respectively presented in Fig. 13(a), Fig. 14(a), and Fig. 15(a). After incorporating material damping, the vibration attenuation calculated by the theoretical model decreases significantly, bringing it closer to the experimental results.

To further illustrate the acceleration response of the structure both inside and outside the bandgap, sinusoidal fixed-frequency excitation is applied at the excitation point, and the time-domain acceleration response of the two ends of the specimen is recorded, respectively. Fig. 16(a), (c), and (e) shows the acceleration responses inside the bandgap for the CSP, CSMP-1, and CSMP-2 and Fig. 16(b), (d), and (f) shows the acceleration responses outside the bandgap, respectively. It can be observed that the output acceleration inside the bandgap exhibits a substantial attenuation compared to the input acceleration, while the output acceleration outside the bandgap has almost no attenuation compared to the input acceleration, which further confirms that the flexural wave propagation inside the bandgap is suppressed. Consequently, it can be concluded that the prediction results of the non-equivalent high-precision

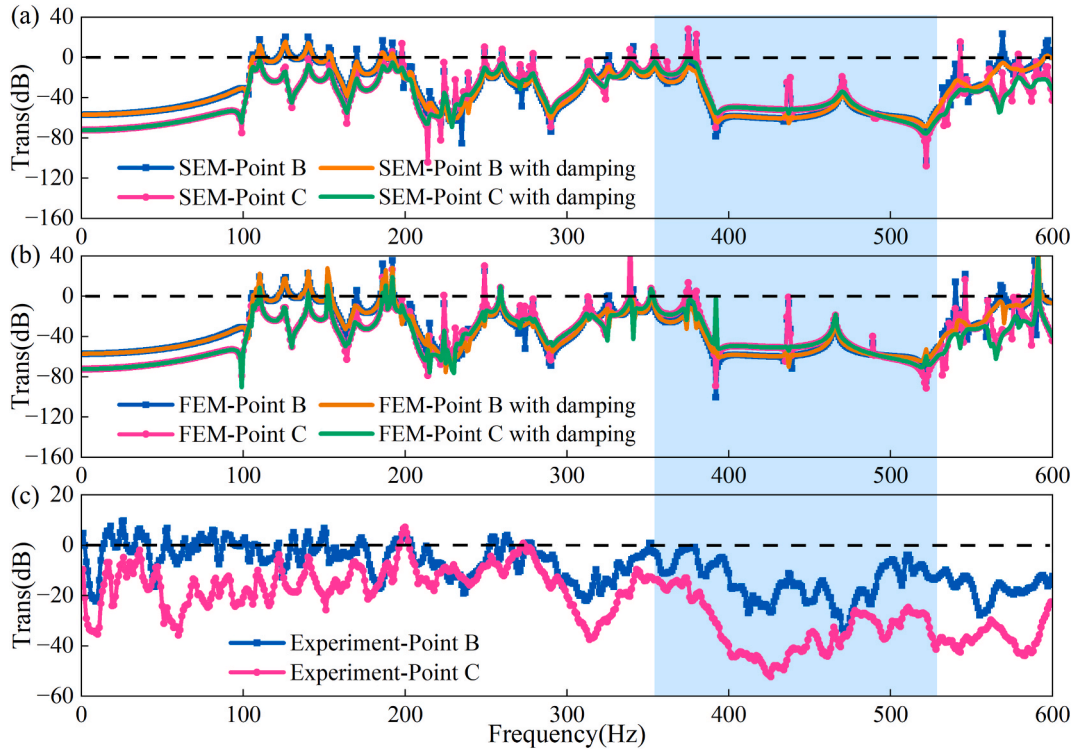


Fig. 13. Comparison of transmission characteristics of the CSP: (a) SEM; (b) FEM; (c) Experiment.

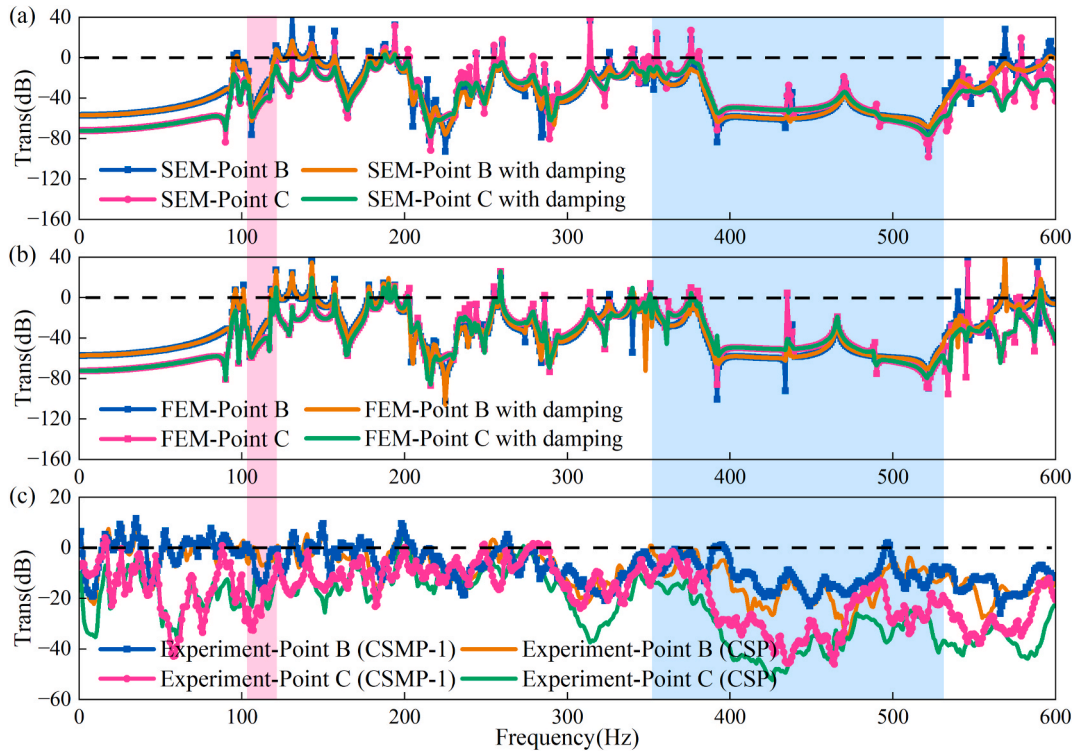


Fig. 14. Comparison of transmission characteristics of CSMP-1: (a) SEM; (b) FEM; (c) Experimental comparison between CSP and CSMP-1.

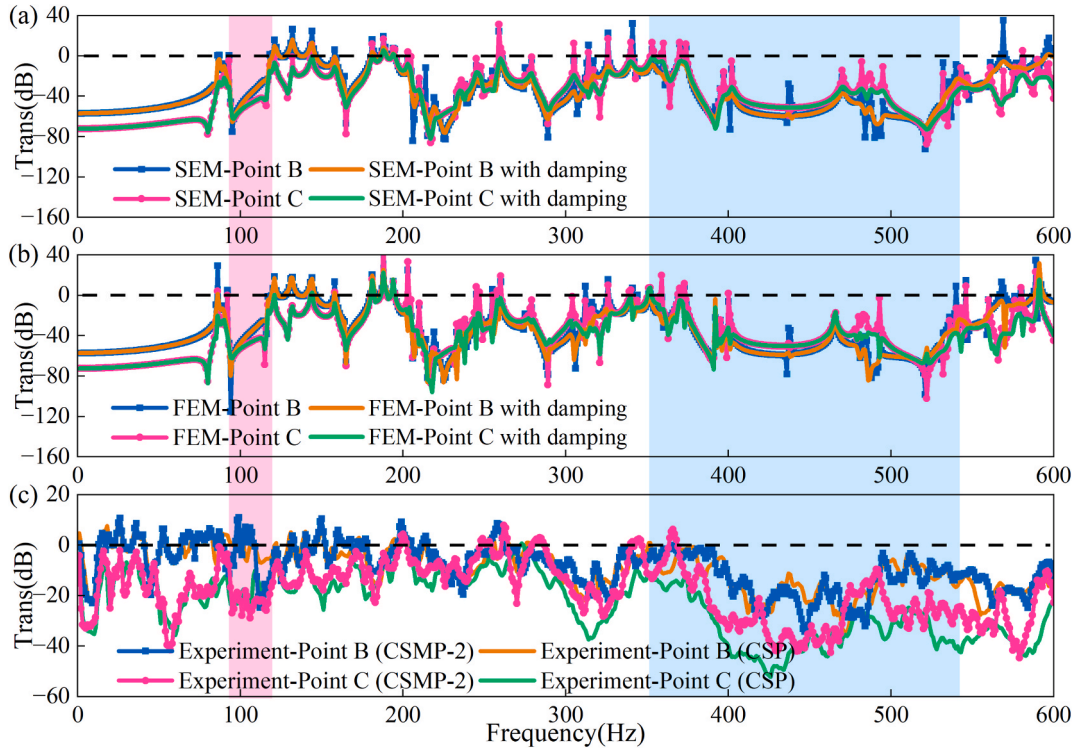


Fig. 15. Comparison of transmission characteristics of CSMP-2: (a) SEM; (b) FEM; (c) Experimental comparison between CSP and CSMP-2.

theoretical wave dynamic model are consistent with those of numerical simulation and experimental test, which verify the correctness of the proposed theoretical model.

3.4. Effects of the structural parameters on the flexural wave bandgap

In order to investigate the effects of geometric parameters on the starting and cutoff frequencies of the flexural wave bandgap, the band structures of a series of proposed CSMP are calculated, as shown in Fig. 17. All calculations are based on the theoretical wave dynamic model established in this paper, and only a single variable is changed during the calculation process.

The sum of the thickness of the upper and lower facesheets is maintained at 3 mm and the bandgaps of CSMP with different thickness ratios are predicted, as shown in Fig. 17(a). With the increase of the thickness ratio, the starting and cutoff frequencies of the first band gap gradually increase, while the onset and cutoff frequencies of the second band gap remain essentially unchanged. For the first band gap, according to the low-frequency bandgap mechanism, increasing the thickness ratio significantly increases the stiffness K_1 of the connecting plate while increasing the mass M_1 only slightly. Consequently, the coupling frequency between the local resonator and the connecting plate increases, leading to an increase in the starting frequency of the first bandgap. Increasing the thickness ratio, on the other hand, slowly increases the stiffness K_2 of the CSP without changing the mass M_2 . Consequently, the coupling frequency between the local resonator and the CSP slowly increases, leading to a slow increase in the cutoff frequency of the first band gap. For the second band gap, since changes in the thickness ratio have little effect on the periodicity of the CSP, changes in the thickness ratio have little effect on the second band gap.

As the angle of the corrugated-core increases, the starting and cutoff frequencies of both the first and second bandgaps shift toward higher frequencies, as shown in Fig. 17(b). For the first bandgap, according to the low-frequency bandgap formation mechanism, as the angle of the corrugated-core increases, the stiffnesses K_1 and K_2 of the connecting plate and CSP increase significantly, while the masses M_1 and M_2 remain essentially unchanged. This results in an increase in the coupling frequencies of the local resonator with the connecting plate and CSP, leading to an increase in both the starting and cutoff frequencies of the first bandgap. For the second band gap, as the angle of the corrugated-core increases, the overall bending stiffness of the CSP increases, increasing the flexural wave modal frequency in the CSP, which in turn increases the starting and cutoff frequencies of the second bandgap.

As shown in Fig. 17(c), increasing the height of the CSMP has little effect on both bandgaps. For the first bandgap, according to the low-frequency bandgap formation mechanism, increasing the height does not significantly change the stiffness K_1 and mass M_1 of the connecting plate. Therefore, the coupling frequency between the local resonator and the connecting plate remains essentially unchanged, resulting in a roughly unchanged starting frequency of the first bandgap. However, increasing the height increases both the stiffness K_2 and mass M_2 of the CSP, leading to a gradual increase in the coupling frequency between the local resonator and the CSP, resulting in a gradual increase in the cutoff frequency of the first bandgap. Similarly, for the second bandgap, height changes have a

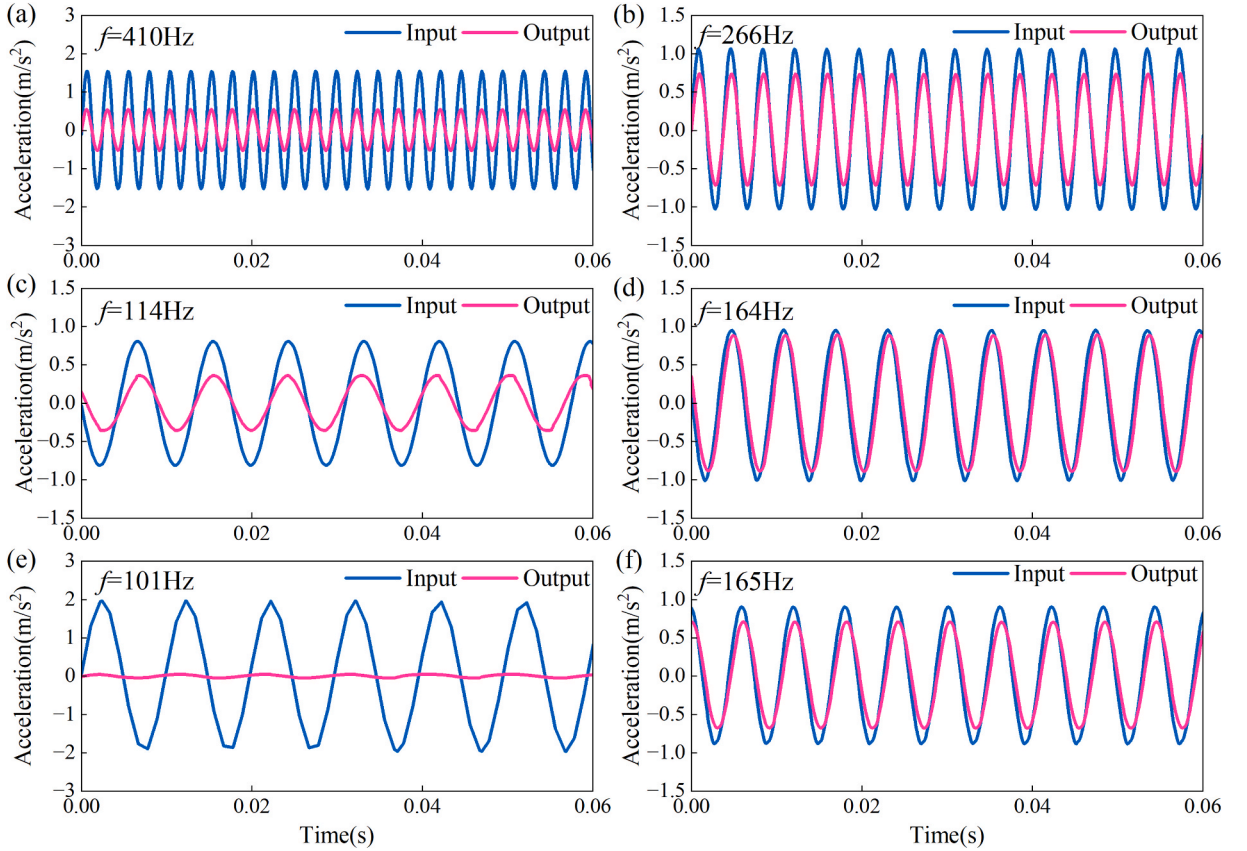


Fig. 16. Acceleration response of sinusoidal excitation: (a) inside and (b) outside the bandgap of CSP; (c) inside and (d) outside the bandgap of CSMP-1; (e) inside and (f) outside the bandgap of CSMP-2.

minimal impact on the periodicity of the CSP, as changes in height have little effect on the second bandgap. Compared to the thickness ratio and the angle, the height of the CSMP has a smaller impact on the bandgap, allowing for flexible adjustment to meet space requirements in engineering applications.

Fig. 17(d) shows the effect of the mass ratio of the local resonator to the CSP on the bandgap. As the mass ratio increases, the starting and cutoff frequencies of the first bandgap decrease, while the starting and cutoff frequencies of the second bandgap remain almost unchanged. The increase in the mass ratio means an increase in mass m_r , while the stiffness k_r remains unchanged. Therefore, the coupling frequencies of the local resonator with the connecting plate and CSP decrease, resulting in a decrease in the starting and cutoff frequencies of the first bandgap. Since the periodicity of the CSP remains unchanged, the starting and cutoff frequencies of the second bandgap remain unchanged.

Therefore, the flexural wave bandgap and vibration isolation characteristics can be effectively adjusted by reasonably changing the design parameters, which can be used for the low-frequency broadband optimization design of the CSMP.

4. Conclusions

In this paper, the elastic wave propagation and low-frequency bandgap characteristics of a lightweight high-strength CSMP with local resonators under simply supported boundary conditions is studied. Based on the spectral element method, a non-equivalent high-precision theoretical wave dynamic model of CSMP is established to calculate the band structure and transmission characteristics. Numerical calculation and experimental tests are conducted to verify the proposed theoretical model. The following conclusions can be drawn:

- (1) Compared to the traditional CSP and typical sandwich metastructure, the proposed CSMP with only 1.77% mass ratio local resonators can exhibit a low-frequency bandgap, demonstrating significant flexural waves attenuation and excellent vibration isolation performance. The starting and cutoff frequencies of the low-frequency band gap are mainly attributed to the vibration coupling of the local resonator with the connecting plate and CSP, respectively.

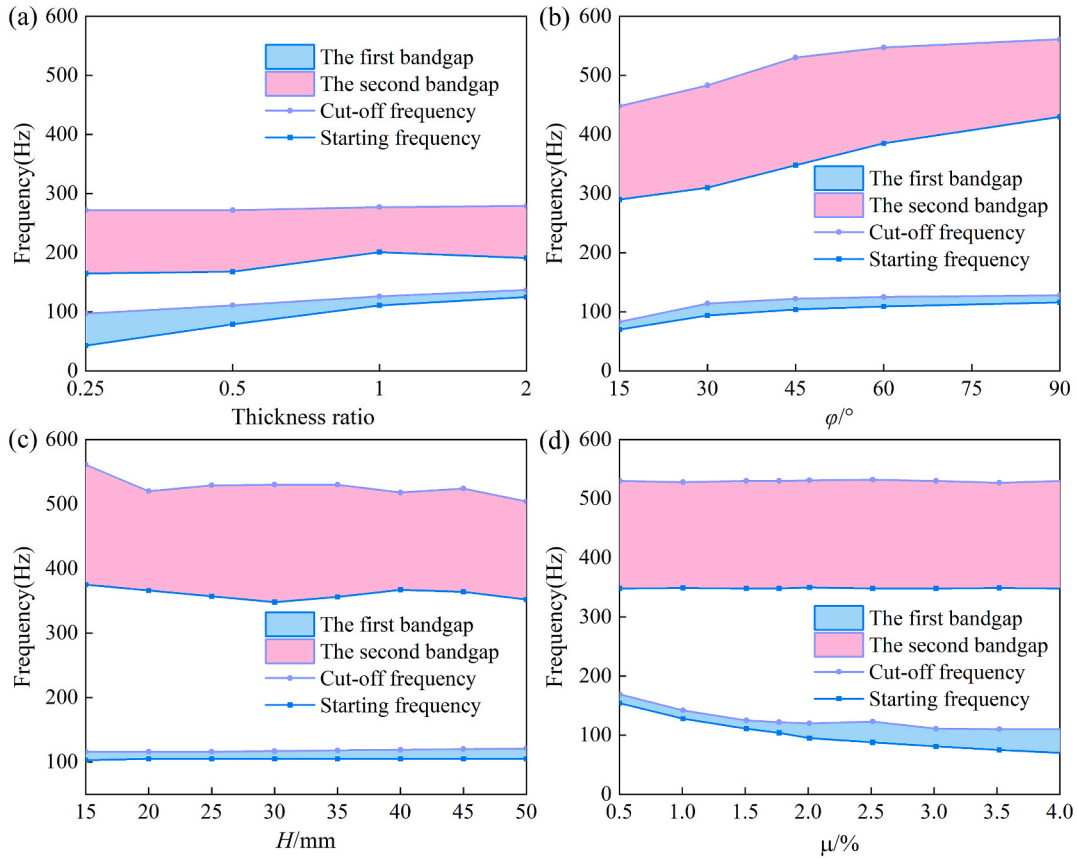


Fig. 17. Effect of the structural parameters on the bandgap: (a) The thickness ratio of the upper and lower plates; (b) The angle of the corrugated-core; (c) The height; (d) The mass ratio of the local resonator to CSP.

- (2) The prediction results of the non-equivalent high-precision theoretical wave dynamic model are consistent with those of numerical simulation and experimental test, which verify the correctness of the proposed theoretical model. This work proposes a novel efficient computational method to study the low-frequency bandgap characteristics of sandwich metastructures.
- (3) The flexural wave bandgap can be effectively adjusted by reasonably changing the design parameters of the CSMP, which can be used for the low-frequency broadband optimization design.

CRediT authorship contribution statement

Dingkang Chen: Writing – original draft, Validation, Methodology, Investigation, Formal analysis. **Xunyu Li:** Writing – review & editing, Investigation, Formal analysis. **Xiaobin Li:** Writing – review & editing, Supervision, Formal analysis. **Li Cheng:** Writing – review & editing, Supervision, Funding acquisition, Formal analysis. **Kian Meng Lim:** Writing – review & editing, Supervision, Formal analysis. **Yinggang Li:** Writing – review & editing, Supervision, Project administration, Investigation, Funding acquisition, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

The detailed expression for the matrix Φ_{in} in Equation (7) is:

$$\Phi_{in} = \begin{bmatrix} \lambda_1 & \lambda_2 & k_{ynm} & k_{ynm} \\ k_{ynm} & k_{ynm} & \lambda_3 & \lambda_4 \\ \lambda_1 e^{\lambda_1 L} & \lambda_1 e^{\lambda_2 L} & k_{ynm} e^{\lambda_3 L} & k_{ynm} e^{\lambda_4 L} \\ k_{ynm} e^{\lambda_1 L} & k_{ynm} e^{\lambda_2 L} & \lambda_3 e^{\lambda_3 L} & \lambda_4 e^{\lambda_4 L} \end{bmatrix} \quad (A.1)$$

where

$$\begin{aligned} \lambda_{1,2} &= \pm \sqrt{k_{ynm}^2 - k_L^2}, \lambda_{3,4} = \pm \sqrt{k_{ynm}^2 - k_S^2} \\ k_L &= \omega_n \sqrt{\frac{(1-\nu^2)\rho}{E}}, \quad k_S = \omega_n \sqrt{\frac{2(1+\nu)\rho}{E}} \end{aligned} \quad (A.2)$$

where k_L and k_S are the wavenumbers of in-plane longitudinal and shear waves, respectively.

The detailed expression for the matrix G_{in} in Equation (9) is:

$$G_{in} = \begin{bmatrix} -\alpha(\lambda_1^2 - \nu k_{ynm}^2) & -\alpha(\lambda_2^2 - \nu k_{ynm}^2) & -\alpha(1-\nu)k_{ynm}\lambda_3 & -\alpha(1-\nu)k_{ynm}\lambda_4 \\ -2\beta k_{ynm}\lambda_1 & -2\beta k_{ynm}\lambda_2 & -\beta(k_{ynm}^2 + \lambda_3^2) & -\beta(k_{ynm}^2 + \lambda_4^2) \\ \alpha(\lambda_1^2 - \nu k_{ynm}^2)e^{\lambda_1 L} & \alpha(\lambda_2^2 - \nu k_{ynm}^2)e^{\lambda_2 L} & \alpha(1-\nu)k_{ynm}\lambda_3 e^{\lambda_3 L} & \alpha(1-\nu)k_{ynm}\lambda_4 e^{\lambda_4 L} \\ 2\beta k_{ynm}\lambda_1 e^{\lambda_1 L} & 2\beta k_{ynm}\lambda_2 e^{\lambda_2 L} & \beta(k_{ynm}^2 + \lambda_3^2)e^{\lambda_3 L} & \beta(k_{ynm}^2 + \lambda_4^2)e^{\lambda_4 L} \end{bmatrix} \quad (A.3)$$

where

$$\alpha = Eh/(1-\nu^2), \beta = Eh/[2(1+\nu)] \quad (A.4)$$

Data availability

Data will be made available on request.

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