



Full Length Article

Low-frequency broadband acoustic transmission through air–water interface using sonic black hole-based metasurface

Zhengyang He[✉], Yuanze Li, Han Liao, Wanglin Qiu, Li Cheng[✉], Xiang Yu^{*✉}

Department of Mechanical Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong, China

ARTICLE INFO

Keywords:

Air-to-water transmission
Sonic black hole
Pressure amplification
Slow sound
Effective parameter inversion

ABSTRACT

Acoustic transmission across the air–water interface is limited by the extreme impedance mismatch between two media. Although recent acoustic metamaterials have enabled cross-interface transmission, their performance is typically restricted to high frequencies or narrow bandwidths. To address this challenge, we propose a sonic black hole (SBH) metasurface for compact, low-frequency, and broadband acoustic transmission across the air–water interface. A transfer-matrix method and finite-element simulations are developed to analyze the transmission characteristics and underlying mechanism. The results show that the metasurface substantially enhances transmission and reduces the transmission loss of the bare interface over a broad low-frequency band. In particular, a pronounced transmission band is predicted near 870 Hz, while the thickness is only 0.0418 m, corresponding to $\lambda/9.34$ in air. An effective-parameter inversion is introduced to reveal continuous impedance variation and strong terminal pressure amplification through slow-wave compression. Thermoviscous analysis indicates that dissipation remains well controlled. Experiments validate this concept, showing a maximum transmitted-level enhancement of 18.7 dB at 1070 Hz and significant transmission from 549 to 1319 Hz. These results demonstrate that SBH metasurface provides a feasible route for compact, low-frequency, and broadband acoustic air–water transmission, with potential applications in cross-medium communication, sensing, and energy transfer.

1. Introduction

Covering about 71 % of Earth's surface, oceans, lakes, and rivers form a vast operating environment where scientific discovery, resource stewardship, and infrastructure increasingly depend on timely information exchange across the water surface. In practice, many emerging systems already straddle this boundary, including underwater acoustic sensor networks, autonomous and remotely controlled vehicles, and surface gateways that relay data to satellites and terrestrial backhaul [1–3]. Yet the most direct form of two-way transmission remains fundamentally attractive: bidirectional acoustic transmission across air–water interface. As conceptually illustrated in Fig. 1(a), such transmission would enable above-surface platforms to communicate with submerged nodes without deploying bulky submerged relays, extend ocean observation networks toward true “internet-scale” coverage, and support real-time control of underwater robots in littoral and inland settings [2–4]. In this scenario, a UAV or a shore-based station can transmit low-frequency acoustic commands through a surface-deployed SBH metasurface to robotic fish or submerged sensor nodes. Such cross-interface links may support environmental monitoring, underwater robotic inspection, and the retrieval of sensing or status

* Corresponding author.

E-mail address: lucien.yu@polyu.edu.hk (X. Yu).

information from underwater devices. Beyond communication, efficient air–water acoustic exchange could unlock new modalities for cross-medium acoustic sensing and situational awareness, where airborne or surface systems interact with underwater sound fields, as well as new opportunities for energy transfer and harvesting in distributed marine electronics [5,6].

Unfortunately, the very interface that separates the underwater world from the atmosphere presents a formidable acoustic barrier. For airborne or underwater sound impinging on a flat air–water interface, the overwhelming contrast in acoustic impedance (about 3600~3700 times) between the two media makes the interface behave almost like a perfect reflector, so that only a minute fraction of the incident acoustic energy is transmitted across it under ordinary conditions [7]. In principle, classical impedance-matching strategies can mitigate the reflection by inserting a matching layer between the two media [8]. However, for the air–water case, the required layer would need an effective impedance far beyond that of common gases and close to the geometric-mean target, which is roughly tens of times higher than air and is rarely available as a simple, homogeneous material [9]. Even if such a material were attainable, the matching layer must have a thickness on the order of a quarter wavelength at the operating frequency, which quickly becomes impractically bulky in the audible and low-kilohertz regimes that are most attractive for robust propagation and long-range signaling [10,11]. These combined constraints, extreme property requirements and wavelength-scale thickness, explain why efficient, compact, and broadband air–water sound transmission remains challenging despite the maturity of impedance-matching concepts in acoustics [10,12].

Acoustic metamaterials and metasurfaces provide a feasible route to engineer “effective” acoustic properties that are difficult or impossible to obtain from homogeneous media, because subwavelength unit cells can synthesize tailored inductance, compliance, and thus an interface impedance designed for a desired transmission pattern [13,14]. To address the severe impedance mismatch at the air–water boundary, a growing body of work has proposed interface-mounted metamaterials that enhance cross-medium transmission by creating strong local reactance and controllable impedance transformation. Representative strategies include membrane–cavity metasurfaces that boost water-to-air transmission in the audio regime through engineered resonant meta-atoms [11], fluid-type metasurfaces that achieve tunable wide-angle and multi-frequency performance by coupling resonant responses in water and air domains [15], and hybrid or gradient designs that simultaneously improve transmission and shape the transmitted wavefront [16]. Bubble-based approaches further exploit the exceptionally strong monopolar resonance of gas inclusions to realize pronounced transmission enhancement with deeply subwavelength thickness [17]. In parallel, topology optimization and inverse design have been

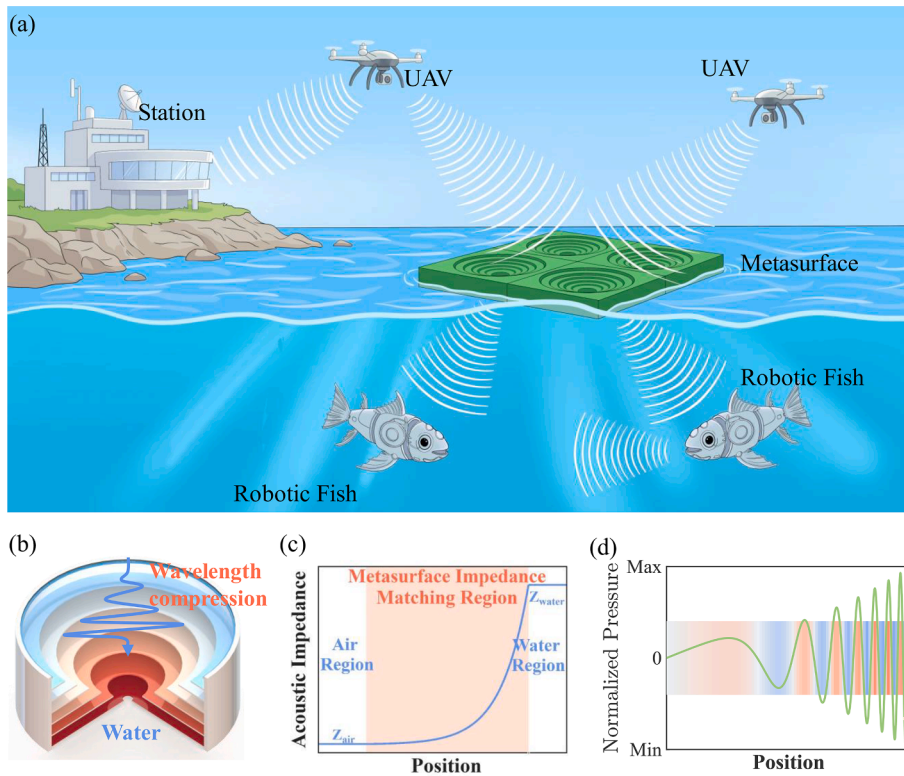


Fig. 1. Wireless acoustic communication via air-water metasurface. (a) Function demonstration of wireless signal communication across air-water via passive SBH-based metasurface. (b) Geometric characteristics of the SBH unit, showing gradual wavelength compression and terminal energy concentration toward the water side. (c) Conceptual evolution of the effective acoustic impedance along the propagation direction, indicating gradual impedance transformation from the air region toward the water region. (d) Schematic of the compressed acoustic field near the terminal section, illustrating the shortening of the effective wavelength and the resulting enhancement of local pressure for improved cross-medium transmission.

used to search discrete unit cells for extreme transmission targets that would be difficult to derive analytically [10,18,19].

From the perspective of practical deployment, low-frequency and broadband operation is often desired, because higher-frequency acoustic waves generally suffer stronger attenuation and scattering in both air and water, whereas lower-frequency components are typically more robust for transmission, control, and sensing over longer distances [1,20]. In this context, it is of considerable interest to explore alternative mechanisms that can support more continuous transmission enhancement over a broad low-frequency band, rather than relying primarily on isolated high-frequency or narrowband resonant peaks.

To overcome these challenges, Sonic Black Hole (SBH) structures can provide a useful route toward low-frequency, broadband air–water acoustic transmission. It is a retarding waveguide metamaterial that achieves slow-wave compression and terminal energy concentration through a power-law geometric profile. In practical duct realizations, SBHs are often implemented as a cascade of rigid rings and thin cavities whose inner radius shrinks toward the termination, so that the waveguide gradually becomes harder to “acoustically fill”. As the wave travels into this graded region, its phase velocity decreases continuously, which effectively compresses long wavelengths into a much shorter physical distance and enables compact manipulation of low-frequency sound. In the ideal limit, the wave speed tends toward zero and the wave becomes trapped, providing the physical picture of “black hole” effect. This mechanism can be interpreted equivalently using transfer-matrix descriptions of the discrete ring–cavity structure [20] or as a homogenized “metafluid” with a power-law varying effective parameter profile that reproduces the same retarding dynamics [21]. Beyond the slow-wave picture, SBHs are also attractive because the same gradient process naturally produces (i) an impedance evolution along the propagation direction and (ii) energy concentration near the terminal region, where acoustic energy density can become strongly localized. These theoretical benefits have been examined and demonstrated through analytical and experimental studies of one-dimensional SBHs [22], as well as detailed investigations into the physical mechanisms and damping pathways that govern practical waveguide SBH performance [23]. Together, these works establish SBHs as a distinctive class of gradient slow-sound metamaterials capable of compact low-frequency control through wavelength compression, impedance evolution, and terminal energy focusing. Most SBH research to date has targeted noise control and broadband low-frequency absorption, where the slow-sound induced energy localization is paired with dissipative elements to convert trapped acoustic energy into heat. A representative case is to combine SBH with micro-perforated boundaries, using SBH for wave retarding and energy focalization while the micro-perforations provide robust dissipation, enabling compact broadband low-frequency absorption in ducts [24]. More recent designs extend this philosophy with multi-unit architectures, folded cavities, and coupled resonant features to widen absorption bandwidth and improve low-frequency efficiency [25]. Along this line, Wu et al. coupled the SBH with a membrane-type acoustic metamaterial, demonstrating that the hybrid configuration can combine SBH-enabled broadband dissipation with tunable low-frequency resonances to achieve compact broadband and low-frequency sound absorption [26]. In parallel, structural modifications such as perforated boundaries have also been explored specifically to strengthen slow-wave generation and enhance SBH effects without requiring an impractically large number of rings [27].

Importantly, SBHs are increasingly being explored not only as absorbers but also as energy-enhancement platforms for sensing and signal processing, leveraging slow sound and local field buildup to raise detectable signal amplitudes. For example, in-duct SBH concepts have been proposed to amplify incident pulse signals for improved sensing under low signal-to-noise conditions [28], and coiled SBH configurations have been developed for contactless sensing and physical signal processing with applications in mechanical fault diagnosis [29,30]. These emerging directions underscore a broader point: SBHs offer a scalable route to compact, low-frequency wave manipulation where slow-sound compression and terminal localization can be harnessed either for dissipation-driven absorption or for constructive enhancement in measurement and transduction.

In this research, we propose an SBH metasurface for enhancing acoustic transmission across the air–water interface. The central idea is to exploit the gradient slow-wave effect of the SBH to progressively compress the wavelength, increase the effective impedance toward the terminal region, and concentrate acoustic energy near the outlet, as conceptually illustrated in Fig. 1(b)–(d). Based on this concept, we establish both transfer-matrix and finite-element models, analyze the underlying physical mechanisms from the perspectives of effective parameter inversion, terminal pressure amplifier, and thermoviscous dissipation, and further evaluate the influence of key geometric parameters. Finally, experiments are conducted to validate the proposed design and to demonstrate its feasibility for low-frequency, broadband cross-medium acoustic transmission. The remainder of this paper is organized as follows. Section 2 presents the structural design of the SBH metasurface and the corresponding theoretical and numerical modeling framework. Section 3 investigates the physical mechanism underlying enhanced transmission, including effective parameter inversion, pressure amplifier, and thermoviscous effects. Section 4 discusses the acoustic characteristics of the proposed structure, with emphasis on parametric dependence. Section 5 reports the experimental validation. Finally, the main conclusions are summarized in Section 6.

2. Design and modelling of SBH for air-to-water acoustic transmission

2.1. SBH unit design

To enhance acoustic transmission across the air–water interface, the metasurface considered in this work is designed not merely as a resonant transmitting element, but as a graded coupling structure that reshapes the incident field before it reaches the interface. The key idea is to realize two functions simultaneously: first, the metasurface should provide a progressive impedance transformation along the propagation direction, so that the abrupt mismatch between the air-side acoustic field and the terminal coupling region can be alleviated in a continuous manner. Second, it should concentrate acoustic energy toward a compact terminal region, where the compressed wave can interact more effectively with the water surface through a subwavelength outlet. From these perspectives, the desired metasurface is essentially a spatially graded acoustic device that combines impedance evolution with terminal energy focusing.

A natural candidate to realize these physical properties is the SBH design, which has demonstrated significant pressure amplification and impedance matching characteristics in air. Owing to its power-law geometric gradient, an SBH can progressively retard the wave, compress the wavelength, and intensify the acoustic field near its terminal section. These features directly match the two requirements described above: the gradual narrowing geometry provides a continuous evolution of the effective acoustic impedance, while the slow-wave effect promotes local field buildup near the end region. Based on this consideration, the metasurface proposed here is constructed by arranging an array of air-side SBH units positioned above the water surface, so that each unit acts as a compact slow-wave meta-atom for cross-medium acoustic coupling.

A schematic of the SBH unit is shown in Fig. 1(b). The unit has a total axial length of L and an outer radius equal to the waveguide radius R . The interior consists of an array of rigid annular rings with gradually shrinking inner radii, so that the cross-sectional area available for axial sound propagation decreases toward the tip. In our implementation, the axial direction is taken as the sound-propagation direction, and the local inner radius profile is prescribed by a power-law function from the inlet radius to a finite residual tip radius. The residual radius is intentionally kept nonzero to leave a controllable terminal acoustic aperture. The SBH unit is positioned above the water surface, with the entire graded structure embedded in air. The terminal small opening of the SBH faces the water surface and is separated from it by a short air spacing. This layout preserves the slow-wave manipulation inside the air-side SBH while enabling strong acoustic coupling to the water through a compact end region.

2.2. Transfer-Matrix Method (TMM) for SBH unit

To quantitatively characterize the transmission of airborne sound through the proposed SBH unit and across the air–water interface, a one-dimensional TMM is established in the low-frequency plane-wave regime. Following the modified TMM framework developed for SBH waveguides [31,32], the acoustic field in each section is represented by the state vector $\Psi = [p \ U]^T$, where p is the cross-section-averaged acoustic pressure and U is the volume velocity. The total response of one SBH unit is obtained by cascading the transfer matrices of all discrete layers and the terminal coupling section located above the water surface.

For theoretical modelling and subsequent optimization, each SBH unit is parameterized by a compact set of geometric variables, including the total length L , inlet radius R_{in} , residual tip radius r_{tip} , number of stepped sections N , ring thickness d_r , and the profile exponent n governing the geometric gradient, as illustrated in Fig. 2(a). Additional parameters describe the terminal coupling region, including the stand-off air-gap thickness h_{gap} between the SBH tip and the water surface and, when applicable, the thickness t_{plate} of the rigid perforated plate forming the outlet aperture. This parameterization is directly used in both the TMM formulation and the full-wave numerical model and also maps naturally to practical fabrication constraints.

In the present design, the SBH channel is discretized into N physical layers along the axial direction. Each layer is further modeled as the product of three sub-matrices, consistent with the physical decomposition shown in Fig. 2: a short uniform ring segment, a tapered core segment, and a shunt branch accounting for the side cavity coupled to the main passage. The transfer matrix of the i -th layer is written as

$$\mathbf{T}^{(i)} = \mathbf{T}_1^{(i)} \mathbf{T}_{2,m}^{(i)} \mathbf{T}_3^{(i)}, \quad (1)$$

and the SBH part is

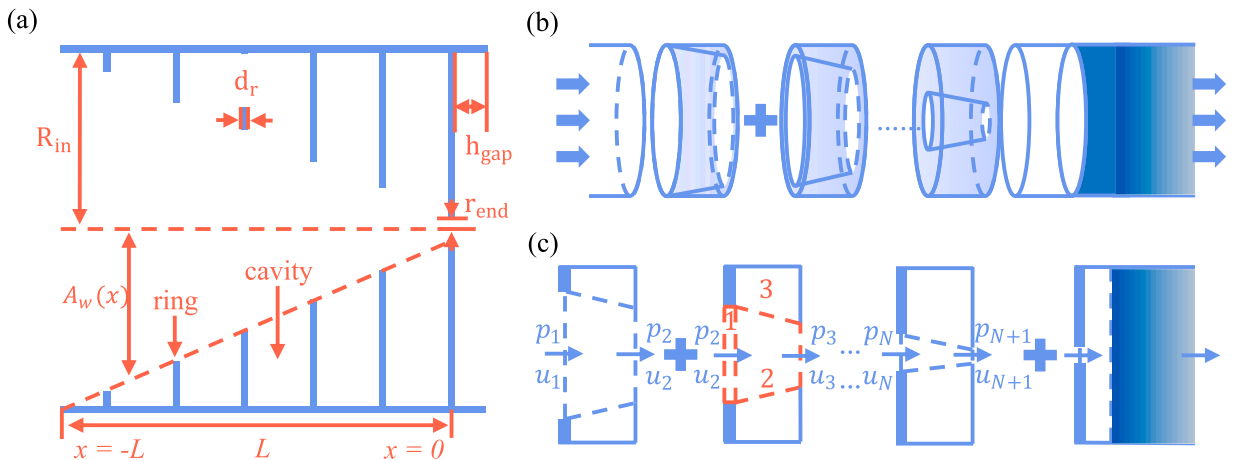


Fig. 2. Geometry and transfer-matrix decomposition of the proposed SBH unit. (a) Cross-section profile of a unit SBH waveguide structure. (b) Three-dimensional illustration of the discretized SBH structure and its terminal coupling to the water interface. (c) Equivalent one-dimensional decomposition of the i -th layer used in the TMM.

$$\mathbf{T}_{SBH} = \prod_{i=1}^N \mathbf{T}^{(i)}. \quad (2)$$

Here, $\mathbf{T}_1^{(i)}$ describes propagation through the ring section with thickness d_r and cross-sectional area $S_i = \pi r_i^2$,

$$\mathbf{T}_1^{(i)} = \begin{bmatrix} \cos(k_a d_r) & j \frac{Z_{0,a}}{S_i} \sin(k_a d_r) \\ j \frac{S_i}{Z_{0,a}} \sin(k_a d_r) & \cos(k_a d_r) \end{bmatrix}, \quad (3)$$

where $k_a = \omega/c_a$ is the wavenumber in air and $Z_{0,a} = \rho_a c_a$ is the acoustic impedance per unit area of air. In our implementation, a small imaginary part may be introduced in c_a as a controllable damping regularization term.

The matrix $\mathbf{T}_{2,m}^{(i)}$ is the modified transfer matrix for the tapered segment of length d_c , which connects the local radius r_i to r_{i+1} . This form is adopted from the modified SBH-TMM treatment to improve the accuracy of conical sections [31], and can be written as

$$\mathbf{T}_{2,m}^{(i)} = \begin{bmatrix} \frac{r_{i+1}}{r_i} \cos(k_a d_c) + \frac{1}{k_a l_{z,i}} \sin(k_a d_c) & j \frac{Z_{0,a}}{S_i} \frac{r_{i+1}}{r_i} \sin(k_a d_c) \\ j \frac{S_i}{Z_{0,a}} \left[\left(\frac{r_{i+1}}{r_i} + \frac{1}{k_a^2 l_{z,i}^2} \right) \sin(k_a d_c) - \frac{d_c}{k_a l_{z,i}} \cos(k_a d_c) \right] & \frac{r_i}{r_{i+1}} \left[\cos(k_a d_c) - \frac{1}{k_a l_{z,i}} \sin(k_a d_c) \right] \end{bmatrix}, \quad (4)$$

with $l_{z,i} = \frac{r_i d_c}{r_i - r_{i+1}}$.

The side-cavity effect of each SBH layer is incorporated as a shunt admittance,

$$\mathbf{T}_3^{(i)} = \begin{bmatrix} 1 & 0 \\ Y_\omega^{(i)} & 1 \end{bmatrix}, \quad (5)$$

where $Y_\omega^{(i)} = S_t^{(i)}/Z_\omega^{(i)}$, and $S_t^{(i)}$ is the conical slant-wall area coupled to the side cavity, given by

$$S_t^{(i)} = \pi(r_i + r_{i+1}) \sqrt{(r_i - r_{i+1})^2 + d_c^2}. \quad (6)$$

The side-cavity input impedance $Z_\omega^{(i)}$ is modeled by the axisymmetric annular-waveguide solution, which yields the Bessel-function form

$$Z_\omega^{(i)} = j Z_{0,a} \frac{J_0(k_a r_i) Y_1(k_a R_{in}) - J_1(k_a R_{in}) Y_0(k_a r_i)}{J_1(k_a r_i) Y_1(k_a R_{in}) - J_1(k_a R_{in}) Y_1(k_a r_i)}. \quad (7)$$

where $J_n(\cdot)$ and $Y_n(\cdot)$ denote Bessel functions of the first and second kinds, respectively, and R_{in} is the inlet radius of the annular cavity. This formulation follows the standard SBH side-cavity representation used in recent analytical models [31].

After the SBH section, the terminal air region above the water surface is modeled as a perforated-neck-cavity coupling stage. In the present metasurface unit, the terminal plate contains a central circular hole of radius r_{tip} , and the plate thickness t_{plate} together with end corrections defines an effective neck length

$$L_{eff} = t_{plate} + \Delta L, \quad (8)$$

where ΔL denotes the total end correction of the circular perforation. Following the classical short-orifice approximation, the total correction is taken as $\Delta L = 16r_{tip}/3\pi$.

The perforation is represented as a lumped series acoustic inductance, i.e.,

$$Z_{neck} = j\omega\rho_a \frac{L_{eff}}{S_{tip}}, \quad (9)$$

$$\mathbf{T}_{neck} = \begin{bmatrix} 1 & Z_{neck} \\ 0 & 1 \end{bmatrix}, \quad (10)$$

where $S_{tip} = \pi r_{tip}^2$. This lumped-neck treatment and the use of end corrections are consistent with the classical perforation acoustic-impedance literature [33–35]. This treatment provides a first-order lumped approximation for the terminal perforation. It should be noted that the lower side of the hole is connected to a thin confined air gap rather than a free half-space; therefore, the present end-correction model is adopted as a simplified approximation, while the compliance of the confined terminal air region is accounted for separately through the gap-compliance model below. The thin air layer between the perforated plate and the water surface is modelled as a shunt compliance

$$C_{gap} = \frac{V_{gap}}{\rho_a c_a^2}, \quad (11)$$

$$Y_{gap} = j\omega C_{gap}, \quad (12)$$

where $V_{gap} = S_{out}h_{gap}$ is the cavity volume and $S_{out} = \pi R_{in}^2$ is the cross-sectional area of the wide section.

The water domain is treated as a semi-infinite plane-wave load at the same cross-section, so its acoustic admittance is written as

$$Y_{water} = \frac{S_{out}}{\rho_w c_w}. \quad (13)$$

At the terminal coupling node, the pressure is continuous across the air-side node and the water load, while the volume velocity satisfies conservation, i.e., the nodal volume velocity is partitioned into the thin-gap compliance branch and the water-load branch. Under the lumped-gap approximation, this gives the terminal relation

$$U_t = (j\omega C_{gap} + Y_{water})p_t. \quad (14)$$

The overall transfer matrix from the inlet plane to the terminal coupling node is then written as

$$T_{all} = T_{SBH} T_{neck} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \quad (15)$$

with the incident pressure amplitude normalized as $p_{inc} = 1$, and using the inlet characteristic admittance $Y_{in} = S_{in}/(\rho_a c_a)$, the inlet reflection coefficient is obtained from the two-port relation as

$$R = \frac{Y_{in} [A + B(j\omega C_{gap} + Y_{water})] - [C + D(j\omega C_{gap} + Y_{water})]}{Y_{in} [A + B(j\omega C_{gap} + Y_{water})] + [C + D(j\omega C_{gap} + Y_{water})]}. \quad (16)$$

The pressure at the terminal coupling node is therefore

$$p_t = \frac{1 + R}{A + B(j\omega C_{gap} + Y_{water})}. \quad (17)$$

Because

$$\tau = |p_t|^2 \frac{Z_{c,a}}{Z_{c,w}}, Z_{c,a} = \frac{\rho_a c_{0,a}}{S_{in}}, Z_{c,w} = \frac{\rho_w c_w}{S_{out}}, \quad (18)$$

and the TL is finally calculated as:

$$TL = -10 \log_{10}(\tau). \quad (19)$$

2.3. Transmission results and Finite Element Method (FEM) validation

To demonstrate the proposed modelling framework and to provide a representative baseline for subsequent parametric studies, one SBH configuration is selected and evaluated using the transfer matrix model developed in Section 2.2. The geometric parameters of the SBH unit are taken from Table 1. In the theoretical calculation, the thermal-viscous dissipation in the air-filled SBH region is incorporated in an effective manner by introducing a complex sound speed, i.e., $c_a = c_{0,a}(1 + j\eta)$, where η is a phenomenological loss factor. In the present case, $\eta = 0.03$ is used to approximately account for the overall dissipation associated with the narrow channels and side cavities [32,36]. Loss factor is an effective parameter representing the overall dissipation level of the SBH region. This treatment preserves the computational efficiency and simplicity of the TMM while capturing the principal attenuation effect required for comparison with the full thermoviscous FEM results.

To validate the TMM predictions, a FEM model is established in COMSOL Multiphysics, as shown in Fig. 3(a). For computational

Table 1
Structure parameters size of unit SBH.

Symbol	Value
Layer number N (-)	10
SBH thickness L (m)	0.04
Outer radius R_{in} (m)	0.04
SBH end radius r_{tip} (m)	0.002
Rings' thickness d_r (m)	0.0008
End cavity thickness h_{gap} (m)	0.001
SBH profile exponent n	1
Plate thickness t_{plate} (m)	0.0008
Cavity depth d_c (m)	0.0032

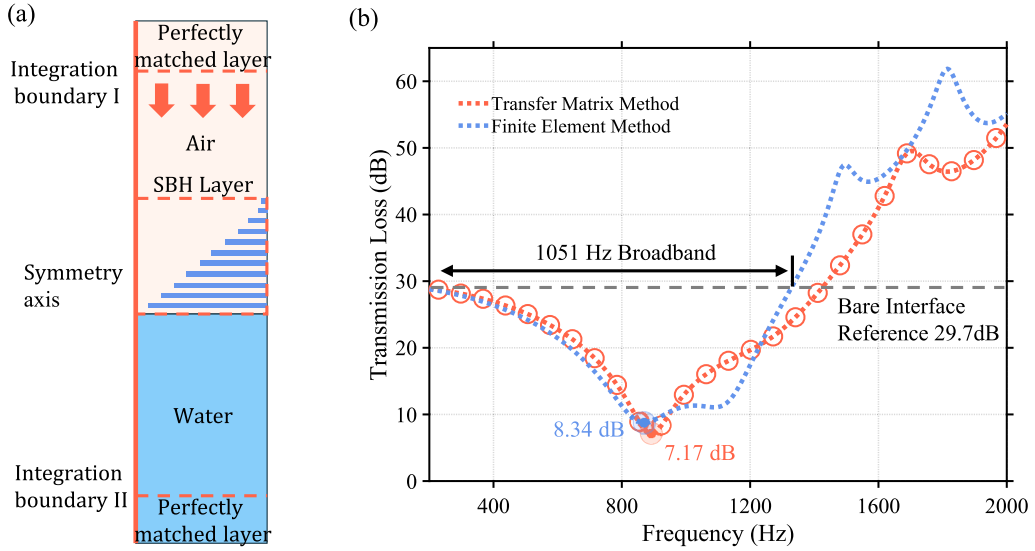


Fig. 3. (a) Schematic of the two-dimensional axisymmetric FEM model used to validate the proposed SBH-enabled air–water transmission. (b) Transmission loss spectra predicted by the TMM and FEM, together with the bare air–water interface reference.

efficiency, a 2D axisymmetric model is adopted, and its results were verified to be in close agreement with those of the corresponding 3D model (see [Appendix A](#) for details). The upper domain is air while the lower domain is water, and perfectly matched layers are applied at the top and bottom ends to suppress spurious reflections. The left boundary is set as the symmetrical axis, while the right boundary is defined as a sound-hard boundary to emulate a one-dimensional waveguide environment. The SBH structure is placed in the air domain above the water surface according to the configuration described in [Section 2.1](#). To accurately resolve viscous and thermal boundary-layer effects, the SBH region is modeled using the *Thermoviscous Acoustics, Frequency Domain* interface, whereas the remaining air and water regions are modelled using the *Pressure Acoustics, Frequency Domain* interface. The incident field is imposed through a *Background Pressure Field* in the air domain, corresponding to a plane wave of unit amplitude (1 Pa) incident from the air side toward the water side. Integration boundaries are defined in the air and water domains to evaluate the reflected and transmitted acoustic power, from which the power transmission coefficient and TL are obtained in the same manner as in the TMM formulation. In addition to the SBH-enhanced case, a bare air–water interface is also calculated as a reference to highlight the transmission enhancement enabled by the proposed structure.

The comparison in [Fig. 3\(b\)](#) confirms, from both the TMM prediction and the FEM simulation, that a properly configured SBH unit can effectively facilitate acoustic transmission across the air–water interface. For the bare interface, the TL remains nearly constant at approximately 29.7 dB over the considered frequency range, governed by the severe acoustic-impedance mismatch between air and water with approximately 3600 in the characteristic impedance ratio. This baseline TL implies that only a very small fraction of the incident acoustic power can penetrate the interface, which is about 10^{-3} in power transmission coefficient, highlighting the intrinsic difficulty of direct air–water acoustic coupling. After introducing the SBH unit, the COMSOL results show a clear transmission enhancement over a broad frequency range from 20 to 1330 Hz, relative to the bare-interface reference. In particular, within 750 – 1170 Hz, the average TL is reduced to below 15 dB, demonstrating the relatively broadband nature of the SBH-assisted transmission mechanism rather than a sharply localized narrowband resonance. The minimum TL occurs at 870 Hz, where $TL \approx 8.3\text{ dB}$, corresponding to a significant reduction of 21.2 dB compared with the bare interface. These results indicate that the SBH structure can still substantially improve air–water acoustic transmission even when thermal-viscous dissipation is considered.

It is also observed that the TMM and FEM results agree well in the low-frequency regime, whereas the discrepancy becomes more pronounced at higher frequencies. This trend is attributed primarily to the limitations of the present one-dimensional TMM approximation, particularly for SBH configurations with a small number of rings, where the discretized geometry and local field evolution are not captured with sufficient accuracy. In addition, multidimensional effects and more detailed thermal-viscous field distributions resolved in FEM are not fully represented in the TMM model. Nevertheless, the TMM correctly predicts the overall transmission-enhancement trend and the main low-frequency operating band and therefore provides a useful and efficient framework for design analysis and parametric optimization.

3. Results and discussion

3.1. Effective parameter analysis

To reveal the physical mechanism by which the SBH structure improves air–water acoustic transmission, the wave propagation behavior inside the SBH is further analyzed by introducing a new effective parameter inversion method. Following the transfer-matrix

formulation in Section 2.2, the entire SBH region is discretized into N axial layers. Each layer corresponds to one physical SBH unit and is represented by the cascaded transfer matrix of the ring section, the shrinking duct section, and the side-cavity contribution. Since the axial size of each discretized layer is much smaller than the wavelength in the frequency range of interest, each layer can be treated, under the long-wavelength assumption, as a locally homogenized equivalent fluid medium.

Under the plane-wave assumption and using the state vector $[p, U]^T$, where p is the averaged acoustic pressure and U is the volume velocity, the transfer matrix of the i -th discretized layer is written as

$$T_i = \begin{bmatrix} T_{11}^i & T_{12}^i \\ T_{21}^i & T_{22}^i \end{bmatrix}. \quad (20)$$

Because each discretized SBH layer is generally unsymmetrical in the propagation direction, T_{11}^i and T_{22}^i are not strictly identical. Therefore, following the common homogenization treatment for unsymmetrical acoustic cells, the two diagonal terms are averaged when retrieving the effective phase constant and effective sound speed. Accordingly, the effective parameters for each layer of the medium can be formulated as follows [37–39]:

$$c_{\text{eff}}^i = \frac{\omega(d_r + d_c)}{\arccos\left(\frac{T_{11}^i + T_{22}^i}{2}\right)} = \frac{\omega(d_r + d_c)}{\arcsin\sqrt{-T_{12}^i T_{21}^i}}, \quad (21)$$

$$\rho_{\text{eff}}^i = S_{\text{ref}} \sqrt{\frac{T_{12}^i}{T_{21}^i}} / c_{\text{eff}}^i, \quad (22)$$

where S_{ref} is the reference cross-sectional area used in the retrieval. In this work, S_{ref} is taken as the inlet area $S_{\text{in}} = \pi R_{\text{in}}^2$, so that the retrieved layer-wise effective impedance and density are directly consistent with the two-port transfer-matrix framework used in Section 2.2. In addition to c_{eff}^i and ρ_{eff}^i , the layer-wise effective acoustic impedance is also examined to characterize the gradual impedance evolution along the SBH. This equivalent-medium representation allows the distributed slow-wave effect, density increase, and impedance growth inside the SBH to be quantified in a compact and physically interpretable manner before analyzing the terminal coupling mechanism.

The retrieved k_{eff}^i , c_{eff}^i , ρ_{eff}^i and Z_{eff}^i are complex quantities. For the effective wavenumber k_{eff}^i , the real part governs the phase variation during wave propagation, whereas the imaginary part represents the corresponding dissipation. The other effective parameters are also retained in their complex forms throughout the retrieval procedure. In the following analysis, the real parts of the retrieved parameters are presented to characterize the spatial evolution of the slow-wave and impedance-transformation effects, while the complex-valued quantities are retained throughout the retrieval procedure.

Special attention is required to the branch selection associated with the inverse cosine used in retrieving the effective parameters. In the numerical implementation, the complex principal value of

$$\theta_i = \cos^{-1}[(T_{11}^i + T_{22}^i)/2], \quad (23)$$

is first evaluated. Since the inverse cosine is multivalued, the admissible solutions differ by integer multiples of 2π . For the first SBH layer, the principal branch is adopted as the initial solution. For each subsequent layer, candidate branches $\theta_i + 2\pi m$ are examined and the branch that preserves the continuity of $\text{Re}(\theta_i)$ with respect to the preceding layer is selected. This layer-to-layer phase unwrapping avoids artificial discontinuities in k_{eff} caused by branch switching along the graded SBH profile. The sign of the square root involved in the impedance retrieval is similarly selected such that the real part of the effective impedance remains non-negative.

It should be emphasized that the retrieved quantities are local effective propagation parameters rather than intrinsic material constants of the entire SBH structure. The SBH is not treated as a globally homogeneous medium. Instead, each physical graded unit is first described by its full local transfer matrix, which explicitly retains the ring section, the shrinking duct geometry, and the associated side-cavity loading, and is subsequently represented by a locally homogeneous equivalent layer. The terminal coupling components, including the terminal aperture, air gap, and water loading, are not included in this homogenization but explicitly included in the overall TMM. This local-equivalent-medium interpretation is consistent with the equivalent-fluid treatment previously adopted for graded acoustic black-hole structures, in which the position-dependent effective density and compressibility are used to describe the progressive reduction of local wave speed along a graded perforation [40]. To provide an additional independent assessment of the present retrieval, the layer-wise effective sound speed of the same SBH structure was also evaluated using the formulation of Umnova et al. This formulation does not rely on the transfer-matrix inversion procedure adopted in the present method and therefore provides an independent reference for the expected slow-wave behavior. As detailed in Appendix B, the two approaches show consistent progressive reduction in effective sound speed along the SBH profile, with only a slight difference near the terminal region.

It should be noted that the applicability of the local homogenization is affected by the long-wavelength and plane-wave assumptions. For the plane-wave approximation, the channel diameter should remain smaller than the acoustic wavelength so that higher-order modes are not significantly excited. Using the maximum channel diameter at the SBH inlet, this condition can be estimated as $f < 1.841c_0/(2\pi R_{\text{in}})$ [41]. With $R_{\text{in}} = 0.04\text{m}$, the corresponding frequency is approximately 2.51 kHz, which is well above the frequency range considered in the present study. Furthermore, the axial dimension $l_i = d_r + d_c$ of an individual SBH layer should

remain substantially smaller than the corresponding local effective wavelength, i.e., [40]

$$l_i / \lambda_{eff}^i \ll 1, \lambda_{eff}^i = 2\pi / \text{Re}(k_{eff}^i). \quad (24)$$

The local homogenization condition is well satisfied here due to the relatively low-frequencies considered. At higher frequencies, the local wavelength shortens and individual cavities show stronger dispersions, requiring separate analyzing tools and is beyond the scope of the present work.

Fig. 4 presents the retrieved effective parameters of the SBH section in a layer-wise staircase form. The retrieval is performed at 892 Hz, which corresponds to the minimum transmission loss predicted by the TMM and therefore represents the characteristic operating condition of the final SBH configuration. As shown in Fig. 4(b), the retrieved effective density increases substantially along the SBH axis, rising from about 1.2 kg/m³ near the inlet to approximately 27.95 kg/m³ near the SBH terminal region, with the most pronounced increase occurring in the final layers. This trend originates from the combined effects of the shrinking main channel and the side-cavity loading. As the local propagation cross-section decreases toward the SBH tip, the motion of the confined air column becomes increasingly inertia-dominated, which manifests itself as a larger effective density. Meanwhile, the side cavities introduce additional dynamic loading and further strengthen this inertial response. As a result, the local acoustic propagation deviates progressively from that in free air, and the equivalent medium becomes much denser toward the terminal region.

This increase in effective density is accompanied by a monotonic decrease in the effective sound speed, as shown in Fig. 4(c), where c_{eff} drops from normal air sound speed at the inlet to about 66.70 m/s near the terminal small opening. From the equivalent-medium viewpoint, the effective sound speed satisfies $c_{eff} = \sqrt{K_{eff}/\rho_{eff}}$. Therefore, the pronounced rise in effective density, together with the cavity-induced modification of the local compressibility, leads directly to a strong reduction in sound wave speed. Physically, this means that the acoustic wavelength is progressively compressed as the wave propagates downstream, allowing low-frequency sound to be manipulated within a compact axial length. The staircase profile observed in Fig. 4 reflects the finite-layer discretization used in the retrieval, rather than a truly discontinuous medium; it is the homogenized representation of the continuously graded SBH structure.

In parallel, the effective acoustic impedance increases from about 0.82×10^5 Pa·s/m³ to 3.71×10^5 Pa·s/m³, as shown in Fig. 4(d). Although a substantial mismatch with the water-side load still remains, the SBH clearly drives the air-side acoustic state away from that of free air and toward a much higher impedance level. More importantly, this increase is gradual rather than abrupt, meaning that the

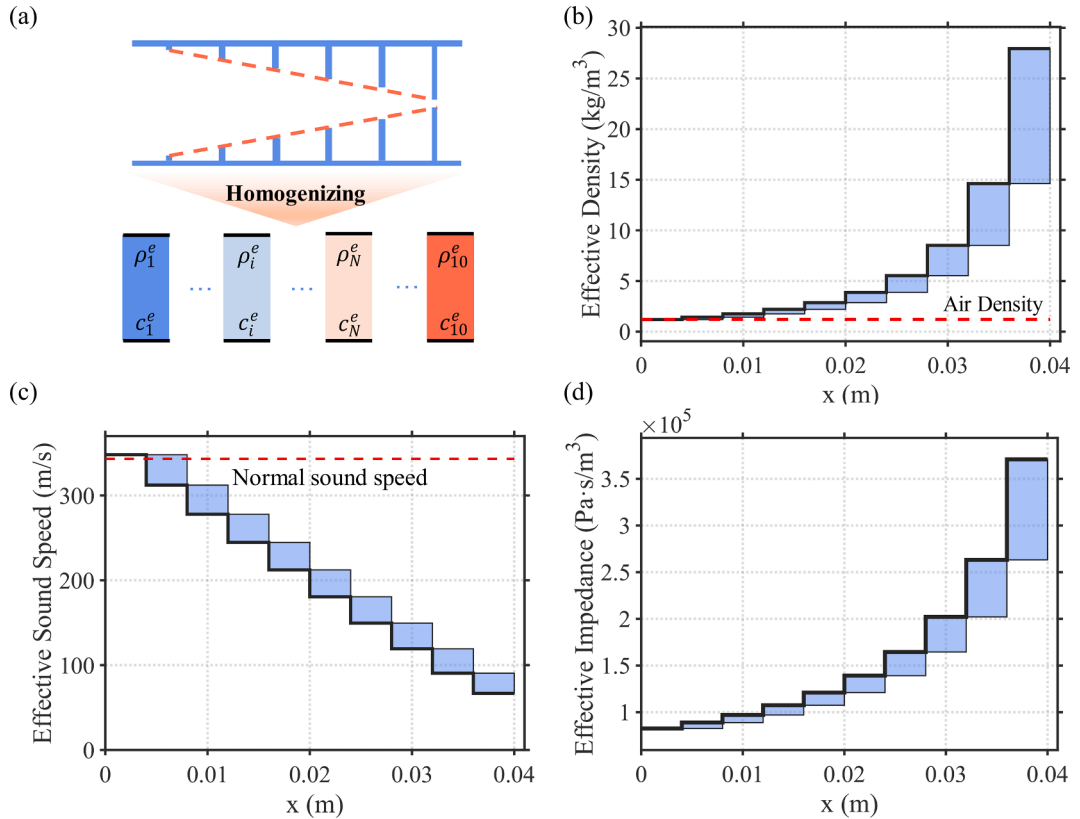


Fig. 4. Homogenized effective-medium interpretation of the discretized SBH unit. (a) Conceptual illustration of mapping the multi-ring SBH structure into an equivalent layered medium. (b) Retrieved layer-wise effective density ρ_{eff} . (c) Retrieved layer-wise effective sound speed c_{eff} along the axial direction. (d) Corresponding layer-wise effective acoustic impedance Z_{eff} .

impedance contrast between neighboring layers is significantly reduced. Consequently, reflections between adjacent layers are suppressed, allowing the wave to propagate progressively deeper into the SBH while continuously undergoing wavelength compression and energy accumulation. Therefore, from the viewpoint of equivalent-medium evolution, the SBH functions as a graded slow-wave transformer that simultaneously lowers the effective sound speed and elevates the effective density and impedance, thereby mitigating the severe impedance discontinuity at the air–water interface.

3.2. Pressure amplification analysis

While the equivalent-medium analysis in Section 3.1 reveals the gradual evolution of c_{eff} , ρ_{eff} , and Z_{eff} , it also provides a direct explanation for the pressure-amplification effect of the SBH. In the idealized metafluid description, the local effective density and sound speed satisfy

$$\rho_{\text{eff}}(x) = \rho_{\text{air}} \frac{S_{\text{in}}}{S(x)} = \rho_{\text{air}} \frac{R_{\text{in}}^2}{r^2(x)}, c_{\text{eff}}(x) = c_{\text{air}} \sqrt{\frac{S(x)}{S_{\text{in}}}} = c_{\text{air}} \frac{r(x)}{R_{\text{in}}}. \quad (25)$$

Therefore, as the wave propagates toward the narrowing end of the SBH, the effective density increases while the effective sound speed decreases, which means that the local wavelength is progressively compressed. At the same time, the effective acoustic impedance also increases as the section shrinks. Under a sufficiently smooth gradient, where internal reflections are suppressed, the incoming wave adapts continuously to this impedance variation. For a forward-propagating wave carrying finite acoustic power, the increase in local impedance implies a reduction in sound speed amplitude and a corresponding amplification of pressure amplitude. In this sense, the SBH acts as a graded slow-wave transformer that progressively converts an extended incident field into a compressed high-pressure field near its terminal region.

This interpretation is also consistent with the governing wave equation in [21,42]

$$\frac{d}{dx} \left(\frac{1}{\rho_{\text{eff}}(x)} \frac{dp}{dx} \right) + \frac{\omega^2}{\rho_{\text{eff}}(x) c_{\text{eff}}^2(x)} p = 0, \quad (26)$$

in which spatial variation of $\rho_{\text{eff}}(x)$ and $c_{\text{eff}}(x)$ directly control the axial pressure variation. As the sound wave enters the region of higher effective inertia and lower sound wave speed, the pressure field is forced to vary over increasingly shorter axial scales, so that energy becomes localized near the terminal part of the structure. If the leakage condition at the water-coupling end is compatible with this slow-wave compression process, the accumulated acoustic energy is released through the terminal aperture rather than being reflected upstream, thereby producing a pronounced pressure-amplification effect at the outlet.

With this mechanism in mind, Fig. 5(a) shows the simulated absolute acoustic pressure fields at three representative frequencies, namely 400, 870, and 1815 Hz. These frequencies correspond to distinct transmission regimes in Fig. 3(b), where 870 Hz is the frequency at which the TL reaches its minimum. At 400 Hz, the pressure field varies relatively smoothly along the SBH channel, and no pronounced localization is observed near the terminal aperture. This indicates that, although SBH modifies the propagation path at low frequency, the wave compression is not sufficient to generate strong terminal focusing. As a result, the coupling enhancement to the water side remains moderate.

At 870 Hz, a very different pattern is observed, where the acoustic pressure becomes strongly concentrated near the SBH terminal region, immediately upstream of the water coupling section. In this case, the wavelength-compression effect and the terminal coupling condition act synergistically, leading to efficient transmission of acoustic energy to the interface. This is the regime in which the SBH

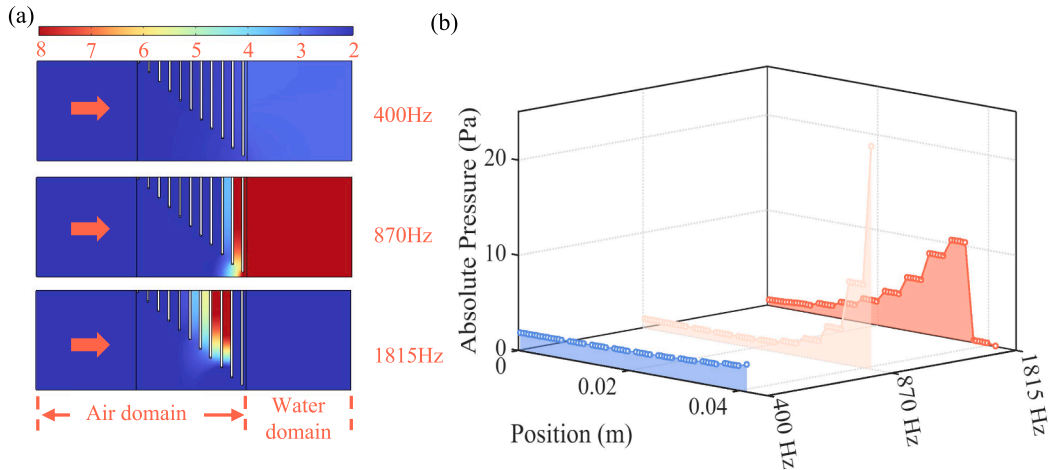


Fig. 5. (a) FEM snapshots of the absolute acoustic pressure field in the coupled air–SBH–water configuration at three representative frequencies (400 Hz, 870 Hz, and 1815 Hz). (b) Corresponding axial distributions of absolute pressure along the symmetry axis for the same frequencies.

most effectively converts the incident airborne plane wave into high local pressure at the terminal aperture, thereby improving the acoustic transmission into water. The correspondence between the strongest terminal localization in Fig. 5(a) and the minimum-TL point in Fig. 3(b) provides direct evidence that terminal pressure amplification is one of the most significant mechanisms for the transmission enhancement.

At 1815 Hz, however, the pressure field no longer peaks at the terminal aperture. Instead, a localized high-pressure region appears in an intermediate ring of the SBH. This behavior is consistent with the so-called rainbow-trapping characteristic of graded slow-wave structures, in which different frequency components are preferentially localized at different axial positions. In the present context, such intermediate localization is unfavorable for cross-interface transmission because a significant portion of the acoustic energy is trapped before reaching the terminal coupling region. Therefore, even though local pressure enhancement still exists, it does not translate into improved transmission into the water domain.

3.3. Relative contribution of the SBH profile and terminal coupling

As described in Section 2.2, the terminal aperture introduces acoustic inertance, the confined air gap provides acoustic compliance, and the water domain imposes an additional radiation load. These elements therefore form a coupled terminal subsystem generating Helmholtz-like response. To distinguish this localized terminal effect from the graded SBH, a comparison study is conducted here in which the graded SBH profile is removed while only leaving the terminal cavity, as shown in Fig. 6(a). The aperture, air gap, water-side loading, dimensions and boundary conditions in the numerical simulations are kept the same.

The corresponding TL spectra are compared in Fig. 6(b). The configuration without the graded SBH profile exhibits a TL minimum of approximately 11.2 dB at 965 Hz, indicating that a terminal cavity alone can also produce localized transmission-enhancement. When the graded SBH profile is introduced, the minimum TL decreases further to approximately 8.8 dB and the effective frequency shifts from 965 Hz to 870 Hz, with a much broader frequency coverage. These differences indicate that the terminal section can contribute to resonance-induced transmission, but the magnitude, low-frequency and broadband performance are not comparable to those of the complete SBH profile.

The axial acoustic pressure distributions in Fig. 7(a) further distinguish the two mechanisms. The acoustic pressures were plotted along the symmetry axis from the air-side entrance to the water-side terminal. The pressure amplitude was normalized by the incident acoustic pressure, while the corresponding phase was unwrapped along the propagation direction. As shown in the upper panel of Fig. 7(a), the normalized pressure amplitude of the complete metasurface at 870 Hz increases progressively from approximately 0.91 at the air-side entrance to approximately 9.98 near the water-side. The pressure increases along the graded region and becomes more pronounced near the terminal section, indicating that the incident field is progressively transformed before reaching the end. In comparison, the configuration without the SBH profile reaches a normalized terminal pressure of approximately 4.12 at 870 Hz. At its own minimum-TL frequency of 965 Hz, the terminal pressure increases to approximately 5.49. Thus, the terminal section can produce local pressure concentration, but its terminal pressure remains substantially lower than that of the complete metasurface. This comparison demonstrates that the terminal section can locally amplify the pressure, while the graded SBH substantially strengthens its excitation through upstream wave transformation.

The unwrapped phase distributions in the lower panel of Fig. 7(a) provide further details. For the complete metasurface at 870 Hz, the pressure phase varies continuously along the graded SBH, with a total phase-delay magnitude of approximately 128° . This distributed phase accumulation is consistent with progressive slow-wave compression inside the graded SBH. By contrast, the configuration without the SBH profile exhibits a total phase variation of only approximately 18° at 870 Hz. At 965 Hz, the structure without SBH profile exhibits a larger phase variation of approximately 80° , but most of this change occurs near the terminal region rather than along the full propagation path. The complete metasurface therefore exhibits a distributed slow-wave compression,

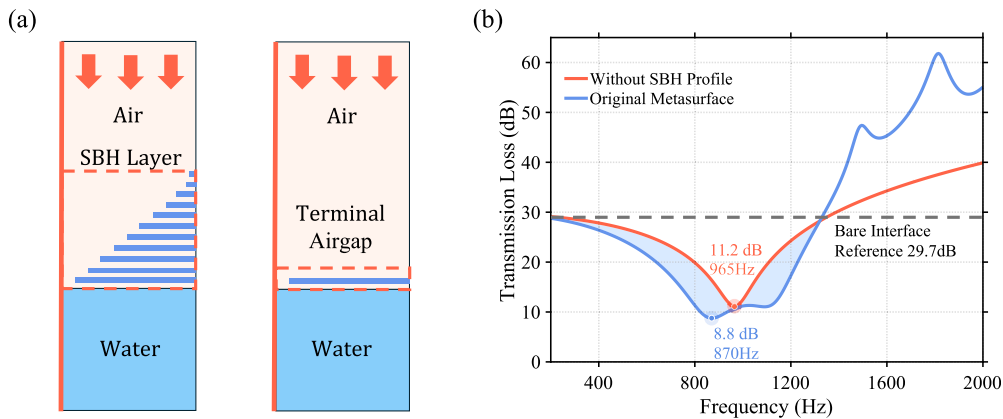


Fig. 6. Comparison of the configurations with and without the SBH profile. (a) Schematics of the complete SBH metasurface and the configuration without SBH profile. (b) Transmission-loss spectra of the complete metasurface and the configuration without SBH profile. The horizontal dashed line denotes the transmission loss of the bare air–water interface.

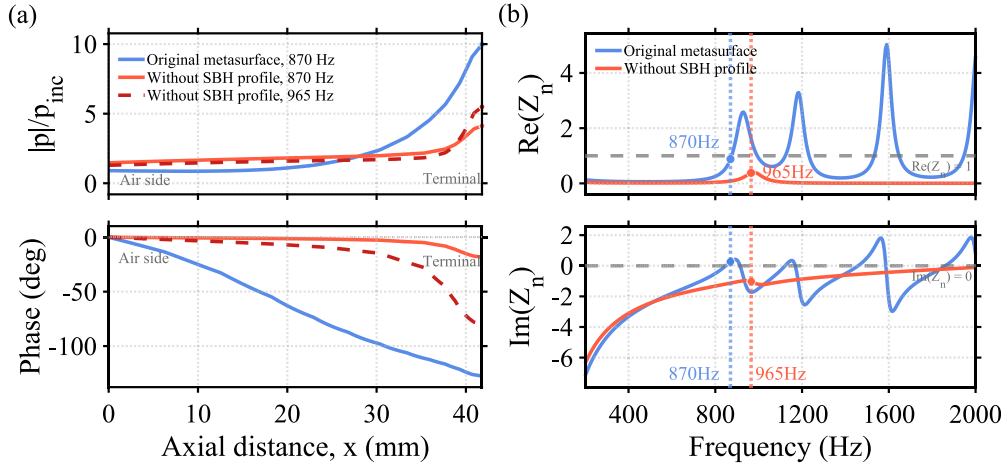


Fig. 7. (a) Axial distributions of the normalized acoustic-pressure amplitude (upper panel) and unwrapped pressure phase (lower panel). (b) Real and imaginary parts of the normalized input impedance for the complete metasurface and the configuration without SBH profile.

whereas the terminal section primarily produces a localized coupling response.

The normalized input impedances shown in Fig. 7(b) provide further support for this interpretation. The input impedance was evaluated at the air-side entrance of the SBH channel. For the configuration without the SBH profile, the impedance was evaluated at the same axial position to maintain a consistent phase reference. At 870 Hz, the input impedance of the complete metasurface is approximately $Z_{n, original} \approx 0.880 + 0.260i$, whereas that of the configuration without the SBH profile is approximately $Z_{n, without SBH} \approx 0.083 - 1.134i$. The complete metasurface therefore exhibits a substantially larger resistive component and a much smaller magnitude

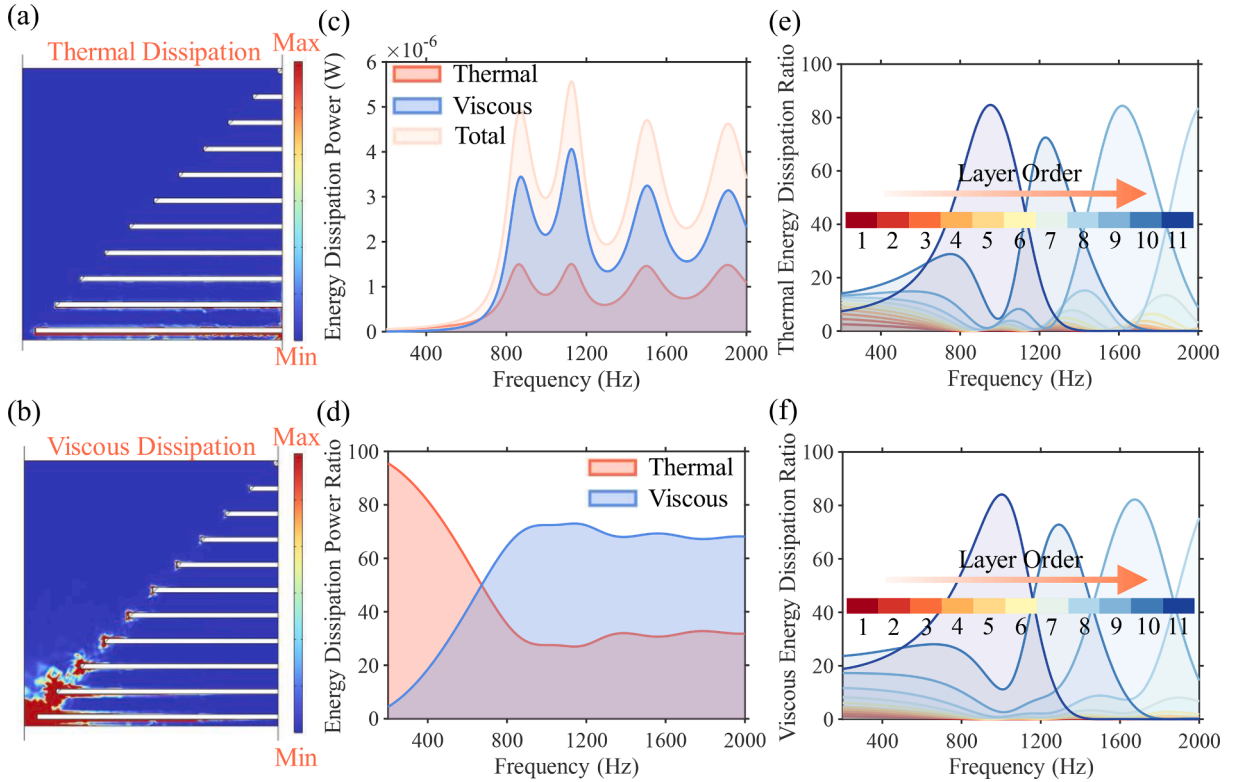


Fig. 8. Energy dissipation power distribution in spatial and frequency domain (a) Spatial distribution of thermal energy dissipation (b) Spatial distribution of viscous dissipation (c) Energy dissipation power distribution along with the individual decomposed viscous and thermal dissipation powers in analysed frequency domain (d) The proportion of energy dissipation power contributed by each component relative to the total energy dissipation power (e) Thermal energy dissipation power of each SBH layer and the end cavity (f) Viscous energy dissipation power of each SBH layer and the end cavity.

of reactance. In particular, the large negative imaginary part of the configuration without the SBH profile indicates a predominantly compliant input response at 870 Hz. The introduction of the graded SBH markedly modifies the input impedance and brings it much closer to the nominal matching condition of $1 + 0i$, consistent with the lower transmission loss observed near 870 Hz.

At 965 Hz, where the configuration without the SBH profile reaches its own minimum TL, its normalized impedance is approximately $Z_{n, \text{without SBH}} \approx 0.379 - 1.031i$. The response remains strongly reactive and is still far from the nominal matching condition. Therefore, the TL minimum at 965 Hz is more appropriately attributed to the localized terminal coupling response rather than to overall impedance matching of the complete system. By comparison, the complete metasurface exhibits additional frequency-dependent variations in both resistance and reactance, indicating that the graded SBH substantially modifies the input-impedance response over the investigated frequency range.

Taken together, Figs. 6 and 7 show that the transmission enhancement arises from the cooperative action of the graded SBH and the terminal coupling section. The terminal section provides a locally tuned coupling path across the air–water interface. The graded SBH acts upstream as a distributed slow-wave and impedance-transforming section, producing progressive phase retardation, wavelength compression, pressure amplification, and input-impedance transformation before the wave reaches the terminal region. Their interaction shifts the principal transmission response toward a lower frequency and produces stronger terminal excitation and lower transmission loss than the terminal section alone.

3.4. The effect of thermoviscous loss

Although the proposed SBH unit is designed to enhance sound transmission across the water–air interface through slow-wave-induced impedance matching and terminal pressure amplification, thermoviscous dissipation in the air domain cannot be neglected in reality. This is particularly important in the present configuration because the acoustic field is strongly compressed near the terminal small aperture, where large velocity gradients and strong near-wall interactions are expected. In contrast to SBH-based absorbers, where thermoviscous loss is often intentionally enhanced, the objective here is different: the thermoviscous effect should be controlled so that the transmission-enhancement mechanism is preserved while parasitic dissipation is minimized.

To further clarify the role of thermoviscous effects in the proposed SBH-based air–water transmission, Fig. 8 presents both the spatial distributions and the frequency-dependent evolution of the dissipative power. As shown in Fig. 8(a) and 8(b), the thermal and viscous losses at 870 Hz are both concentrated in the narrow near-wall regions of the SBH channels, but their spatial patterns are clearly different. Thermal dissipation is mainly distributed along the thin boundary layers adjacent to the rigid surfaces and in the terminal confined region, whereas viscous dissipation is more strongly localized around the lower layers and the strongly compressed terminal part of the SBH, where the particle-velocity gradient becomes particularly large. This difference indicates that the slow-wave compression induced by the SBH not only enhances acoustic energy concentration but also redistributes the two loss mechanisms in distinct ways.

The corresponding frequency-domain dissipation powers are summarized in Fig. 8(c), while Fig. 8(d) gives the relative proportions of thermal and viscous contributions. It can be seen that the total thermoviscous dissipation is not monotonic with frequency, but exhibits several pronounced peaks, indicating that the dissipative process is closely linked to the resonant and slow-wave characteristics of the SBH. At lower frequencies, thermal dissipation constitutes the dominant part of the total loss. This behavior can be attributed to the nearly isothermal compression and expansion of air in the confined channels, where heat exchange between the oscillating fluid and the rigid walls remains significant. As the frequency increases, however, the contribution of viscous dissipation rises rapidly and eventually exceeds the thermal component. Physically, this trend reflects the fact that wave compression in the narrowing SBH channels progressively strengthens the local velocity gradient, so that shear-related dissipation becomes the main loss channel in the medium-to-high frequency range. Therefore, the thermoviscous loss in the present system evolves from a predominantly thermal mechanism at low frequencies to a more viscous-dominated mechanism as the operating frequency moves upward.

A more detailed description is provided by the layer-resolved results in Fig. 8(e) and 8(f), which show the thermal and viscous dissipation power ratios of the ten SBH layers together with the terminal air cavity. Before the first pronounced dissipation peak, the loss contributions are relatively distributed, and a gradual increase in dissipation ratio can be observed from the entrance-side layers toward the terminal region. This indicates that, in the lower-frequency regime, the acoustic field still extends over a relatively large portion of the structure, so that no single layer completely dominates the dissipation process, although the terminal part already carries a larger share of the loss. At the first significant peak, however, the end cavity becomes the dominant dissipative component, suggesting that the strongest energy concentration initially occurs in the confined terminal air gap rather than in one of the ring layers themselves. Beyond this first peak, the dissipation no longer remains broadly distributed but becomes increasingly concentrated in a few terminal layers. More specifically, the dominant contribution is transferred from the end cavity to the last several SBH layers, indicating that the main dissipative region is confined to the tip-side part of the structure. This behavior implies that the enhanced slow-wave compression does not simply produce a uniform increase of loss toward the termination; instead, it gives rise to a sequence of frequency-selective localized dissipative states, each associated with a particular resonant section near the terminal region. In this sense, the last cavity and the adjacent deep layers together constitute the principal dissipation zone of the SBH. A further distinction can be made between thermal and viscous dissipation. The thermal dissipation ratio in Fig. 8(e) exhibits comparatively broader peaks and smoother transitions between dominant layers, indicating that heat-exchange-related loss is distributed over a somewhat wider region. By contrast, the viscous dissipation ratio in Fig. 8(f) is more sharply localized, with stronger layer selectivity and narrower dominant branches. This difference shows that viscous loss is more sensitive to the local velocity-gradient amplification induced by wave compression, whereas thermal loss retains a relatively more distributed character even when the dissipation is centered around the terminal part. Therefore, the layer-resolved results confirm that thermoviscous attenuation in the present SBH is governed

primarily by the end cavity and the last few layers, while the dominant dissipative component changes with frequency in a discrete, resonant manner.

Taken together, Fig. 8 shows that thermoviscous dissipation in the present SBH metasurface is not merely a parasitic effect, but also a useful indicator of the underlying slow-wave and energy-focusing mechanisms. The results demonstrate that acoustic energy is progressively compressed along the SBH and eventually concentrated near its terminal part, where both thermal and viscous losses are intensified. At the same time, the dissipation remains sufficiently limited over the main enhancement band, allowing a substantial portion of the incident energy to be transferred across the water–air interface.

The quantitative influence of thermoviscous dissipation is shown in Fig. 9(a) by comparing the TL predicted by the lossless pressure-acoustics model and the thermoviscous model. In both cases, the SBH unit substantially outperforms the bare air–water interface, demonstrating that the proposed configuration can indeed enable strong cross-interface transmission. However, thermoviscous dissipation degrades the peak transmission performance at the optimal frequency. Without thermoviscous losses, the minimum TL is reduced to 4.56 dB, indicating a near-perfect transmission state. When thermoviscous effects are included, the minimum TL increases to 8.34 dB. Despite this increase, the transmission remains dramatically improved relative to the bare-interface reference, confirming that the SBH mechanism is robust even in the presence of realistic dissipative effects. In other words, thermoviscous dissipation reduces the achievable transmission enhancement but does not eliminate the slow-wave-enabled coupling across the impedance-mismatched interface.

Fig. 9(b) further reveals the corresponding energy redistribution by comparing the reflection and transmission coefficients with and without thermoviscous dissipation. In the lossless pressure-acoustics case, the strong transmission peak is accompanied by a corresponding reduction in reflection, indicating efficient impedance matching and energy injection into the water domain. After thermoviscous effects are introduced, the transmission peak is suppressed and the reflection coefficient is further reduced, showing that part of the energy is no longer converted into transmitted power. Instead, a finite fraction is dissipated inside the SBH air domain through thermal and viscous boundary-layer losses, especially near the terminal focusing region identified in Fig. 5. This comparison highlights the central design trade-off in transmission-oriented SBH structures: stronger wavelength compression and pressure amplification improve interface coupling, but they also increase local thermoviscous dissipation. Therefore, practical optimization should aim to retain the slow-wave gradient-impedance effect while limiting excessive boundary-layer loss in the terminal region.

4. Parameter analyses

After fully underlying the main mechanisms governing enhanced transmission, the effects of the key geometric parameters on TL response are further examined. Fig. 10 summarizes the frequency–parameter maps obtained from the validated FEM model by sweeping (i) the terminal stand-off distance, i.e. end-cavity height, between the SBH outlet and the water surface, (ii) the SBH outer radius, (iii) the SBH total length, and (iv) the end-hole radius. Across all cases, the dominant trend is that the low-TL region (blue region) is highly tunable, shifting in frequency and bandwidth with geometry, which is consistent with the SBH acting as a graded slow-wave transformer whose effective impedance variation and terminal coupling are controlled by its axial and radial scales.

As shown in Fig. 10(a), increasing the end-cavity height causes the minimum-TL region to shift gradually toward lower frequencies, while the corresponding transmission band becomes narrower. This trend indicates that the thin air layer between the SBH outlet and the water surface acts as a compliant terminal element. A larger cavity height increases the effective compliance of this terminal region, thereby lowering the characteristic coupling frequency and moving the optimal transmission regime to a lower band. At the same time, stronger compliance also makes the coupling condition more frequency-selective, which explains the progressive reduction in bandwidth. A similar but more pronounced downward shift is observed when increasing the SBH outer radius, as shown in Fig. 10 (b). In this case, the effective transmission region moves rapidly toward lower frequencies, while its bandwidth also decreases.

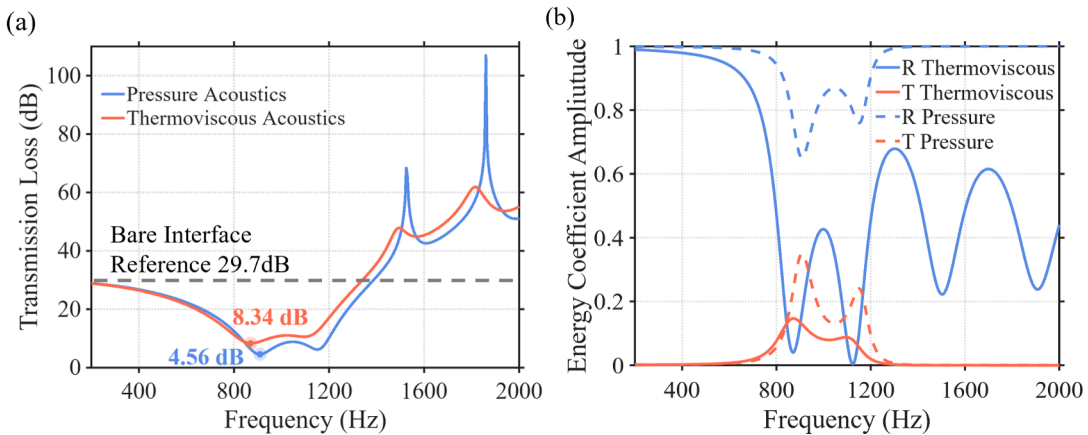


Fig. 9. Effects of thermoviscous energy dissipation (a) Energy transmission loss with and without thermoviscous energy dissipation (b) Energy reflection coefficient and transmission coefficient with and without thermoviscous energy dissipation.

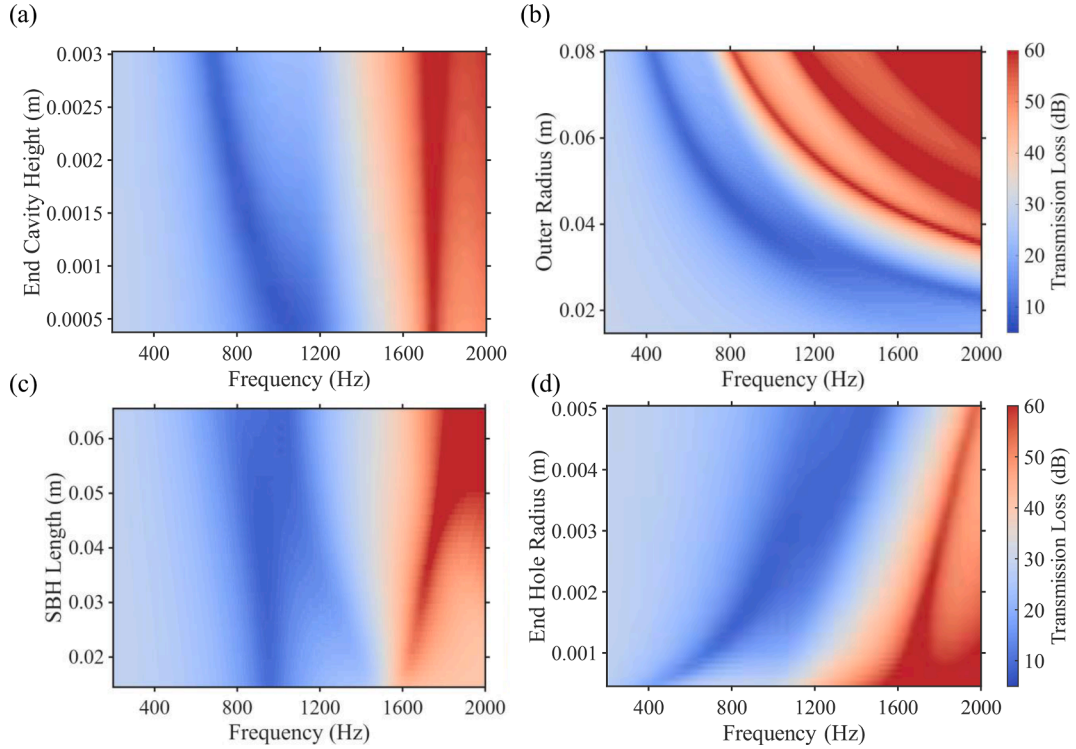


Fig. 10. Parametric sweeps of the predicted transmission loss as a function of frequency for key geometric parameters of the SBH-enabled air–water coupler: (a) terminal air-gap height, (b) SBH outer radius, (c) SBH total length, and (d) SBH tip radius. Color denotes TL (dB).

Physically, a larger outer radius strengthens the SBH slow-wave effect by enlarging the contrast between the wide entrance region and the compressed terminal region, thereby allowing longer wavelengths to be progressively compressed within the structure. This effectively lowers the characteristic operating frequency. However, the stronger geometric gradient and more pronounced wave compression also make the terminal energy-focusing process more selective, so the enhanced transmission tends to be concentrated within a narrower band.

The influence of SBH length is shown in Fig. 10(c). With the number of layers kept fixed, increasing the total length enlarges the axial extent of each layer and modifies the side-cavity thickness distribution, which in turn changes the distributed slow-wave and impedance-transformation process. As the length increases, the low-TL region shifts gradually toward lower frequencies, and, unlike the previous two cases, the transmission band becomes broader. This suggests that a longer SBH provides a more gradual impedance evolution and a more distributed wavelength-compression process, which improves the continuity of energy transfer over frequency. Nevertheless, increasing the length also reduces structural compactness, so in practical design a balance must be struck between transmission performance and deployability. Finally, Fig. 10(d) shows that decreasing the end-hole radius shifts the favorable transmission region toward lower frequencies, but again at the expense of bandwidth. The end hole directly controls the terminal

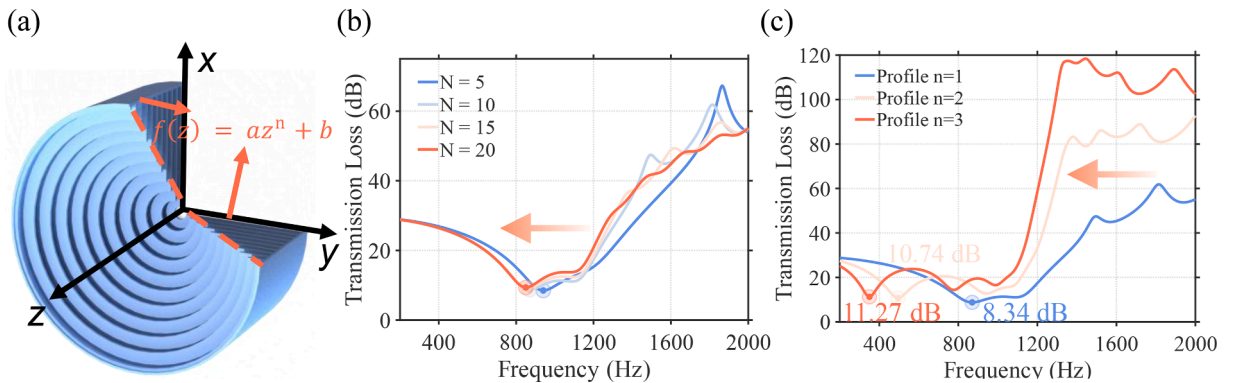


Fig. 11. Influence of SBH discretization and profile on transmission loss. (a) Schematic of the SBH geometry and the adopted power-law profile $f(z) = az^n + b$. (b) TL spectra for different numbers of discretized rings N . (c) TL spectra for different profile exponents n .

acoustic inertance and the leakage condition into the water-coupling region. A smaller aperture increases the inertive loading and strengthens terminal pressure buildup, which is beneficial for low-frequency operation because it allows stronger energy localization near the outlet. However, the same increase in inertance also makes the outlet more frequency-sensitive, so the effective transmission band becomes narrower.

Fig. 11 explores two structural descriptors that are specific to the discretized SBH realization, namely the number of rings N and the power-law exponent that controls how rapidly the inner radius shrinks along the axis. Unlike the continuous idealization, changing N modifies not only the effective slow-wave gradient but also the distribution of narrow passages and solid boundaries, and therefore the strength of thermoviscous dissipation in practical implementations. In particular, increasing N typically introduces more interfaces and a larger boundary-layer area within the same overall length, which tends to enhance viscous and thermal losses and can raise the achievable TL “floor” even when the slow-wave effect is favorable. Conversely, using fewer rings reduces boundary-layer area but can intensify local gradients and produce stronger frequency selectivity. Meanwhile, Fig. 11(c) shows that the profile exponent also plays a decisive role by controlling how rapidly the effective waveguide area and impedance evolve toward the outlet. Steeper profiles concentrate the gradient near the terminal region and tend to produce stronger but more frequency-selective enhancement, while gentler profiles distribute the transformation over a longer distance, favoring broader-band improvement and more robust performance. These parametric trends provide practical guidance for designing compact SBH units that balance low-frequency enhancement, bandwidth, and manufacturability for cross-interface sound transmission.

5. Acoustic transmission experiment

5.1. Experiment-matched FEM simulation

Before conducting the experiment, an additional three-dimensional FEM simulation matching the experimental configurations was conducted, as illustrated in Fig. 12(a). The idealized two-dimensional axisymmetric model used in preceding sections represents a single SBH unit under simplified plane-wave conditions. In contrast, the present model incorporates three-dimensional structures with finite-size according to the subsequent experiment, providing reference results that more closely match the actual experimental measurements. Specifically, a confined water domain of $194 \text{ mm} \times 194 \text{ mm} \times 230 \text{ mm}$ enclosed by a 3-mm-thick glass tank was considered as the source field, for which the density, Young’s modulus, and Poisson’s ratio of the glass were set to 2210 kg/m^3 , 72 GPa and 0.22 respectively [43]. An underwater acoustic transducer was positioned at the center of the tank bottom and modeled as a cylindrical source with a radius of 37.5 mm and a height of 8.5 mm . Above the water surface, a finite 2×2 SBH-based metasurface was introduced, with the geometry of each SBH unit identical to that listed in Table 1 and an overall lateral dimension of approximately $164 \text{ mm} \times 164 \text{ mm}$. The surrounding airborne region was represented by a hemispherical domain with a radius of 0.6 m , sufficiently large to describe the radiated sound field above the metasurface while minimizing the influence of the outer computational boundary.

To reduce the computational cost of the three-dimensional model, the SBH-based metasurface was modeled using the *Pressure Acoustics, Frequency Domain* interface, consistent with the surrounding air and water domains. The thermoviscous dissipation within the narrow SBH channels was instead incorporated through the same effective loss treatment adopted in the TMM model, with a loss factor of $\eta = 0.03$. To account for the acoustic-structural interaction between the confined water and the glass tank, the tank walls were modeled using the *Solid Mechanics* interface. Regarding the boundary conditions, a Spherical Wave Radiation condition was applied to the outer hemispherical surface of the air domain to minimize artificial acoustic reflections from the computational boundary. The upper surface of the underwater transducer was prescribed with a normal velocity of 1 m/s to provide the acoustic excitation from the water side, while the bottom surface of the glass tank was constrained using a Fixed Constraint. Finally,

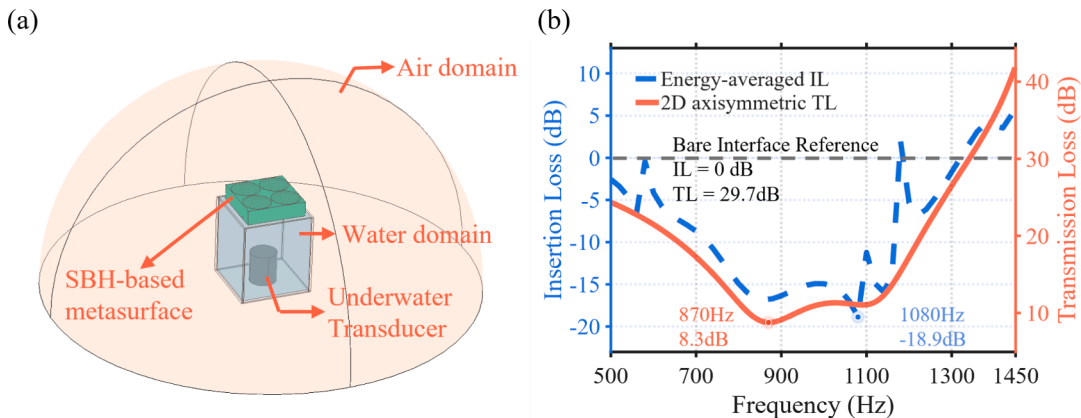


Fig. 12. Experiment-matched finite-element simulation of the water–air acoustic transmission configuration. (a) Three-dimensional FEM model reproducing the principal experimental configuration; (b) Comparison between the averaged insertion loss (IL) obtained from the experiment-matched model and the transmission loss (TL) predicted by the idealized two-dimensional axisymmetric model. The bare-interface references corresponding to $\text{IL} = 0 \text{ dB}$ and $\text{TL} = 29.7 \text{ dB}$ are also indicated.

Acoustic–Structure Boundary conditions were imposed on the glass surfaces in contact with the water and air domains, allowing the acoustic pressure field and structural vibration of the tank to be fully coupled.

To provide a performance index that is directly comparable with the pressure-based experimental measurements, the transmitted airborne response was evaluated using insertion loss (IL). A sampling plane was defined 15 cm above the SBH-based metasurface, on which 36 uniformly distributed sampling points arranged in a 6×6 grid were defined. The sound pressure level (SPL), denoted by $L_{p,i}$, was obtained at each sampling point. Since SPL is a logarithmic quantity, the 36 values were not directly averaged in the decibel scale. Instead, they were first converted into quantities proportional to the local mean-square acoustic pressure, and the spatially averaged SPL was calculated as

$$\bar{L}_p = 10 \log_{10} \left(\frac{1}{36} \sum_{i=1}^{36} 10^{L_{p,i}/10} \right). \quad (27)$$

This averaging procedure was applied separately to the bare-interface reference case and the case with the SBH-based metasurface, yielding $\bar{L}_{p,ref}$ and $\bar{L}_{p,SBH}$, respectively. The insertion loss was then defined as

$$IL = \bar{L}_{p,ref} - \bar{L}_{p,SBH}. \quad (28)$$

According to this sign convention, $IL = 0$ dB indicates no change relative to the bare water–air interface, whereas a negative IL indicates that the introduction of the SBH-based metasurface increases the spatially averaged received airborne SPL and therefore enhances water-to-air acoustic transmission.

Fig. 12(b) compares the averaged IL obtained from the experiment-matched model with the TL predicted by the idealized two-dimensional axisymmetric model. Although the two quantities are defined differently, both results exhibit a similar frequency-dependent transmission behavior, characterized by a broad low-frequency enhancement region and a pronounced optimum within this band. In the experiment-matched model, the IL decreases progressively and reaches a minimum of approximately -18.9 dB at 1080 Hz, indicating a substantial increase in the spatially averaged airborne response after introducing the SBH-based metasurface. Compared with the minimum-TL frequency of approximately 870 Hz predicted by the idealized axisymmetric model, the characteristic enhancement frequency shifts slightly upward to approximately 1080 Hz when the finite experimental configuration is reproduced. The observed frequency shift is likely due to the differences in structural-acoustic coupling and excitation-receiver conditions between the two numerical configurations. In the experiment-matched model, the finite water volume and the elastic glass tank modify the incident conditions and acoustic–structural coupling, whereas the idealized plane-wave axisymmetric model represents a much simpler acoustic condition. In addition, the finite 2×2 metasurface could introduce edge effects. These effects would collectively modify the terminal coupling condition of the SBH and slightly shift the frequency at which the strongest transmission enhancement occurs.

5.2. Experiment settings

Following the experiment-matched numerical analysis presented in Section 5.1, experiments were conducted using the corresponding physical configuration to provide a direct comparison between the numerical and measured responses. To validate the proposed SBH-enabled water–air transmission mechanism and the preceding numerical analysis, experiments were conducted using the setup shown in Fig. 13. A 2×2 SBH metasurface was fabricated by three-dimensional printing according to the geometric parameters listed in Table 1. The sample was mounted above the water surface of a glass tank with dimensions of $200 \text{ mm} \times 200 \text{ mm} \times$

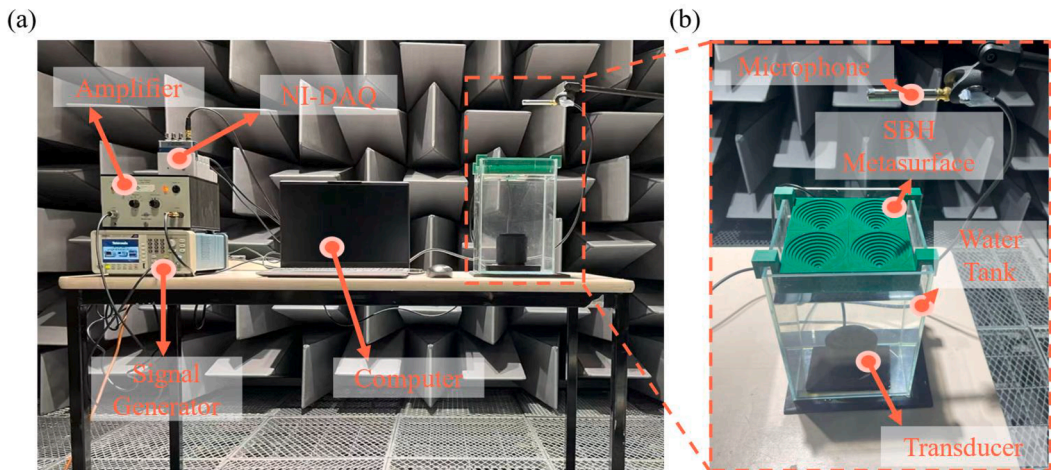


Fig. 13. Experimental settings of acoustic transmittance behavior across the air-water interface. (a) Photograph of the experimental platform. (b) Photograph of the water tank and the SBH metasurface.

300 mm. An underwater transducer (DARAVOC JX00) positioned inside the tank was used as the acoustic source to excite sound from the water side toward the air domain. The transmitted airborne signal was received by a 1/2-inch microphone (Xinsheng CHZ-211) placed above the metasurface. The distance between the SBH metasurface and microphone is 15 cm, consistent with the sampling height adopted in the experiment-matched FEM configuration. To reduce the influence of standing waves and other local fluctuations at a single measurement point, a 150 mm × 150 mm area above the metasurface is scanned. The microphone collected the transmitted sound pressure at 36 measured points arranged in a 6 × 6 array. The same spatial sampling arrangement was adopted in the experiment-matched FEM model to ensure a consistent comparison between the numerical and experimental results.

For the direct comparison between the experiment-matched simulation and the experiment, the transmission enhancement is expressed in terms of insertion gain (IG), which is the sign-reversed form of the IL introduced in Section 5.1. The IG is defined as the difference between the spatially averaged SPLs with and without the SBH-based metasurface and can be equivalently expressed in terms of the measured acoustic pressures as [10,19]

$$IG = \bar{I}_{p,SBH} - \bar{I}_{p,ref} = 10 \log_{10} \left(\frac{\frac{1}{36} \sum_{i=1}^{36} 10^{I_{p,i,SBH}/10}}{\frac{1}{36} \sum_{i=1}^{36} 10^{I_{p,i,ref}/10}} \right) = 20 \log_{10} \sqrt{\frac{\sum_{i=1}^{36} |p_{i,SBH}|^2}{\sum_{i=1}^{36} |p_{i,ref}|^2}}, \quad (29)$$

where $p_{i,SBH}$ and $p_{i,ref}$ denote the acoustic pressures at the i -th sampling point with and without the SBH metasurface, respectively. Accordingly, $IG > 0$ indicates an enhancement of the received airborne acoustic response, whereas $IG = 0$ indicates no change relative to the bare-interface reference. The excitation signal was generated by a signal generator (Tektronix AFG1022X), amplified by a power amplifier (B&K Type 2706), and the microphone output was acquired by a National Instruments data acquisition system (NI 9234) and post-processed using in-house MATLAB routines.

To characterize both the spectral transmission behavior and the temporal waveform response, two sets of measurements were conducted. First, a swept-frequency experiment was performed over the range from 500 Hz to 1450 Hz to obtain the frequency-dependent transmission response of the metasurface. Second, time-domain measurements were carried out at a fixed frequency of 900 Hz to assess the transmitted waveform characteristics. In all cases, the acoustic pressure measured by the microphone was used as the principal experimental observable. For reference, the bare water–air interface without the SBH metasurface was tested under the same excitation and measurement conditions. To improve the reliability of the results, each measurement was repeated 30 times to reduce possible errors. To reduce external disturbances and improve measurement fidelity, the entire setup was placed inside an anechoic chamber. Moreover, 1.5 cm-thick rubber vibration-isolation pads were inserted beneath both the glass tank and the transducer assembly to suppress structural vibration of the setup and to minimize spurious airborne radiation unrelated to the intended water–air transmission process. These measures ensured that the measured enhancement in airborne acoustic response could be attributed primarily to the introduced SBH metasurface.

5.3. Experimental results

The experimental results are presented in Fig. 14. Fig. 14(a) compares the IG obtained from the experiment-matched FEM model with that measured experimentally, where both results are evaluated using the same spatial averaging procedure defined in Eq. (29).

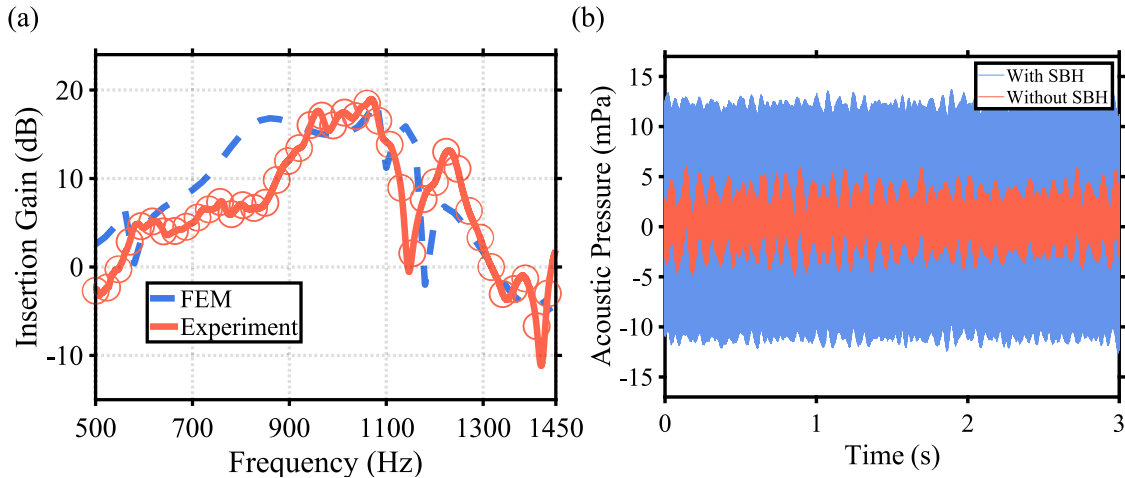


Fig. 14. Experimental verification of the acoustic transmission enhancement across the water–air interface. (a) Comparison of the insertion gain obtained from the experiment-matched FEM model and the experimental measurement; (b) Measured time-domain acoustic pressure signals at 900 Hz over a 3 s acquisition period, comparing the cases with and without the SBH metasurface.

Overall, the numerical and experimental spectra exhibit good agreement in both the frequency-dependent trend and the magnitude of the transmission enhancement. In both cases, the introduction of the SBH-based metasurface produces a pronounced positive IG over a broad low-frequency range, confirming that the transmitted airborne acoustic response is consistently enhanced relative to the bare water–air interface. The experimental IG reaches a maximum of approximately 18.7 dB at 1070 Hz, while the experiment-matched FEM model predicts a maximum enhancement of approximately 18.9 dB at 1080 Hz. Some local discrepancies and spectral fluctuations remain, particularly outside the principal enhancement region, which can be attributed to fabrication tolerances, uncertainties in the actual source response and material properties, residual experimental disturbances, and the simplified treatment of losses and boundary conditions in the numerical model. Nevertheless, the overall quantitative agreement provides stronger validation of the predicted water–air transmission enhancement.

The time-domain measurements at 900 Hz, shown in Fig. 14(b), provide further evidence of the transmission enhancement. Over the 3 s acquisition window, the received acoustic signal without the metasurface remains relatively weak, whereas the introduction of the SBH metasurface enables the microphone to capture a substantially stronger pressure fluctuation. This result is consistent with the frequency-domain observation that the structure improves water-to-air acoustic coupling in the low-frequency band. It should be noted that the experimental spectra do not exactly reproduce the numerical curve, which can be attributed to several practical factors, including fabrication tolerances of the printed metasurface, unavoidable deviation from ideal boundary conditions in the experimental setup, and the relatively limited low-frequency signal-to-noise performance of the underwater transducer. Nevertheless, the combined frequency- and time-domain measurements still provide convincing evidence that the proposed SBH metasurface offers a feasible route for enhancing low-frequency, broadband acoustic transmission across the water–air interface.

6. Conclusion

In this work, an SBH-based metasurface was proposed and investigated for enhancing acoustic transmission across the water–air interface. To address the severe impedance mismatch between water and air, the design was developed to improve cross-medium impedance matching and energy transfer, thereby enabling effective low-frequency, broadband, and compact transmission. By introducing an air-side SBH structure above the water surface, the proposed metasurface provides a new route for manipulating interfacial transmission through slow-wave compression and graded impedance evolution, rather than relying solely on conventional narrowband metamaterials. The primary conclusions of this work are summarized as follows.

- (1) A combined theoretical and numerical framework was established to analyze the proposed SBH metasurface. The transfer-matrix model and finite-element simulations consistently showed that the SBH metasurface can substantially reduce the transmission loss of the bare water–air interface over a broad low-frequency range. To clarify the mechanism, an effective-parameter inversion method was employed to retrieve the layer-wise equivalent sound speed, density, and acoustic impedance. The results showed that, toward the terminal region, the effective density increases while the effective sound speed decreases, indicating a pronounced slow-wave compression effect. Meanwhile, the effective acoustic impedance rises continuously along the SBH, which suppresses strong internal reflections and enables acoustic energy to propagate deeper into the structure and concentrate near the outlet. This terminal pressure amplification significantly enhances the coupling to the water side. In addition, thermoviscous dissipation accompanies the compression process, especially in the narrow terminal region, but remains sufficiently controlled within the main enhancement band so that effective cross-interface transmission can still be achieved.
- (2) The proposed design exhibits a clear compactness advantage. According to the numerical results, the strongest transmission enhancement occurs around 870 Hz, while the total metasurface thickness is only 0.0418 m, corresponding to approximately 0.11 wavelengths in air at that frequency (about $\lambda/9.34$). This confirms that the present approach is capable of realizing meaningful low-frequency transmission enhancement in a deeply subwavelength form factor. Furthermore, the parametric studies clarified the influence of the terminal cavity, outer radius, total length, tip radius, layer number, and geometric profile, thereby providing useful guidelines for practical design and optimization.
- (3) Experimental measurements performed with a 2×2 3D-printed SBH metasurface validated the essential predictions of the model. The measured airborne response above the water surface showed clear enhancement compared with the bare interface, with a maximum increase of about 18.7 dB and a sustained improvement over a relatively broad frequency range. Time-domain measurements at 900 Hz further confirmed that the metasurface enables the microphone to capture substantially stronger transmitted signals. Although quantitative discrepancies remain between simulations and experiments due to fabrication tolerances, non-ideal boundary conditions, and source limitations, the overall agreement demonstrates the feasibility of using SBH metasurfaces to improve water–air acoustic transmission.

Overall, the present research demonstrates that SBH-based metasurfaces provide a new and feasible route for compact, low-frequency, and broadband air–water acoustic transmission. The combination of gradual impedance variation, slow-wave compression, and terminal pressure amplification makes this design particularly attractive for practical air–water acoustic links, with potential acoustic applications in cross-medium communication, sensing, and energy transfer. Furthermore, this work deepens the understanding of the acoustic consequences of the SBH-induced slow-wave effect and provides a new framework for extending SBH-based concepts to other wave-manipulation and sound amplification-related applications.

CRediT authorship contribution statement

Zhengyang He: Writing – original draft, Software, Methodology, Data curation, Conceptualization. **Yuanze Li:** Writing – review & editing, Software, Data curation, Conceptualization. **Han Liao:** Writing – review & editing, Visualization, Investigation. **Wanglin Qiu:** Writing – review & editing, Visualization, Investigation. **Li Cheng:** Writing – review & editing, Supervision, Resources. **Xiang Yu:** Writing – review & editing, Supervision, Resources, Methodology, Funding acquisition, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

This research is supported by the National Natural Science Foundation of China (grant No. 12402106) and Research Grants Council of Hong Kong under grants N_PolyU553/23, C5085-25G and T42-513/24-R.

Appendix A

Three-dimensional validation of the axisymmetric model

To validate the 2D axisymmetric approximation adopted in the main text, three 3D FEM models were established under the same normal-incidence plane-wave excitation, as shown in Fig. A1(a). First, a 3D circular single-unit model was constructed as the direct rotational counterpart of the 2D axisymmetric geometry. Second, a 3D rectangular single-unit model was developed to reproduce the external geometry used for additive manufacturing while retaining the same internal SBH profile. Finally, four rectangular units were arranged into a finite 2×2 array with the same dimensions as the fabricated metasurface. These models successively assess the effects of dimensional reduction, the external printed housing, and inter-unit coupling under plane-wave conditions.

The corresponding power-based TL spectra are compared in Fig. A1(b). All four models exhibit closely matched spectral trends and consistently predict the principal low-TL band near 850–900 Hz. In particular, the close agreement between the 3D rectangular single-unit model and the 2×2 array indicates that the array arrangement and inter-unit coupling have only a limited influence on the principal low-frequency response under plane-wave excitation. Although small deviations appear at higher frequencies, they do not alter the main transmission-enhancement band. These results confirm that the 2D axisymmetric model captures the dominant low-frequency transmission characteristics of the SBH metasurface and is therefore suitable for the detailed mechanism, thermoviscous, and parametric analyses presented in the main text.

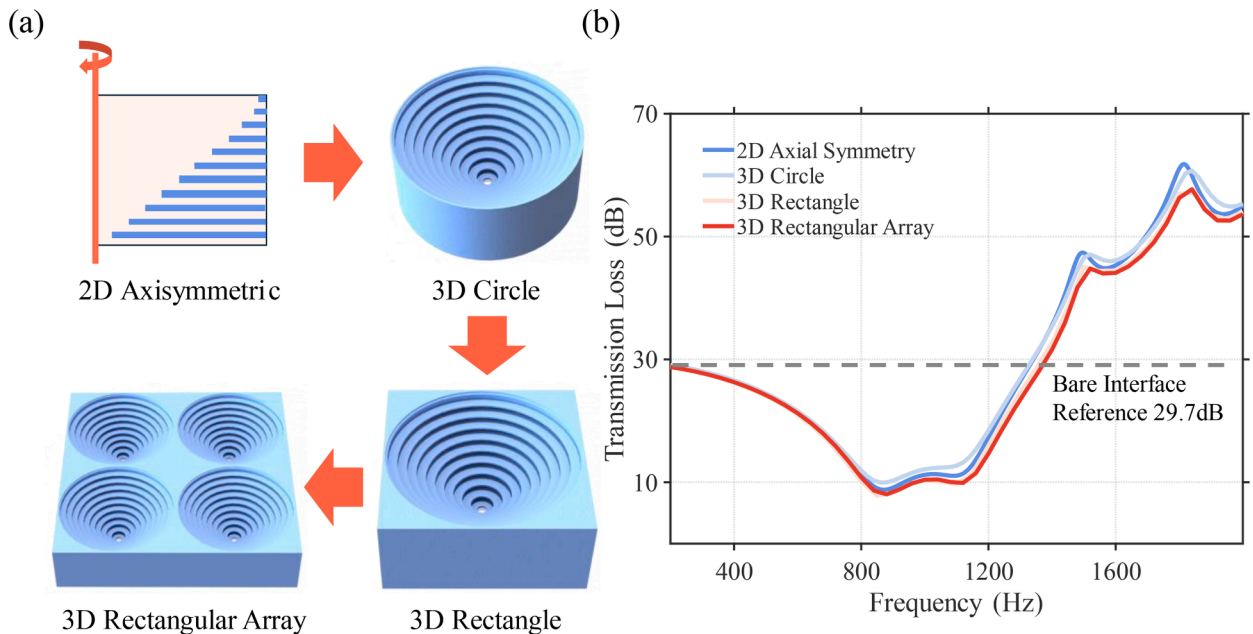


Fig. A1. Validation of the two-dimensional axisymmetric model under normal-incidence plane-wave excitation. (a) Two-dimensional axisymmetric model and the corresponding three-dimensional circular unit, rectangular unit, and finite array. (b) Comparison of their power-based transmission-loss spectra with the bare air-water interface reference.

Appendix B

Cross-validation of the retrieved effective sound speed

To further assess the physical reliability of the effective-parameter retrieval adopted in the present work, the layer-wise effective sound speed obtained from the present TMM-based retrieval was independently compared with that calculated using the equivalent-fluid formulation developed by Umnova et al. [40]. The comparison was performed at $f = 892$ Hz, corresponding to the minimum-transmission-loss frequency predicted by the final TMM configuration.

As described in the main text, the present approach represents each graded SBH section by its local transfer matrix and retrieves the corresponding complex propagation constant, from which the local effective phase velocity is determined. In contrast, the formulation of Umnova et al. describes the graded acoustic black hole as a locally equivalent fluid with position-dependent dynamic density and compressibility. In this formulation, the thermoviscous response of the central perforation and the acoustic loading introduced by the lateral cavities are incorporated into the local effective constitutive properties through the corresponding pore response and wall admittance. The resulting complex local sound speed is evaluated from

$$c_{\text{eff},U}(\mathbf{x}) = [\rho_{\text{eff},U}(\mathbf{x})C_{\text{eff},U}(\mathbf{x})]^{-1/2}. \quad (\text{B1})$$

The same SBH geometry and corresponding local perforation radii were used in the two calculations. Since both formulations generally yield complex-valued effective quantities in the presence of losses, the real parts of the effective sound speeds are compared in Fig. B1.

As shown in Fig. B1, the two independent formulations predict essentially the same progressive reduction in effective sound speed from the wide inlet toward the narrow end of the SBH. The agreement is particularly good over most of the graded region, where both approaches reproduce the pronounced slow-wave evolution associated with the gradually decreasing central-channel radius. This consistency is noteworthy because the two methods determine the effective sound speed through fundamentally different procedures: the present method retrieves the local propagation characteristics from the transfer matrix of each graded section, whereas the method of Umnova et al. evaluates the wave speed from independently formulated dynamic density and compressibility.

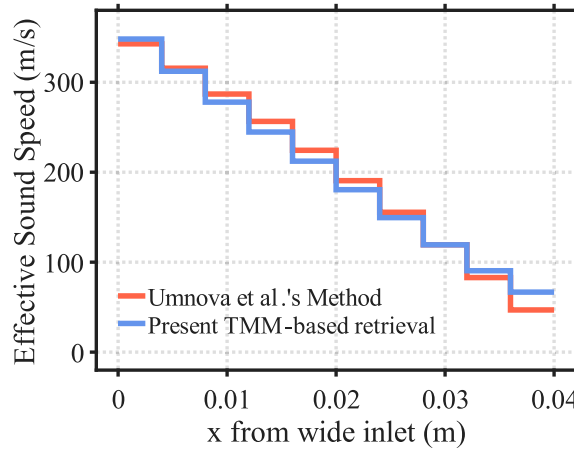


Fig. B1. Comparison of the layer-wise effective sound speed obtained using the present TMM-based retrieval and the independent equivalent-fluid method of Umnova et al. at 892 Hz.

A slight deviation is observed in the terminal section. This difference is physically reasonable because the local geometric variation becomes strongest close to the SBH tip. In the present configuration, the final graded section connects a radius of approximately 5.8 mm to the 2.0-mm terminal aperture, resulting in a substantially larger relative change in cross-sectional dimension than in the upstream sections. The present TMM explicitly retains this finite geometric transition within the local transfer matrix, whereas the equivalent-fluid formulation characterizes each local region primarily through its local perforation geometry and the associated effective constitutive properties. Moreover, Umnova et al. pointed out that the accuracy of the local equivalent-fluid description decreases near the narrow end of an acoustic black-hole structure when the central-perforation dimension becomes comparable to that of the lateral cavities and the required scale separation becomes weaker. The increased discrepancy in the terminal section is therefore consistent with the different local homogenization assumptions of the two formulations rather than indicating a failure of the slow-wave description.

Overall, despite their different theoretical formulations, both approaches predict the same monotonic spatial reduction in the local effective phase velocity and comparable values over the majority of the graded SBH region. The independent comparison therefore supports the physical interpretation of the retrieved c_{eff} and confirms that the progressive slow-wave behavior identified using the present TMM-based retrieval is not an artifact of the inverse transfer-matrix procedure.

Appendix C

Experimental repeatability and uncertainty analysis

To assess the repeatability of the experiment and quantify the associated measurement uncertainty, 30 repeated measurements were conducted under the same nominal experimental conditions. The insertion gain (IG) for each measurement was calculated according to the definition given in the main text. At each frequency, the mean IG was obtained by averaging the results of the repeated measurements. The corresponding measurement range was determined from the minimum and maximum IG values among these repeated measurements.

To further quantify the measurement variability, the sample standard deviation at each frequency was calculated as

$$SD(f) = \sqrt{\frac{\sum_{j=1}^n [IG_j(f) - \overline{IG}(f)]^2}{n-1}}, n = 30, \quad (C1)$$

where $IG_j(f)$ is the insertion gain obtained from the j -th repeated measurement, $\overline{IG}(f)$ is the corresponding mean value, and $n = 30$ is the number of repeated measurements.

Fig. C1 presents the statistical results obtained from the 30 repeated measurements. The solid blue curve represents the mean IG, while the shaded region indicates the full range between the minimum and maximum values measured at each frequency. The red curve shows the corresponding frequency-dependent standard deviation. The repeated measurements exhibit consistent spectral trends over the investigated frequency range. The calculated standard deviation remains below 1.5 dB throughout the investigated frequency range. This variation is relatively small compared with the magnitude of the measured transmission enhancement. These results demonstrate good repeatability of the experimental measurements.

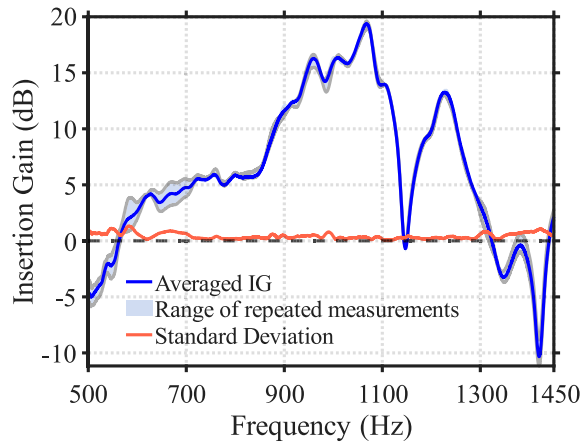


Fig. C1. Experimental repeatability and uncertainty analysis based on repeated measurements.

Data availability

Data will be made available on request.

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