



Underwater sound absorption using rubber enhanced sonic black hole

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ABSTRACT

The absorption of sound waves underwater is important for the development of underwater vehicles and equipment. However, achieving broadband sound absorption in such an environment presents significant challenges. In recent years, Sonic Black Hole (SBH) has attracted growing attention owing to its excellent ability for sound absorption in air, yet its effectiveness and potential for underwater applications remain unexplored. This paper proposes a novel slow-sound structure designed for broadband underwater sound absorption. We first demonstrate that directly transferring the original airborne SBH to water is not feasible. This is due to weak dissipation in water and strong vibro-acoustic coupling between structures and fluids, which hinders the SBH effect. To overcome these issues, rubber insertions are incorporated into the cavities of the SBH. Analytical and numerical models are developed to predict sound absorption characteristics. Our findings indicate that rubber not only enhances energy dissipation, but also greatly reduces vibrations and structure-borne sound transmission across the SBH structure, thereby preserving the advantageous features of SBH. The results confirm the feasibility of broadband absorption, and detailed analyses of the underlying mechanisms are provided. Finally, experiments conducted in a water-filled impedance tube verified the excellent performance and the accuracy of the proposed modeling methods. This study advances the understanding of underwater SBH physics and its potential for sound absorption, potentially paving the way for the development of optimized underwater acoustic devices and opening new avenues for practical applications.

1. Introduction

The demand for broadband sound absorption in underwater environments is critical for various applications, including reducing noise emissions and improving the performance of underwater equipment [1–3]. Effective sound absorption is vital for achieving acoustic stealth in military applications, as well as for the design of Unmanned Underwater Vehicles (UUVs) and acoustic sensing systems [4–6]. Furthermore, underwater noise from human activities exerts adverse effects on aquatic life, prompting the implementation of noise mitigation measures [7]. Traditionally, viscoelastic materials such as rubber are employed as underwater sound-absorbing materials due to their acoustic impedance being close to water and their high loss factor. However, these materials exhibit several limitations, including inadequate resistance to water pressure and corrosion, bulky structural dimensions and poor acoustic performance, restricting their ability for practical use [4,5,8].

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Considering the limitations of rubber, the development of compact, high-performing and robust underwater acoustic structures has attracted overwhelming attention, as evidenced by the persistent efforts and the flourishing advances in the field. However, compared with the airborne sound absorber [9–13], the underwater sound absorber is required to control sound waves with larger wavelengths, address the non-negligible structural-acoustic coupling effect, and attain greater damping. These requirements pose practical limitations. To solve these problems, a wide range of sound absorbing materials and structures have been proposed over the past decades. For example, Fu et al. synthesized a nanocomposite material using chemical synthesis methods which exhibited an average sound absorption coefficient exceeding 0.8 [14–17]. To enhance sound absorption, incorporating air cavities and incorporating metal local resonators into the rubber layer are two commonly adopted approaches. The cavities create significant impedance mismatch across the rubber and air interface, converting incident longitudinal waves to transverse waves, thereby improving the sound absorption capacity. Metal local resonators not only provide a similar wave conversion mechanism, but also introduce additional resonance modes, leading to new resonance peaks in the absorption curve. Various resonant structures and design methods have been developed, often incorporating conical structures inside to improve impedance matching, or employing hollow configurations to enhance the scattering of sound waves [18–22]. The combination of these two structures and the effect of design parameters have been thoroughly studied. Several innovative designs by wrapping the structure or shaping it into a spiral form have been proposed [23–27]. However, the cavity structure is highly sensitive to water pressure, leading to deteriorated performance under high-pressure conditions. Additionally, locally resonant structures exhibit a relatively narrow sound absorption peak, limiting their effectiveness across a wide frequency range [28–31].

To improve low-frequency performance, commonly used meta-structures for airborne sound absorption, including Helmholtz resonators [32–34], Micro-perforated Plates (MPP) [35–37], and membrane-type resonators [38–39] have been explored for underwater applications. For Helmholtz resonators, the resulting sound absorption bandwidth is usually narrow. In the context of MPP, Wang et al. [35] proposed a coiled structure to realize broadband underwater sound absorption. However, their study did not consider the structural-acoustic coupling effect in water. Kim [36] and Li [37] revealed that structural-acoustic coupling can lead to sharp absorption peaks, but the intrinsic damping within water is still low. Similar characteristics have also been observed in membrane-based systems. Besides, Yu et al. [40–41] incorporated parallel steel plates into a rubber layer and discovered that significant energy dissipation occurs through shear deformation at the interface between the steel plates and the rubber matrix. Cai et al. [42–43] developed a honeycomb structure filled with rubber and analyzed the effect of different geometric parameters. Furthermore, non-local resonance effect has garnered increasing attention. Multiple resonant units arranged in parallel by leveraging the Fabry–Pérot (FP) mechanism could achieve near-perfect sound absorption across a wide frequency range [44–48]. Based on the principle of acoustic black hole, Zhang et al. [49–51] proposed to incorporate steel plates with varying cross-sections into a viscoelastic material, which enhances the vibration modes within the viscoelastic medium. Besides, based on the principle of wave mode conversion, several topology-optimized elastic structures have been incorporated into a uniform rubber layer to achieve both broadband sound absorption and insulation and resistance to hydrostatic pressure [52–58]. However, the processing and manufacturing of these designs present significant challenges, and their acoustic performance needs to be further improved.

Recently, the concept of the Sonic Black Hole (SBH) has garnered significant attention as a novel approach to broadband sound absorption. SBH systems are typically constructed using an array of rigid partitions within an acoustic waveguide, taking the form of a cylindrical duct with concentric rings of decreasing inner radii, or a rectangular duct with increasing partition heights. SBH operates on the principle of slow-wave propagation, where the unique geometric profile induces a progressive retarding effect that decelerates wave propagation [59]. This results in a continuous reduction in wave speed as the waves travel through the SBH, leading to significant wavelength compression and energy focusing [60]. Designed for sound absorption in air, SBH has demonstrated unique physical characteristics and excellent acoustic performances. Through numerical and experimental studies, the acoustic properties of airborne SBH have been relatively well-understood [61–64]. The existence of slow-sound effect has been verified through transient simulations and experiments [62–65]. Besides, to attain more desirable sound absorption performance, the investigations of internal power-law optimization and periodic arrangement characteristics of SBH are conducted [66–67]. Furthermore, when combined with absorptive acoustic elements such as micro-perforated panel or micro-lattices, SBH exhibits enhanced sound absorption capabilities and potential [68]. However, it is noted that while airborne SBHs have been extensively studied, their applications for underwater sound absorption have not been explored.

To fill this gap, this study investigates the feasibility of constructing a broadband underwater sound absorber based on the innovative concept of SBH. In principle, the slow-sound effect arises from the intrinsic wave propagation properties governed by specific geometries, making it applicable to both air and underwater settings. However, transferring established air SBH structures directly to water presents several challenges. First, strong water-structure interactions may induce flexural vibrations in the partition walls, invalidating the rigid-wall assumption. Second, the reduced impedance difference between water and structure allows sound waves to transmit through the walls, undermining the effectiveness of the slow-wave effect, as it relies on the collective resonance of continuously varying resonators. Third, the inherent damping loss in water is generally lower than that in air, thus achieving effective sound absorption is more difficult.

To address these challenges, we propose a feasible construction for an underwater SBH by integrating rubber materials between the rings of the original SBH, termed as Rubber Enhanced Underwater SBH (REUSBH) hereafter. The inclusion of rubber not only enhances the damping factor but also provides strategic support to the rings, reducing potential flexural vibrations and flanking sound transmission. The increased stiffness and damping of the reinforced structure ensure that the metal rings vibrate less, thereby maintaining the physical mechanism and design merits of original SBH structure. This configuration can thus enhance wavelength compression and energy focusing, enabling effective sound absorption across a broad range of frequencies.

This study provides a comprehensive analysis of such an underwater sound absorber through theoretical, numerical, and

experimental means, aiming to elucidate the underlying physical mechanisms and design benefits. Initial investigations will focus on the acoustic coupling in the water-filled SBH, revealing that substantial sound absorption is not achievable due to the previously mentioned challenges. Specifically, an analytical prediction model is developed to analyze the wave propagation and sound absorption. The rubber is modeled as an equivalent fluid medium, and the accuracy of our model is verified against numerical simulations. The incorporated rubber component is capable of providing shear restoring force and damping effects to the steel ring, which can reduce vibration magnitude and flanking sound transmission, thereby addressing the identified challenges while preserving the original benefits of the slow-wave structure. The addition of rubber will further enhance energy absorption, resulting in a broadband sound absorber. Finally, the underwater acoustic experiments validate the low-frequency broadband high-efficiency absorption ability and the modeling accuracy of the proposed REUSBH.

The paper is organized as follows: Section 2 introduces the deficiencies of water-filled SBH and design procedures of REUSBH. Section 3 presents the theoretical modeling of REUSBH, the corresponding Finite Element Method (FEM) model, and the validations. Section 4 analyzes the mechanisms of REUSBH, emphasizing the necessity of incorporating rubber, and the properties and behaviors of the constructed absorber. Section 5 presents a parametric analysis. Section 6 introduces fabrication of the tested sample and the measured sound absorption performance of the proposed REUSBH using a water-filled impedance tube. Finally, conclusions and remarks are provided in Section 7.

2. Design of underwater SBH

In this section, we first provide a brief introduction to the concept of slow-sound induced absorption in SBH and analyze the two key challenges in constructing underwater SBHs: weak energy dissipation and strong vibro-acoustic coupling. We then propose a feasible structure that integrates rubber with the SBH concept and introduces the modeling framework used for our analysis.

2.1. SBH concept

The fundamental mechanism that governs excellent sound absorption in SBH is the slow-wave effect. An SBH configuration typically consists of a series of rigid rings with a power-law decaying inner radius, which are inserted into a rigid-walled acoustic duct with circular or rectangular cross section [59]. Fig. 1 shows the schematic of a classical SBH, where the 2D axisymmetric layout can be either rotated to form a cylindrical SBH, or extruded to form a rectangular SBH. The tailored geometry, combined with impedance matching at the entrance and smoothly varying impedances along the boundaries, creates a unique physical condition that slows down sound waves. Theoretically, the slow-sound behavior in an ideal SBH is achieved with linear decaying profile and smooth variation of side-wall impedance (achieved through a large number of rings). As a result of the retarded sound propagation, not only is the wavelength compressed, but also a substantial amount of energy becomes concentrated at the end of the structure and is subsequently dissipated.

2.2. Challenges for underwater SBH

We first assume that the airborne SBH is directly adapted to the underwater environment to absorb sound waves. It is anticipated that two challenges could arise. First, previous studies on underwater micro-perforated panels [36] indicated that the thermo-viscous effect in water is weaker compared to air, potentially leading to insufficient energy dissipation and sound absorption. Second, to establish the desired SBH effect, the walls inside the SBH configuration are required to be rigid. Such approximation is only valid when the solid structure has significantly higher acoustic impedance relative to that of the fluid medium, which is true for the case of an airborne SBH. However, in the case of water, the acoustic impedances of common materials used for fabricating the frame, including resin or thin metal rings, show limited differences from that of water. Moreover, the thickness of the ring needs to be as thin as possible.

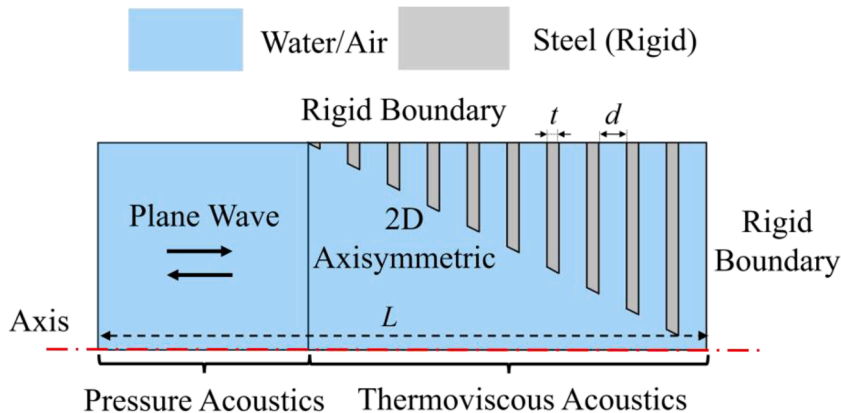


Fig. 1. The 2D axisymmetric simplified diagram of SBH.

This could invalidate the rigid-wall assumption, resulting in non-negligible sound transmission and structural-acoustic coupling effects that jeopardize the SBH effect. Subsequent discussions will provide a deeper analysis of these two challenges.

2.2.1. Weak energy dissipation underwater

We first use numerical simulations to demonstrate the inadequate energy dissipation mechanism for water-filled SBH as compared to airborne ones. The simulations are conducted in commercial software COMSOL Multiphysics. As shown in Fig. 1, all the walls in the SBH are set as rigid boundaries. Geometric parameters of the SBH are listed in Table 1, including H the radius of the main duct, L the length of SBH, t the thickness of each ring and N the ring number. The fluid medium (air or water) within the SBH is modeled by the thermoviscous acoustics module to precisely capture the thermal-viscous effect. The incident acoustic domain is set as pressure acoustics with plane wave conditions. A 2D axisymmetric model is simulated, which incurs much reduced calculation cost as compared to a 3D model while maintaining the same accuracy. Using the thermoviscous acoustics module, the sound absorption coefficients of water and airborne SBH are calculated separately. The results below the duct's cut-off frequency are presented and compared in Fig. 2(a). Given that the sound speeds in water and air are different, a dimensionless wavenumber is used as the horizontal frequency range, defined as $k_0 L / \pi$, where the wavenumber in the medium is $k_0 = \omega / c_0$.

According to Fig. 2(a), the peak and valley positions in the sound absorption coefficient of SBH filled with water and air are consistent, indicating that the inherent resonances and wave interaction behaviors are not strongly dependent on the medium. However, the sound absorption ability of SBH in water is substantially lower than that in air. This discrepancy arises because the thermal and viscous effects in water are generally weaker. To further elucidate this principle, the viscous and thermal boundary layers can be expressed as:

$$\delta_\nu = \sqrt{\frac{2\nu}{\omega}}, \delta_t = \sqrt{\frac{2\kappa}{\omega\rho_0 c_p}} \quad (1)$$

where ω is the angular frequency, ν is the kinematic viscosity of fluid, κ is the adiabatic coefficient and c_p is the isobaric specific heat capacity. Table 2 shows the viscous and thermal parameters of water and air. It is evident that the viscosity of water is lower than that of air, while its specific heat capacity is higher. Consequently, both the viscous boundary layer and the thermal boundary layer in water are much thinner than those in air. Given that a thicker boundary layer generally results in greater energy dissipation, the relatively thinner boundary layer in water indicates that SBH in water may not perform as well as it does in air.

2.2.2. Strong structural-acoustic coupling

Preceding SBH studies in the literature and the numerical results discussed above assume that the rings are rigid. However, in real conditions, the acoustic impedance of steel relative to the fluid medium cannot be infinite. As indicated in Table 3, the acoustic impedance of steel is approximately 30 times greater than that of water, which is more than one hundred thousand times greater than that of air. This suggests that steel rings cannot be simply treated as rigid. Therefore, considering the limited thickness of the steel plate and the finite difference in acoustic impedance between steel and water, the vibration effect of the steel rings under acoustic excitation should be considered.

To further elucidate the effect of ring vibrations, we employ the solid mechanics module to capture the structural-acoustic coupling effect. The thermoviscous settings remain unchanged. The sound absorption coefficient calculations are also shown in Fig. 2(a). When the fluid medium is air, the sound absorption coefficient remains unchanged regardless of whether the steel rings are treated as rigid or elastic, confirming the rigid-wall assumption. In contrast, in water, the coefficient under elastic conditions differs significantly from the rigid case, altering the sound absorption peaks and introducing sharp resonant peaks. As shown in Fig. 2(d), when the steel rings are treated as elastic, the acoustic field inside the water-filled SBH no longer exhibits the rainbow-trapping type localization obtained under the rigid-ring condition. The marked change in the field pattern demonstrates that ring motion and the resulting structural-acoustic coupling cannot be neglected. Consequently, the rings cannot be reasonably approximated as rigid boundaries in this configuration. Additionally, the vibrations of the steel rings cannot be ignored, indicating that the original model for airborne SBH becomes invalid if this vibration cannot be eliminated.

The resonant absorption peaks, energy dissipation power and density, and corresponding ring displacements are shown in Fig. 2(b-c). The results demonstrate that the absorption peaks of the water-filled SBH with elastic rings arise from coupled structural-acoustic resonances. The cavity response builds up the acoustic-pressure and particle-velocity fields inside the SBH, while ring motion provides a compliant boundary and an additional structural dissipation channel. A strong absorption peak therefore appears when the side-cavity response and ring vibration are effectively coupled. This fluid-structure coupling departs from the conventional rigid-wall SBH mechanism. Owing to the low thermoviscous loss in water and the appreciable motion of the rings, the assumptions underlying the conventional SBH model are not satisfied, and design modifications are required to recover the desired black-hole effect underwater.

Table 1
Geometry parameters of SBH.

Parameters	L	H	t	N
Value	100 mm	50 mm	3 mm	10

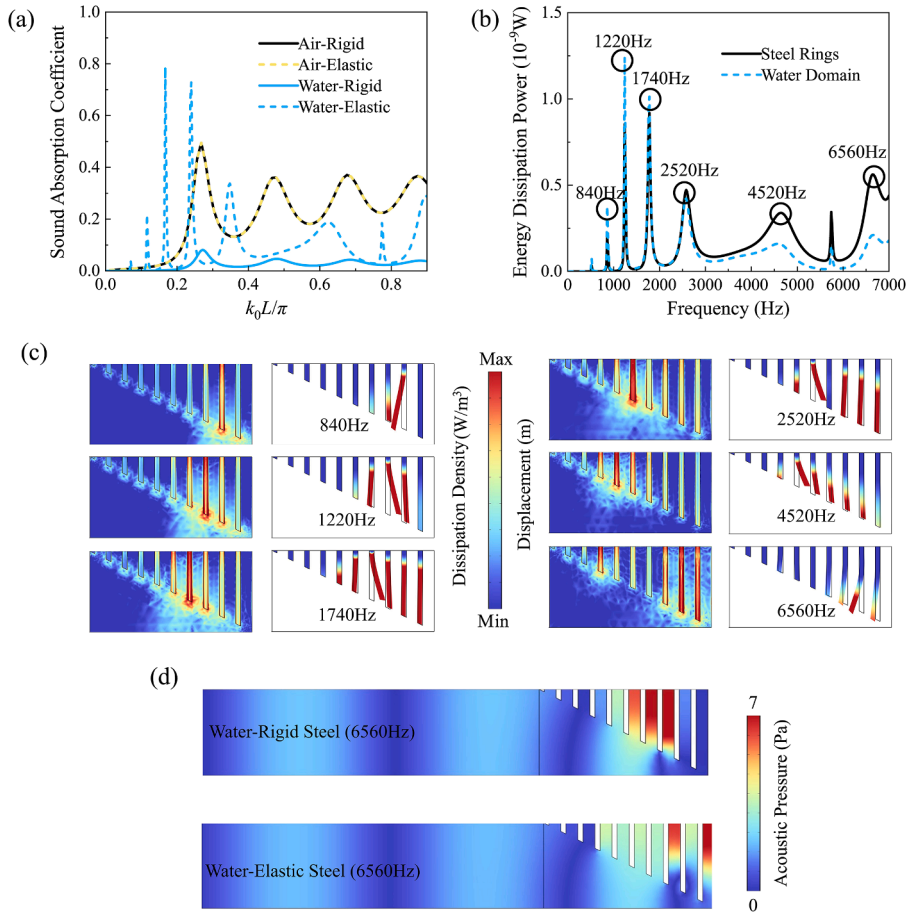


Fig. 2. Effects of the filling medium and steel-ring elasticity on the acoustic response and energy dissipation characteristics of the SBH. (a) The sound absorption coefficients of the SBH filled with air or water are calculated under two conditions: rigid and elastic steel rings. (b) The comparison of the energy dissipation power in the water domain and the steel rings. (c) Energy dissipation density within the water-filled SBH and the displacement of steel rings at the six resonant frequencies. (d) Acoustic pressure field within a water-filled SBH with rigid or elastic steel rings.

Table 2

Density, sound speed, viscous and thermal parameters of water and air at 20 °C.

Physical Parameters	Water	Air
Density ρ_0 (kg/m ³)	1000	1.215
Sound Speed c_0 (m/s)	1483	343.2
Kinematic Viscosity ν (m ² /s)	1.01×10^{-6}	14.8×10^{-6}
Adiabatic Coefficient κ (-)	1.33	1.4
Isobaric heat capacity c_p (J/(kg•K))	4200	1005

Table 3

Physical parameters of steel used in the simulations.

Parameters	Values
Young's modulus (GPa)	210
Density (kg/m ³)	7850
Longitudinal wave speed (m/s)	6100
Poisson's ratio	0.28
Loss factor	0.005

2.3. Proposed Rubber-enhanced Underwater SBH (REUSBH)

Based on the above discussions, the insufficient dissipation loss in water and the undesirable structural-acoustic coupling effect limit the sound absorption performance of water-filled SBH, making it incomparable to that of airborne SBH. Moreover, it challenges the fundamental assumptions and design merits of SBH. To solve this issue, we incorporate rubber into the water-filled SBH, leveraging its stiffness enhancement, viscoelastic damping and outstanding vibration reduction capabilities. Meanwhile, such a design could overcome the inherent limitations of rubber, including single resonance mode and poor resistance to water pressure.

As shown in the configurations in Fig. 3, the REUSBH features a circular cross-section and comprises a series of steel rings inserted into a cylindrical acoustic waveguide. The radius of these rings varies according to a linear relationship, whose geometric parameters remain the same as before. Rubber is used as a filler between each partition to address the shortcomings of water-filled SBH. Specifically, the inclusion of rubber aims to reduce flexural vibrations caused by the strong structural-acoustic coupling among partitions and to enhance energy dissipation within each resonant cavity. This restores the fundamental physical principles of SBH and achieves ideal sound absorption through the strategic arrangement of cascaded resonant cavities [62].

To develop a theoretical model for the designed structure, we initially assume that the partitions satisfy the rigidity condition, neglecting the elasticity of the steel rings. This assumption greatly simplifies the modeling process, and its validity will be confirmed later using FEM simulations. Consequently, the entire structure can be treated as a cascaded arrangement of cavity resonators, with a rigid termination at the end. Following this, sound wave propagation within the structure can be separated into two parts: one along the main duct and the other into the side-branch cavity. The main duct can be segmented into multiple sections based on the number of partitions. The Transfer Matrix Method (TMM) is then employed to model the acoustic field within the REUSBH and calculate the sound absorption coefficient.

3. Modeling and simulations

3.1. TMM modeling

A schematic of the proposed REUSBH is shown in Fig. 4(a), whose geometric values are consistent with those in Table 1. The REUSBH of length L is placed along the x -axis, where the incident sound wave enters from the left at $x = -L$ and propagates through the REUSBH unit until it reaches the termination at $x = 0$. The distance between each ring is denoted as d , while the thickness of the partition is represented by t . The outer radius of the rings is equal to that of the waveguide H . $S(x)$ and $h(x)$ are the cross-sectional area and radius of the annulus of the rings at the position x , and their relationship is $S(x) = \pi h^2(x)$. For the simplest REUSBH with a linear change of cross-sectional profile, the radius $h(x)$ of the inner rings at position x can be approximated by:

$$h(x) = -\frac{H}{L}x \quad (2)$$

Under the framework of TMM, the system is discretized into subcomponents as depicted in Fig. 4(b), where N represents the number of layers segmented by the inner rings. Each layer between two adjacent rings can be further divided into a virtual shrinking duct in the center, and two side-branch cavities on both sides. In addition, the cavity between the rings can be considered as a duct with a uniform cross-sectional area. To obtain the overall sound reflection and absorption characteristics of REUSBH, the sound pressure and volume velocity continuity conditions at each separation interface need to be established.

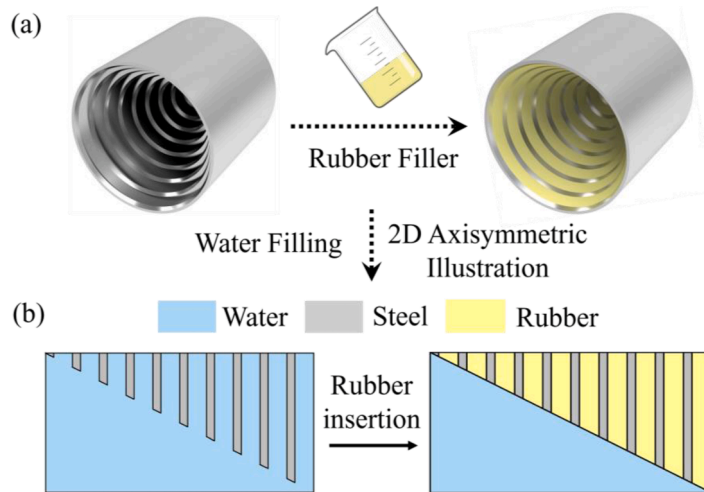


Fig. 3. Design concept and geometric configuration of the proposed REUSBH. (a) The design concept of REUSBH. (b) 2D axisymmetric representation of the REUSBH.

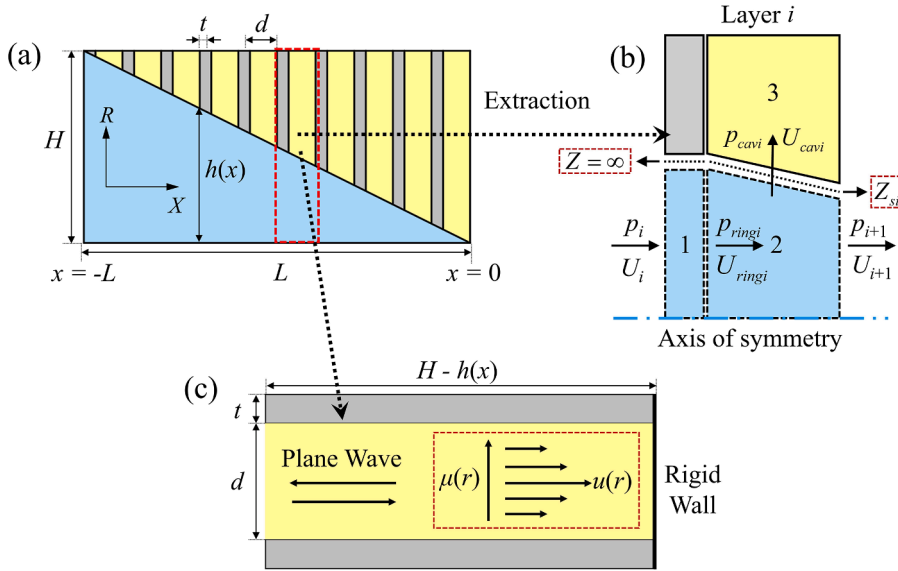


Fig. 4. TMM modeling of the proposed REUSBH. (a) 2D axisymmetric cross-section. (b) TMM modeling of the i -th layer. (c) Side-cavity modeling.

At the entrance, ring end and exit of a specific layer i , the acoustic pressure and volume velocity are expressed by vectors $(p_i \ U_i)^T$, $(p_{ringi} \ U_{ringi})^T$ and $(p_{i+1} \ U_{i+1})^T$, respectively, where the positions are x_i and x_{i+1} . Due to the REUSBH being designed in a precise tapering shape and partially constructed from rubber with dissipative properties, the local high-order waves generated by the abrupt change in cross-section are deemed negligible and only the plane wave modes are considered. Specifically, these high-order waves constitute only a small fraction relative to the main waveguide mode and dissipate more rapidly in highly dissipative media. Under the plane wave assumption, the acoustic pressure can be expressed as:

$$p_i = A_i e^{-jk_0 x_i} + B_i e^{jk_0 x_i} \quad (3)$$

$$p_{ringi} = A_i e^{-jk_0 (x_i + t)} + B_i e^{jk_0 (x_i + t)} \quad (4)$$

$$p_{i+1} = A_i e^{-jk_0 x_{i+1}} + B_i e^{jk_0 x_{i+1}} \quad (5)$$

where A_i and B_i are the amplitudes of the incident and reflected waves in the i -th layer.

Slightly different from previous SBH studies that usually neglect the partition thickness, it is considered in the present REUSBH modeling, and each block is divided into three parts: Part 1 the inner cavity of the ring, part 2 the conical cavity between two adjacent rings, and part 3 the side cavity surrounding the conical cavity. Notably, within part 2, a portion of the volume velocity enters the side-branch cavity (part 3), so the continuity of volume velocities at both the entrance, ring end, and exit can be expressed as:

$$U_i = S_i \frac{A_i e^{-jk_0 x_i} - B_i e^{jk_0 x_i}}{Z_0} \quad (6)$$

$$U_{ringi} = S_i \frac{A_i e^{-jk_0 (x_i + t)} - B_i e^{jk_0 (x_i + t)}}{Z_0} \quad (7)$$

$$U_{cavi} + U_{i+1} = S_{cavi} \frac{p_{i+1}}{Z_{si}} + U_{i+1} = S_{i+1} \frac{A_i e^{-jk_0 x_{i+1}} - B_i e^{jk_0 x_{i+1}}}{Z_0} \quad (8)$$

where S_b , S_{i+1} and S_{cavi} are the cross-section areas of the entrance, exit and side-branch cavity in the i -th layer, Z_{si} describes the surface acoustic impedance of the rubber layers with fixed boundary and Z_0 is the characteristic acoustic impedance of water. To simplify the modeling, Z_{si} can be approximated using the plane wave model and expressed as follows:

$$Z_{si} = -jZ_{eq} \cot[k_{eq}(H - h_{i+1})] \quad (9)$$

where h_{i+1} and h_i are the radii of the inner annulus at the exit and entrance, the relationship between S_i and h_i is $S(x) = \pi h^2(x)$. Besides, $d+t = x_{i+1} - x_i$ is the width of the layers. Z_{eq} and k_{eq} are the effective characteristic acoustic impedance and wavenumber of the fixed boundary rubber layers.

The transfer matrix T_i for describing the sound pressure and volume velocity for each layer can then be written as:

$$\begin{bmatrix} p_i \\ U_i \end{bmatrix} = \mathbf{T}_i \begin{bmatrix} p_{i+1} \\ U_{i+1} \end{bmatrix} = \begin{bmatrix} \cos k_0 t & j \frac{Z_0}{S_i} \sin k_0 t \\ j \frac{S_i}{Z_0} \sin k_0 t & \cos k_0 t \end{bmatrix} \begin{bmatrix} \cos k_0 d & j \frac{Z_0}{S_{i+1}} \sin k_0 d \\ j \frac{S_{i+1}}{Z_0} \sin k_0 d & \cos k_0 d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{S_{\text{cavi}}}{Z_{\text{si}}} & 1 \end{bmatrix} \begin{bmatrix} p_{i+1} \\ U_{i+1} \end{bmatrix} \quad (10)$$

The initial vector $(p_1 \ U_1)^T$ of the entire system at the incident side can then be associated with the final vector $(p_{N+1} \ U_{N+1})^T$ at the REUSBH end. Consequently, the final system transfer matrix $\mathbf{T}$ can be obtained through multiplication of N matrices, as:

$$\begin{bmatrix} p_1 \\ U_1 \end{bmatrix} = \prod_{i=1}^N \mathbf{T}_i \begin{bmatrix} p_{N+1} \\ U_{N+1} \end{bmatrix} = \mathbf{T} \begin{bmatrix} p_{N+1} \\ U_{N+1} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_{N+1} \\ U_{N+1} \end{bmatrix} \quad (11)$$

The termination interface is rigid, which means the volume velocity $U_{N+1} = 0$ for a sound hard boundary. Substituting the boundary condition into Eq. 11, the specific surface acoustic impedance ratio of the entire REUSBH unit can be obtained as:

$$z = \frac{p_1}{U_1} = \frac{S_1}{Z_0} \frac{T_{11}}{T_{21}} \quad (12)$$

In accordance with specific surface acoustic impedance ratio z , the sound absorption coefficient of the REUSBH unit can be expressed as:

$$\alpha = \frac{4\text{Re}(z)}{(1 + \text{Re}(z))^2 + (\text{Im}(z))^2} \quad (13)$$

3.2. Rubber modeling

When the side and bottom boundaries of rubber are set as fixed, the displacement of particles in the rubber layer decreases near the fixed boundary, and the greatest displacement occurs in the central region. Additionally, rubber is a viscoelastic material characterized by a low shear modulus, which induces relative motion of its particles in the tangential direction. This phenomenon is analogous to the movement of air medium within slits and small holes, where acoustic energy is dissipated through viscous effects. Furthermore, the momentum equation for viscoelastic materials, after simplification, exhibits similarities to the equation governing viscous fluids. Consequently, the equivalent viscosity of rubber can be expressed as [41]:

$$\mu_{\text{eq}} = -j \frac{G(1 + j\eta)}{\omega} \quad (14)$$

where G , σ and η are the shear modulus, Poisson's ratio and loss factor of rubber respectively. The shear modulus G can be written as:

$$G = \frac{E}{2(1 + \sigma)} \quad (15)$$

where E is the Young's modulus of rubber.

Clearly, the equivalent viscosity is frequency dependent. Following the above discussion, the rubber layer with fixed boundaries can be considered as an equivalent viscous fluid filling the slit. The schematic diagram is presented in Fig. 4(c). The Johnson-Champoux-Allard (JCA) model can be utilized to determine the equivalent density of the medium within the slit to model the viscous effect, which can be expressed as [69]:

$$\rho_{\text{eq}} = \rho_1 \left[1 - \frac{3j}{\chi^2} \sqrt{1 + \frac{j\chi^2}{9}} \right] \quad (16)$$

where ρ_1 is the density of rubber. χ is the ring spacing to viscous boundary layer thickness, which can be expressed as:

$$\chi = \frac{d}{2} \sqrt{\frac{\omega \rho_1}{\mu_{\text{eq}}}} \quad (17)$$

Consequently, the equivalent characteristic acoustic impedance and wavenumber of this medium can be expressed as follows:

Table 4
Physical parameters of rubber.

Parameters	Values
Young's Modulus E (MPa)	5
Density ρ_1 (kg/m ³)	1100
Poisson's Ratio σ	0.49
Loss Factor η	0.3

$$Z_{eq} = \sqrt{\rho_{eq} S}, k_{eq} = \omega \sqrt{\rho_{eq} / S} \quad (18)$$

where S is the longitudinal wave modulus of rubber, which can be calculated as:

$$S = \frac{E(1 + j\eta)(1 - \sigma)}{(1 - 2\sigma)(1 + \sigma)} \quad (19)$$

The basic parameters are listed in Table 4.

3.3. FEM simulation model for validation

As shown in Fig. 5, a FEM simulation model is established in COMSOL Multiphysics to validate the analytical model. A 2D axisymmetric simulation is considered, which reduces computational cost as compared to a 3D model. The entire acoustic domain is discretized into triangular elements using FEM, where the maximum mesh size is determined based on the minimum wavelength or highest simulation frequency. For the acoustic domain, pressure acoustics modules are employed for modeling. The rubber and steel are modeled using solid mechanics modules. The thermo-viscous effect in water is omitted because its energy dissipation is much less than that provided by rubber. Its associated computational cost can be avoided, which leads to the same simulation results.

In the model, a Perfectly Matched Layer (PML) is implemented at the left boundary to eliminate any potential wave reflections. A background incident field with unity pressure amplitude is defined in front of the REUSBH structure to generate acoustic excitation. The acoustic field and vibration field in the fluid and structural domains within the REUSBH can then be determined, allowing us to calculate the specific surface acoustic impedance and sound absorption coefficient. Both rubber and steel components are modeled using solid mechanics modules, with the interface between the water region and the rubber/steel set as a structural-acoustic coupling boundary. The parameters are consistent with those in Table 4.

3.4. Validation

Based on the proposed theory and the established FEM model, the sound absorption coefficients of the REUSBH are calculated, which are compared in Fig. 6. Note that the FEM model considers two distinct cases, with the steel rings treated as rigid or elastic. According to Fig. 6, the theoretical results exhibit close agreement with the FEM simulation results, which validates the accuracy of the TMM modeling. Meanwhile, REUSBH exhibits broadband and highly efficient sound absorption capabilities without displaying pronounced resonance peaks associated with ring resonance. This indicates that, compared to the water-filled SBH, REUSBH achieves a significant enhancement in energy dissipation performance while effectively suppressing the structural-acoustic coupling effect. It can also be observed that treating the steel rings as either rigid or elastic does not affect the results, which supports our initial assumption. This observation leads to two hypotheses: first, the rubber significantly enhances energy dissipation, surpassing the effects of structural-acoustic coupling and dominating sound absorption performance. Second, the rings behave as rigid bodies with the addition of rubber, allowing the entire structure to achieve the ideal SBH effect. The following section will provide an in-depth analysis of the underlying mechanisms.

4. Mechanism study

4.1. Vibration reduction of rings after inserting rubber

To provide deeper insights, the structural-acoustic coupling effect within REUSBH is analyzed using FEM to quantify the effect of flexural vibration of steel rings and demonstrate the benefit of adding rubber. In the FEM model, the acoustic pressure field within the fluid medium is solved by the governing wave equation [70]:

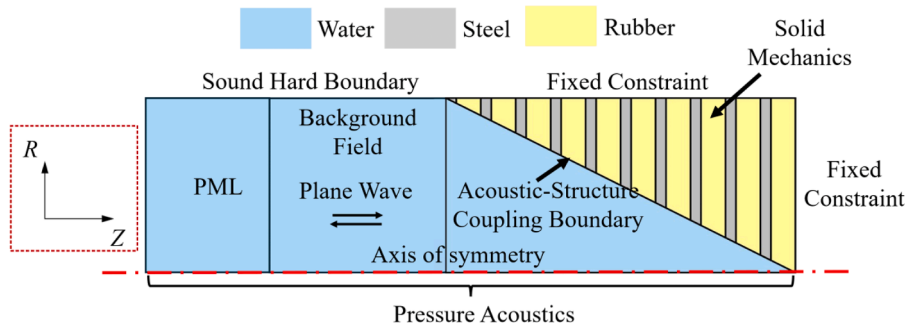


Fig. 5. 2D axisymmetric simulation model for the REUSBH.

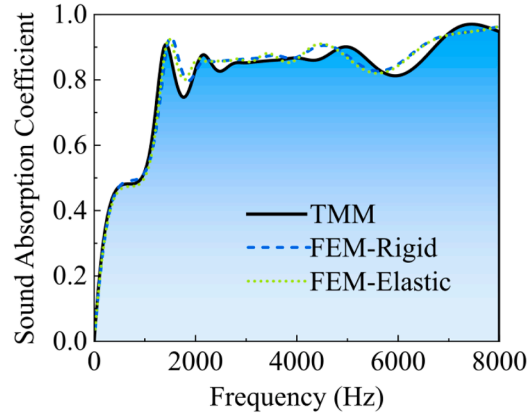


Fig. 6. Theoretical and simulation results of REUSBH.

$$\nabla^2 p = \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} \quad (20)$$

where p is the acoustic pressure and t represents the time and ∇^2 is the Laplace operator.

At the fluid-solid coupling interface, the normal particle vibration velocities in the fluid and structural domains are made equal, which can be expressed as:

$$-\nabla p = \rho_0 \frac{\partial u}{\partial t} \quad (21)$$

where u represents the particle displacement in either the structural or fluid domain.

The coupled equations governing the structural-acoustic interactions can be written as:

$$\begin{bmatrix} \mathbf{K}_s - \omega^2 \mathbf{M}_s & \mathbf{R}_1^T \\ -\rho_0 \omega^2 \mathbf{R}_1 & \mathbf{K}_f - \mathbf{C}_g - \omega^2 \mathbf{M}_f \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} \mathbf{F}_m \\ \mathbf{F}_p \end{bmatrix} \quad (22)$$

where $\mathbf{K}_s$ and $\mathbf{K}_f$ are the stiffness matrices of solid and fluid, respectively, $\mathbf{M}_s$ and $\mathbf{M}_f$ are the mass matrices of solid and fluid respectively, $\mathbf{F}_m$ and $\mathbf{F}_p$ are the external forces of solid and fluid respectively, $\mathbf{R}_1$ represents the coupling matrix of solid and fluid, $\mathbf{C}_g$ is the normal acoustic pressure gradient of the coupling boundary.

Finally, all components of the solid displacement vector are constrained to zero at the fixed boundary. This condition is expressed as

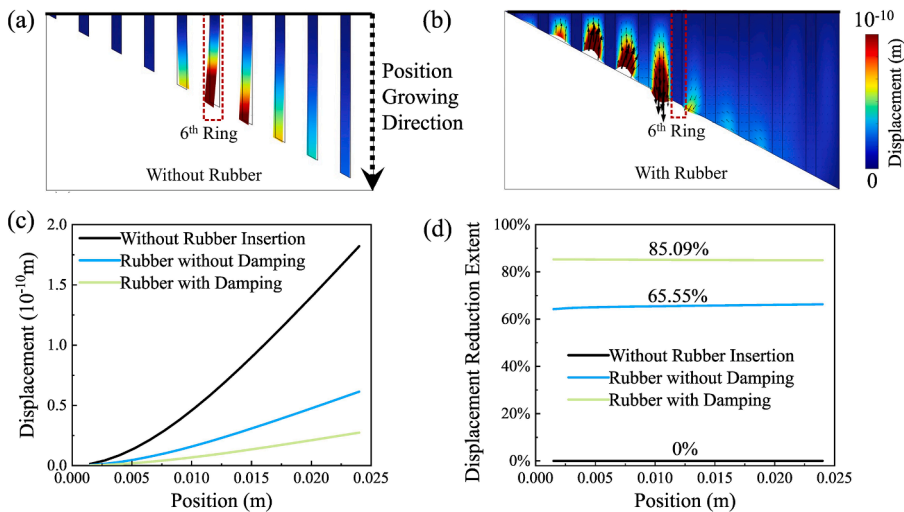


Fig. 7. Effect of rubber insertion on the flexural vibration of the steel rings. (a) Displacement field of the water-filled SBH without rubber at 3000 Hz. (b) displacement field of the REUSBH with rubber at 3000 Hz. (c) displacement of the sixth steel ring under three conditions: without rubber insertion, with rubber insertion but without material damping, and with rubber insertion including material damping. (d) corresponding percentage reduction in the average displacement of the steel rings.

$\mathbf{u} = 0$, with no tangential slip:

$$\mathbf{u} = 0 \quad (23)$$

where $\mathbf{n}$ is the normal vector. The rubber layers and the steel rings are assumed to be closely attached together without any gaps. As an example, the displacement fields for the steel rings in the REUSBH are extracted, and the cases with and without rubber are compared in Fig. 7(a and b). The displacement magnitude of each ring, particularly the sixth one, is greatly suppressed by rubber. This is because the rubber insertion connects all the steel rings to form an integrated sandwich structure, which significantly enhances its bending stiffness. Each steel ring is reinforced and more resistant to deformation. Additionally, the rubber can be viewed as constrained damping layers subjected to compressive and shear deformations. It inherently introduces damping to dissipate vibrational energy [70], which further suppresses the vibration magnitude.

To quantify the effect of vibration reduction, the percentage of displacement reduction ε for the steel ring is calculated as:

$$\varepsilon = 100\% \times (|\delta - \delta_0|/\delta_0) \quad (24)$$

where δ_0 is the average displacement of steel rings without rubber insertion, and δ is the displacement with rubber insertion. As shown in Figs. 7(c-d), the addition of rubber significantly reduces the flexural vibration of the steel rings. In Fig. 7(c), when the loss factor of rubber is zero (without damping), the reduction extent reaches 65.5%, meaning that the higher bending stiffness reduced vibration. In Fig. 7(d), after introducing the loss factor, the percentage of reduction further increases to 85%. This highlights that both higher stiffness and damping contribute to vibration reduction. It is also noted that the ring vibration has decreased far below the vibration amplitude of the rubber, which not only ensures the rigidity assumption in the theoretical framework but also positions the rubber as the primary medium for energy dissipation. Based on the visualized vibration field, the ring vibrations at different frequencies have been evaluated, exhibiting similar behavior. It is expected that less flexural vibration of the internal rings recovers the benefits of SBH effect, thereby leading to higher sound absorption ability.

4.2. Energy flow

The energy flow also serves as an important indicator to elucidate the absorption mechanism. Figs. 8(a-b) depict the energy flow within the REUSBH and the water-filled SBH at 840 Hz and 6560 Hz, respectively. At 840 Hz, the water-filled SBH achieves a resonant absorption peak due to the vibration modes of the last three rings, as can be seen from Fig. 2(c). When no rubber is inserted, the acoustic energy mainly accumulates near the resonance rings, associated with effective energy exchange between the fluid and structure domains. This structural-acoustic coupling effect contributes to resonant sound absorption. When rubber is added, the energy dissipation mainly occurs in the rubber domain. The energy vectors basically do not flow toward the steel rings, as the rubber modulus is closer to water than steel, such that wave propagation in rubber is more favorable. This also reduces the vibration energy contained

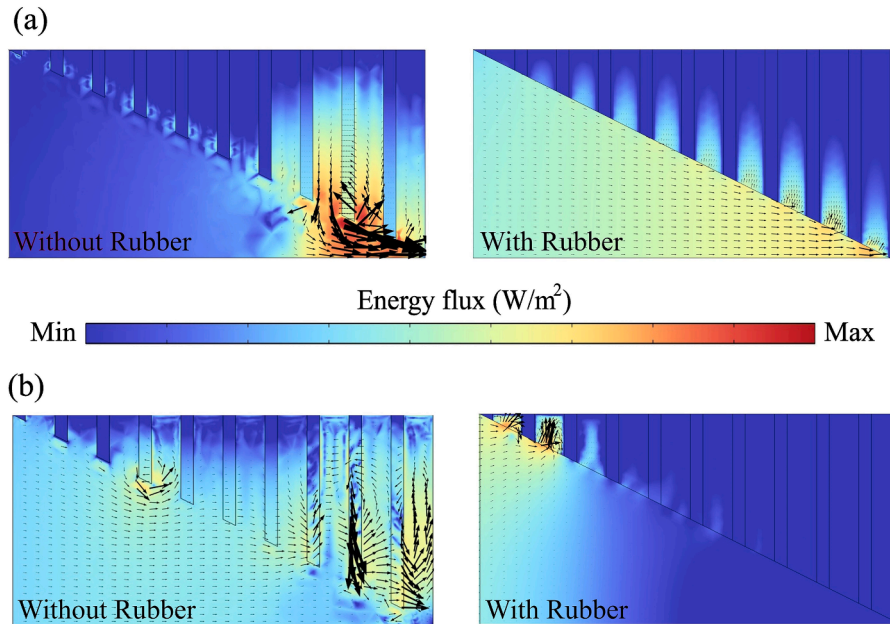


Fig. 8. Comparison of energy-flow characteristics in the water-filled SBH with and without rubber insertion. (a) Energy-flux distributions at 840 Hz. (b) energy-flux distributions at 6560 Hz. The color contours indicate the magnitude of the energy flux, while the arrows indicate the energy-flux vectors.

in the steel rings.

At 6560 Hz, Fig. 8(b) shows a similar trend. A prominent difference lies in the main energy accumulation and dissipation areas. Without rubber, energy mostly focuses near the last several steel rings, while with rubber, the energy accumulation and dissipation areas move to the entrance of REUSBH, indicating a more holistic and uniform effect provided by the entire structure.

4.3. Ideal sound absorbing conditions

Apart from mitigating the adverse effects of ring vibrations, the rubber also significantly enhances the damping properties within the structure. As a viscoelastic material, rubber exhibits a low modulus and high loss factor, enabling it to function like a highly viscous fluid. To analyze the acoustic characteristics of REUSBH, we calculate the specific acoustic impedances and perform the complex frequency plane analysis. The results below the cut-off frequency of the incident duct, calculated as $f_{\text{cut-off}} = 0.29c_0/H = 8640$ Hz. The upper limit of calculation is set at 8000 Hz, are presented in Fig. 9.

Fig. 9(a) compares the sound absorption coefficient of a REUSBH and a rubber absorber alone, where the rubber parameters and dimensions are kept the same. REUSBH is significantly better than the rubber alone, with a sound absorption coefficient exceeding 0.8 at most frequencies and achieving average value close to 0.9. This is because REUSBH homogenizes multiple resonances to establish the slow-sound effect, while the rubber alone has only a singular resonance mode. Moreover, the absorption curve of REUSBH also resembles that of typical airborne SBH, demonstrating the advantage of incorporating rubber into the water-filled SBH and achieving the desired performance. Furthermore, to better understand absorption conditions, the surface acoustic impedance of the REUSBH is presented in Fig. 9(b). The acoustic resistance and reactance vary rapidly in the low frequencies, converge from positive and negative infinity to about 0.5 and 0, respectively. As the frequency increases, they quickly stabilize at a relatively ideal absorption condition, leading to excellent sound absorption.

To better understand the role of the resonance modes, complex frequency analysis is conducted by calculating the reflection coefficients with varying imaginary part of frequency. As shown in Fig. 9(c), eight resonant modes emerge clearly within the calculated frequency range, with all zeros labeled sequentially from A to H. Notably, all zeros lie above the real axis, indicating that all modes are in an overdamped state. The lower frequency modes (A-F) are densely distributed, and the zero-pole pairs are close to the zero-axis, suggesting lower radiation losses. The close proximity of the modes and their overdamped nature ensure a high sound absorption coefficient. Consequently, a platform emerges in the 2000–4000 Hz range. At higher frequencies, G and H resonance modes are sparser, and the distance between the zeros and poles is greater. The radiation loss is greatly compensated by the loss provided by the rubber, leading to a strongly overdamped condition and high sound absorption. The above observations are consistent with the sound absorption coefficient curve, and the characteristics of REUSBH closely resemble those of an airborne SBH, particularly with porous material insertions [71]. It should be recalled that for a sound absorbing structure in air, an overdamped state is typically associated with a normalized surface resistance exceeding 1. For REUSBH, it is observed from the above analysis that its normalized resistance is below 1. This can be attributed to the significant difference in modulus between the incorporated rubber and water, in contrast to the case in air where the modulus of commonly used sound absorbing materials is close to air.

4.4. Analysis of energy dissipation mechanism

The above analyses have mostly confirmed the desired acoustic properties of REUSBH, resembling those previously found in airborne SBHs. To further elucidate the sound absorption mechanism of REUSBH, the responses at the absorption peaks [zeros in Fig. 9 (c)], including the displacement field, shear strain and energy dissipation density within the REUSBH, are presented in Figs. 10–12, respectively. As shown in Fig. 10, the displacement of rubber is significantly higher than that of the steel, affirming the rigidity assumption made in the theoretical model and the vibration reduction analysis in Section 4.1. The main displacement directions of the rubber are visualized, showing that the primary movement is along the lateral R direction, with minimal axial displacement in the Z

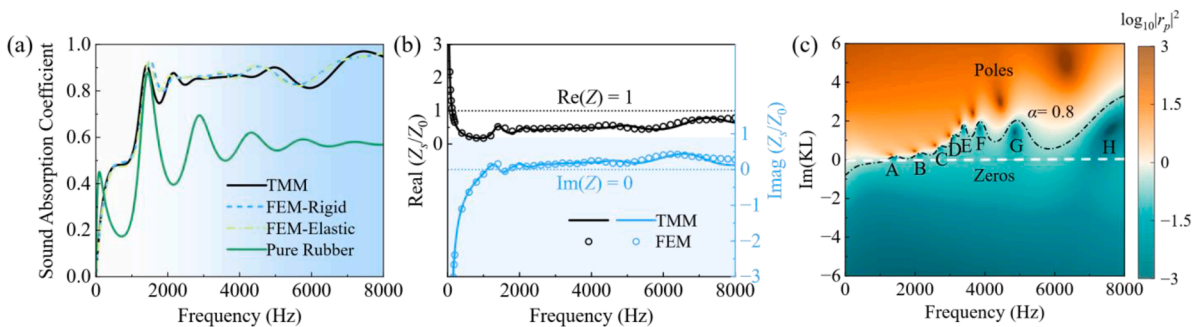


Fig. 9. Sound absorption performance, surface impedance, and reflection coefficient distribution on complex frequency plane of the REUSBH. (a) Theoretical and FEM results for the sound absorption coefficients of the REUSBH and a pure-rubber absorber. (b) theoretical and FEM results for the normalized surface impedance of the REUSBH. (c) complex frequency plane analysis of the REUSBH, showing the zero-pole pairs of the reflection coefficient.

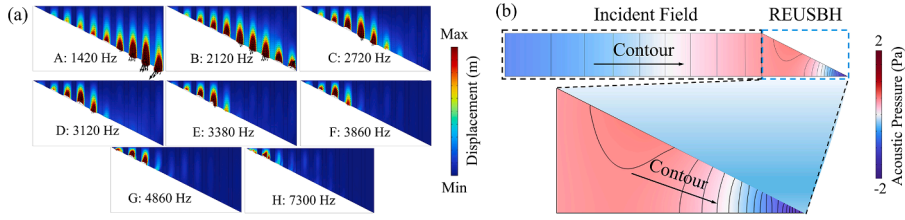


Fig. 10. Displacement distributions and representative acoustic pressure field (1420Hz) of the REUSBH. (a) Displacement magnitude of REUSBH at different frequencies corresponding to zero points mentioned in Fig. 9(c). Arrow flows are used to indicate the displacement directions. (b) Representative acoustic pressure field distribution at 1420 Hz.

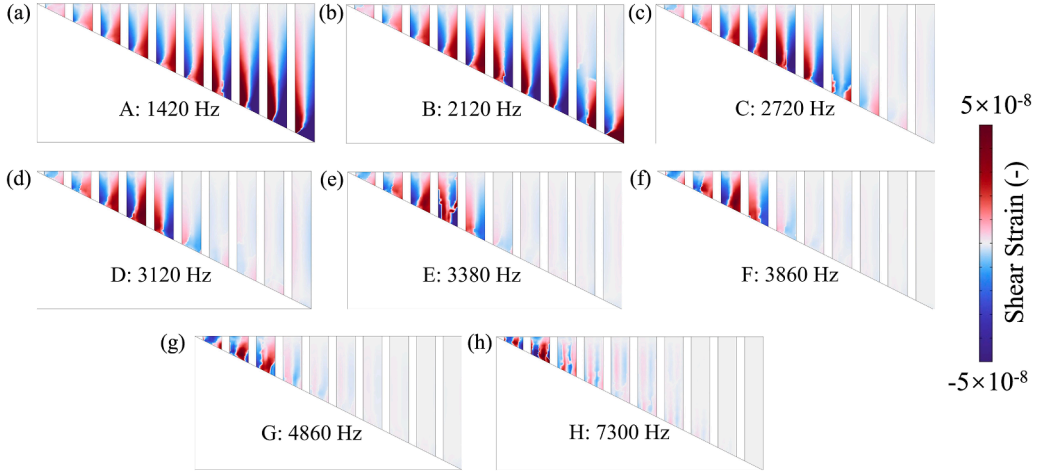


Fig. 11. Shear (R-Z) strain distribution diagrams of REUSBH at eight frequencies corresponding to zero points mentioned in Fig. 9(c): (a) 1420 Hz. (b) 2120 Hz. (c) 2720 Hz. (d) 3120 Hz. (e) 3380 Hz. (f) 3860 Hz. (g) 4860 Hz. (h) 7300 Hz.

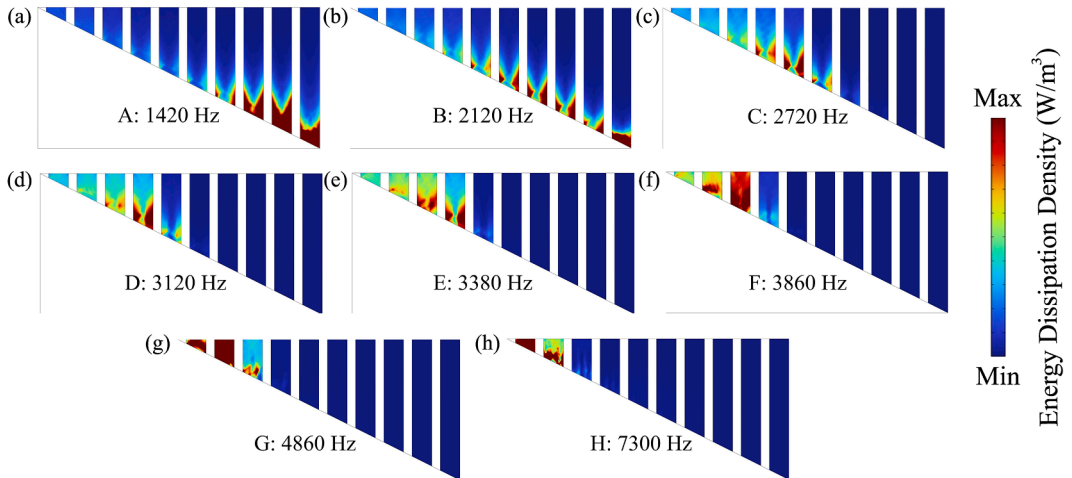


Fig. 12. Energy dissipation density distribution diagrams of REUSBH at eight frequencies corresponding to zero points mentioned in Fig. 9(c): (a) 1420 Hz. (b) 2120 Hz. (c) 2720 Hz. (d) 3120 Hz. (e) 3380 Hz. (f) 3860 Hz. (g) 4860 Hz. (h) 7300 Hz.

direction. Note that the lateral direction of the REUSBH is along the longitudinal motion direction of each rubber layer, thus confirming that local rubber motion contributes to strong energy dissipation and sound absorption. The global and local vibration patterns in the individual rubber layers, conducive to the effective absorption at low and high frequencies, respectively, are also observed clearly, confirming the analysis conducted in Fig. 10.

The displacement diagrams illustrate that the primary vibration mode of the rubber is longitudinal vibration. It also indicates relative motion of the rubber layer against the steel-ring boundaries, where the displacement gradient is higher at their separation interface. This leads to the generation of shear motion and causes significant shear strain. As considerable impedance mismatch also exists between the rubber and the steel ring, wave mode conversion occurs at their interface, facilitating the transformation of longitudinal waves into shear waves. Due to the shorter wavelength of shear waves, they are more susceptible to energy dissipation. To illustrate this, Fig. 11 plots the shear strains within the rubber layers, which are closely related to the energy dissipation density in Fig. 12. The shear strain is higher at the interface between the steel ring and the rubber, indicating that energy dissipation primarily occurs through elastic deformation. The mechanism of energy dissipation primarily stems from the viscoelastic properties of rubber, which converts sound wave energy into heat through internal thermal viscosity and relaxation processes. As frequency increases, the rate of interactions increases so that less displacement is required to exhibit high energy dissipation. This explains the reduced rubber displacement at higher frequencies while the resulting sound absorption is still high. At low frequencies, the strain exhibits a single-mode pattern, which becomes more complicated as the frequency increases, potentially leading to enhanced energy dissipation. In the axial Z direction, certain regions of the rubber layer are in direct contact with water, resulting in localized bending vibrations that contribute to additional energy dissipation.

In airborne SBH, acoustic energy is dissipated through the viscous and heat conduction effects caused by motion gradients within narrow regions. Similarly, REUSBH dissipates acoustic energy through shear strain resulting from impedance mismatch and relative longitudinal motion, as well as normal strain induced by longitudinal displacement. These mechanisms suggest that the sound absorption in both systems shares common characteristics. Furthermore, the slow-wave effect can also be clearly observed. Fig. 10(b) shows the acoustic pressure field associated with the first mode at 1420 Hz (mode A in Fig. 9(c)). The color map shows the real part of the complex acoustic pressure, where the distances between contours denote the wavelength variations. In the incident duct, prior to entering into the REUSBH region, the contour lines are straight, indicating uniform sound speed. After the wave enters the REUSBH, the spacing between adjacent contours becomes narrower toward the tapered termination, indicating strong slow-sound effect with reduced wavelength. Observations at other frequencies of interest and modes show generally similar behavior, confirming the existence and the effect of slow sound. The energy focusing effect is illustrated in Fig. 8. At low frequencies, most of the energy is concentrated in the main duct and gradually accumulates with increasing propagation distance, peaking at the terminal end of the REUSBH. During the transition from global to local effects, energy continues to propagate to the end of the structure, but the primary energy concentration region progressively shifts toward the front portion of the REUSBH.

To gain a better understanding of the energy dissipation mechanism, we calculated the P-wave-related dissipation power and the S-

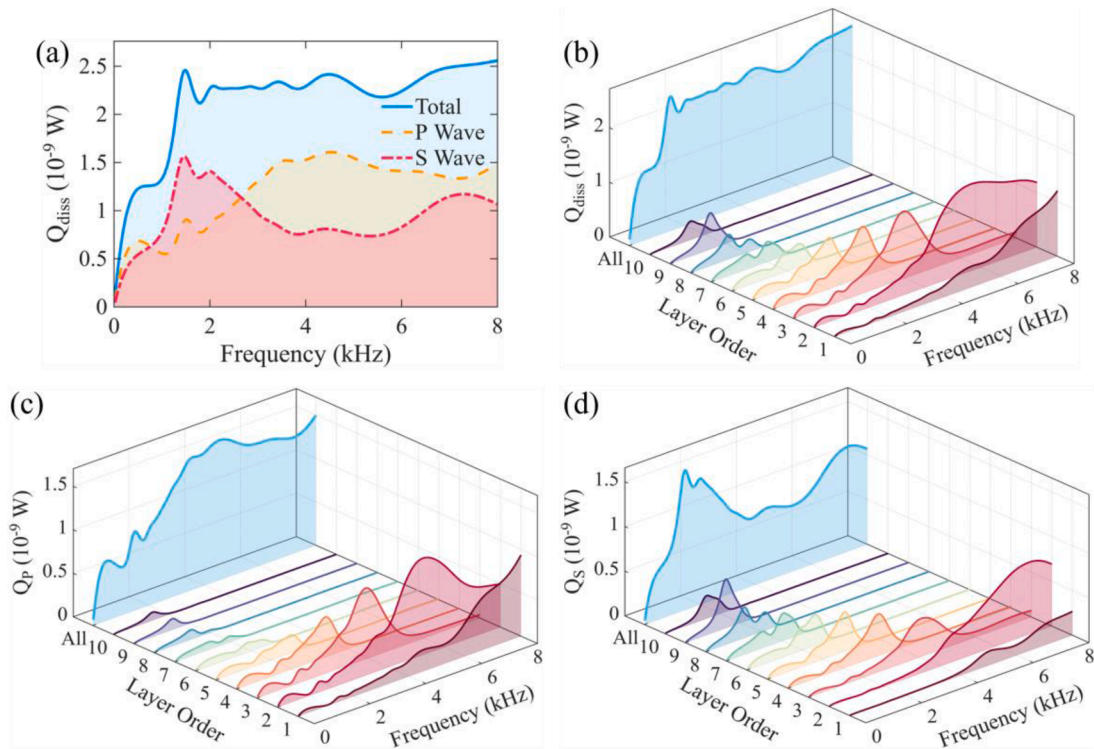


Fig. 13. Frequency-dependent energy dissipation mechanisms and layerwise contributions in the REUSBH. (a) Total dissipation, P-wave-related dissipation and S-wave-related dissipation power within the REUSBH at different frequencies. (b) Energy dissipation powers between REUSBH and individual rubber layers. (c) P-wave-related dissipation power distribution in each layer. (d) S-wave-related dissipation power distribution in each layer.

wave-related dissipation power, respectively. Specifically, the P-wave-related dissipation power Q_P is governed by the complex longitudinal modulus S of the rubber, whereas the S-wave-related dissipation power, Q_S , is governed by the complex shear modulus G of the rubber. The total energy dissipation power Q_{diss} is obtained by summing the two contributions. These parameters can be expressed as follows [72]:

$$Q_P = \int \int \int_V \pi f \text{Im}(S \varepsilon_v \varepsilon_v^*) \quad (25)$$

$$Q_S = \int \int \int_V \pi f \text{Im}(2G \varepsilon_{\text{dev}} : \varepsilon_{\text{dev}}^*) \quad (26)$$

$$Q_{\text{diss}} = Q_P + Q_S \quad (27)$$

where ε_v and ε_{dev} are the volumetric strain energy and deviatoric strain energy, $*$ is the conjugate symbol.

It can be observed from Fig. 13(a) that, in the low-frequency range up to approximately 1.5 kHz, the total dissipation increases rapidly and reaches a pronounced peak. This peak is mainly contributed by Q_S . This behavior can be attributed to the relatively long acoustic wavelength in this frequency range, under which the structure exhibits a strong global response. The acoustic energy can continue to propagate toward the rear part of the REUSBH and accumulate near the terminal region. Meanwhile, the rubber layers near the rear end are deeper and have a larger effective compliance, leading to relatively large displacements under the low-order global mode. Since the rubber in this region is strongly constrained by the steel rings, both longitudinal and tangential motions of the terminal rubber layers become evident, producing significant shear strain near the steel–rubber interfaces. As a result, Q_S dominates in this frequency range. This interpretation is consistent with the displacement fields shown in Fig. 10 and the energy dissipation density distributions shown in Fig. 12.

In the frequency range of approximately 2–5 kHz, Q_P becomes noticeably larger than Q_S , while the total dissipation power remains at a relatively high but smooth plateau. The dominant mechanism in this range is more closely associated with the overlapping localized longitudinal responses of multiple rubber side-branch cavities. In other words, the longitudinal modes of different rubber layers are successively excited, leading to enhanced local pressure fields and volumetric strain. Therefore, Q_P , which is governed by the complex longitudinal modulus, becomes the main dissipation component. Owing to the relatively dense distribution of these localized modal frequencies and the high material damping of the rubber, individual resonance peaks are broadened and overlap with one another. Consequently, the total dissipation power does not appear as a series of sharp peaks, but rather as a broadband plateau. Above 5 kHz, Q_P decreases to some extent or tends to become relatively stable, whereas Q_S gradually increases again. This suggests that, at higher frequencies, the displacement field inside the rubber develops higher-order spatial variations, accompanied by more pronounced wavelength compression. The alternating tensile–compressive deformation near the steel–rubber interfaces becomes more rapid, and the displacement gradient increases, thereby strengthening the shear strain again. The above analysis is also consistent with the displacement fields in Fig. 10 and the energy dissipation density distributions in Fig. 12.

Note that the rubber layers are numbered from entrance to termination along the incident wave direction, according to the schematic shown in Fig. 4(a). The energy dissipation powers (Q_P , Q_S and Q_{diss}) in the individual layers are calculated and their ratios are determined, as presented in Fig. 13(b–d), respectively. In Fig. 13(b–d), at relatively low frequencies below 2200 Hz, all the layers contribute together to the energy dissipation, each providing a portion between 5% and 20%. This means that low-frequency absorption is achieved as an overall effect of all layers, which also suggests that the slow-sound induced absorption is a global effect. The energy dissipation is also stronger near the termination of REUSBH.

At higher frequencies above 3600 Hz, the energy dissipation primarily occurs in the first several layers near the entrance. This indicates that once the slow-sound effect has been established, one of the frontal layers can dominate the sound absorption and the proportion is progressively increasing, in contrast to the global behavior observed at low frequencies. An ideal sound absorption condition is easier to establish. Meanwhile, the shorter resonators at the front have higher eigenfrequencies, thus contributing more to the locally induced absorption at higher frequencies. This phenomenon resembles the well-known rainbow trapping effect. As the sound wave propagates, resonators with matching frequency extract energy progressively from the waveguide, preventing propagation further downward. Therefore, it is concluded that the proposed structure globally retards sound waves. At low frequencies, dissipation is contributed by all layers, stronger at the termination end. At high frequencies, SBH effect is fully established and effective dissipation can be readily achieved in the first several layers. The transition between global and local effects is observed in the mid-frequency range starting at around 3000 Hz.

By comparing Figs. 9(c) and 13(b), we observe that the zero-pole pairs, which are associated with the local modes, can be related to the peak frequencies of the dissipation curves, especially for the first several layers at high frequencies. This again affirms the local sound absorption mechanism in this range. It is worth noting that the positions of zeros are slightly offset from the poles. This is because the poles indicate the original local modes, while the zeros represent frequencies at which minimum reflection occurs. The maximum energy dissipation is dominated by a single layer and influenced by several adjacent layers, resulting in a slight shift in the positions of the zeros.

Overall, the energy dissipation mechanisms of REUSBH share many common features with airborne SBH, particularly because rubber insertion effectively suppresses unwanted structural vibrations and vibro-acoustic coupling. The original airborne SBH dissipates energy via the viscous effect and heat conduction in the narrow regions between the rings, while REUSBH dissipates energy

through shear strain generated by impedance mismatch and the relative motion. In both systems, energy dissipation primarily occurs near the walls, suggesting similar fundamental principles. Additionally, the advantages of the slow-wave effect can be observed, resulting in lower absorption frequency and higher absorption magnitudes. The proposed structure retains the wave propagation characteristics of conventional SBH, leading to gradually reduced wave speeds. As a result, the wavelength compresses, causing energy to accumulate progressively during propagation. Energy dissipation shows global behavior at low frequencies, and the high absorption zone is concentrated at the end of the REUSBH. A transition from global to local effect exists, with the primary regions of energy focusing and dissipation gradually moving from the end to the frontal layers. At high frequencies, the rainbow trapping effect is observed, where energy is predominantly localized and dissipated in the resonating layers. Compared with similar underwater sound absorber studies in Refs. [42–43], the structural composition and modeling methods are relevant, but the underlying sound absorption mechanisms are fundamentally different. In Ref. [42], sound energy is mainly dissipated through large shear deformation induced by the impedance mismatch between the rubber and the steel plate. In Ref. [43], the high stiffness and dense arrangement of the metal rings constrain the vibration of the rubber in the regions near the rings, thereby converting longitudinal waves in the rubber into transverse waves. In our present work, the SBH mechanism, namely the slow-sound effect, is exploited to enhance the energy dissipation in each rubber layer. Despite the similar damping role of the rubber, we demonstrate that integrating it into an SBH solves the limitations that hinder the underwater application of conventional SBH structures, including the detrimental structural-acoustic coupling effect and insufficient underwater damping.

5. Parametric study

The rubber parameters can significantly influence the sound absorption performance of REUSBH. In this section, we provide a concise analysis of key rubber parameters, including Young's modulus, loss factor, Poisson's ratio, and density. The baseline parameters are consistent with those used in previous analyses, with one parameter varied at a time. The absorption coefficient is then plotted as a colormap to illustrate its impact. The regions with absorption coefficient greater than 0.8 are identified and highlighted.

As illustrated in Fig. 14(a), an increase in the Young's modulus of the rubber results in a greater overall stiffness of the system, leading to a shift in effective frequencies toward higher frequencies. This also improves impedance matching between the system and water, thereby maintaining excellent sound absorption performance in the mid to high frequency range. However, to ensure effective low-frequency performance, the rubber selected should be less stiff. Fig. 14(b) shows that a higher loss factor generally leads to enhanced sound absorption, particularly at the resonant frequency. Therefore, rubber with high loss is preferred. The effect of Poisson's ratio as shown in Fig. 14(c) is also significant, as it could change the deformation mode of the constrained rubber layers. However, its impact is less pronounced compared to the Young's modulus and loss factor. Fig. 14(d) depicts the effect of rubber density, where a reasonable range between 600 and 1800 kg/m³ is considered. Unexpectedly, the influence of density is not very significant, with only slight differences at mid-to-high frequencies. This suggests that rubber with varying densities can acoustically match with water over a wide range, allowing selection based on other factors such as weight and strength.

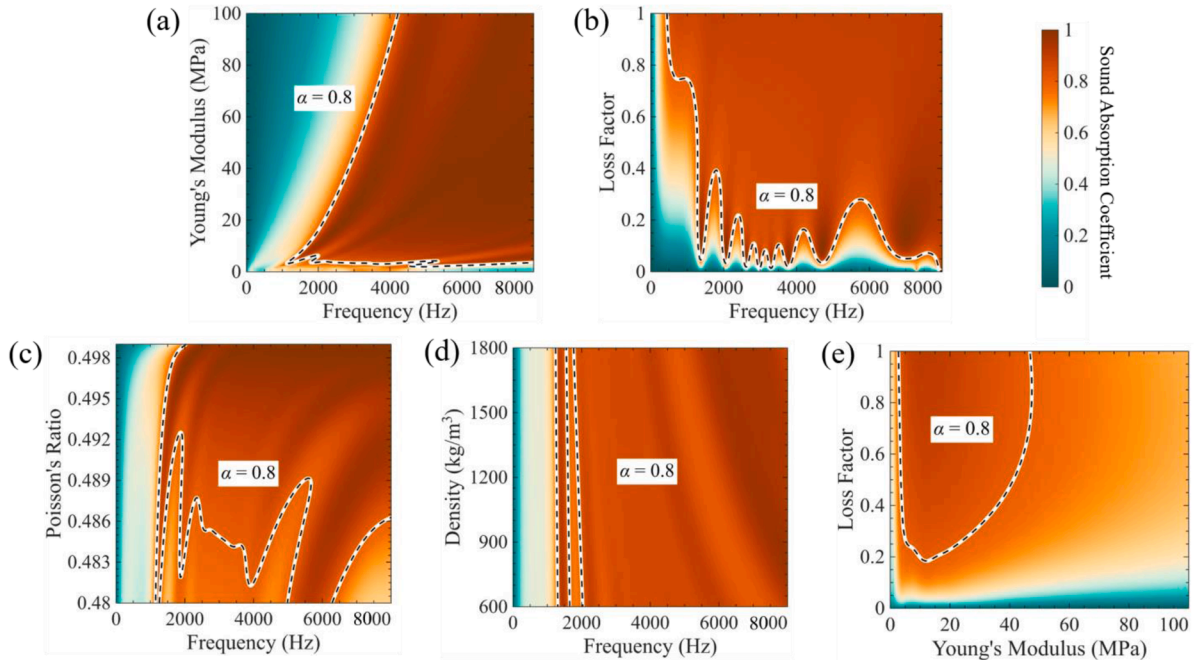


Fig. 14. Effect of physical parameters on sound absorption coefficient for REUSBH. (a) Young's modulus. (b) Loss factor. (c) Poisson's ratio. (d) Density, and (e) Effect of both Young's modulus and loss factor on average sound absorption coefficient of REUSBH in the entire frequency range.

Based on the analyses above, the Young's modulus and loss factor are the most critical factors in determining the sound absorption performance of REUSBH. Both parameters are varied together, and the average sound absorption coefficient across the entire frequency range of interest is compared. Fig. 14(e) illustrates that the optimal conditions occur when the rubber stiffness is relatively low and the loss factor is high. A wide range of parameters can ensure that the average sound absorption coefficient exceeds 0.8 across broadband frequencies, demonstrating the significant potential of the proposed structure. In addition to rubber parameters, the layout and geometric configurations of REUSBH provide abundant design flexibility. Future studies warrant systematic investigations and optimizations using tailored algorithms.

Fig. 15(a) presents the sound absorption coefficients of REUSBHs with different total thicknesses while the number of internal rings is kept unchanged. Increasing the thickness improves the low-frequency absorption and shifts the effective absorption range toward lower frequencies, whereas the high-frequency performance remains generally satisfactory. This trend is attributed to the axial resonance that governs the low-frequency response. Fig. 15(b) shows the results for different numbers of rings at a fixed total thickness. Reducing the number of rings lowers the structural stiffness and shifts the first absorption peak toward lower frequencies. However, it also reduces the number of resonant units, enlarges the frequency intervals between their effective operating ranges, and introduces pronounced absorption valleys at higher frequencies. In contrast, increasing the number of rings improves the mid- and high-frequency absorption continuity, although the low-frequency performance becomes less favorable.

6. Experimental Verification

To verify the modeling accuracy and sound absorption performance of the proposed REUSBH, samples were fabricated and underwater sound absorption tests were conducted using a water-filled impedance tube as illustrated in Fig. 16. The test method follows a combination of the dual hydrophone method and the pulse method [73–74]. The dual hydrophone method is applied below 6000 Hz, whereas the pulse method is used above this frequency. The entire testing system, as shown in Fig. 16(a), is carefully calibrated to ensure high accuracy and repeatability.

During the test, the signal generator produces a signal, which is transmitted to the underwater transducer through a power amplifier. The transducer generates a plane wave within the tube, and the two hydrophones capture the signal to compute the absorption coefficient using the measured transfer function. At high frequencies, the transducer emits pulse signals (with a center frequency of 7 kHz, a 3 dB bandwidth of 3–9 kHz, and a duration of 0.5 ms) and the reflected pulses from the sample surface in the time domain are used to obtain the absorption coefficient. The inner diameter of the tube is 120 mm, and the total length of the tube is 10 m. The dimension of the tested REUSBH sample is slightly modified to fit the diameter of the testing tube, with a diameter $2H$ of 114 mm and a length L of 70 mm. The ring number N is adjusted to 7, with the ring thickness remaining unchanged. The tested sample is 30 mm shorter than the one considered in prior theoretical analysis, and simulation was performed again according to the updated dimensions. The wall thickness of REUSBH was set at 2 mm to ensure rigidity, and its cross-section matched well with the tube with a small gap of 1 mm to balance the water pressure. The outer shell and inner walls of the REUSBH were 3D printed using 304 stainless steel to ensure high precision and structural rigidity.

Then, according to the parametric study, a polyurethane (PU) rubber was selected and filled into the steel frame through vacuum injection technology. In fact, the Young's modulus and loss factor of the rubber may vary with respect to frequency, so an accurate description of these parameters is crucial for the simulation and test results of sound absorption. We used a Dynamic Mechanical Analyzer (DMA) (Model Mettler Toledo DMA1) as shown in Fig. 16(b) to conduct dynamic mechanical parameter analysis on the selected PU rubber. The temperature during the water-filled impedance tube measurements was 10 °C and this temperature is selected as the reference temperature of DMA tests. The dynamic parameters of the PU rubber from 1 to 10000 Hz were inferred using the Time-Temperature Superposition (TTS) principle [75–76], and the results are shown in Fig. 16(c). Clearly, the selected PU rubber has a desired performance, making it suitable for use in REUSBH. Finally, the density and Poisson's ratio of the PU rubber were precisely measured using an electronic scale and a tensile machine, which were 981 kg/m³ and 0.494, respectively.

To avoid the interference caused by bubbles during the experiment, the sample was immersed in water for 24 hours. Subsequently, it was placed in the tube and allowed to stand for an additional 24 hours prior to conducting the test. The results of the underwater sound absorption test are presented in Fig. 17(a). The sound absorption coefficients calculated through theoretical and simulation methods, along with those obtained from experiments, exhibited a high degree of consistency within the analyzed frequency range. These results display only minor discrepancies within a narrow frequency range, yet they all demonstrate that the proposed REUSBH possesses excellent sound absorption performance. These discrepancies primarily stem from processing errors during sample preparation, size constraints, and certain non-rigid boundary effects induced by the gaps on the circumferential sides. Based on the experimental findings, its average sound absorption coefficient above 1000 Hz is 0.9246, and it sustains a sound absorption coefficient exceeding 0.9 above 1700 Hz. In conclusion, the experimental results have verified that the proposed REUSBH exhibits excellent underwater sound absorption performance, and they have also confirmed the correctness of the proposed modeling method.

Moreover, experiments concerning the effect of hydrostatic pressure on the tested sample were carried out, and the results are shown in Fig. 17(b). Increasing the hydrostatic pressure has very limited influence on the sound absorption performance above 4000 Hz, which is desirable. Below this frequency, however, the absorption performance drops as pressure increases. This deterioration can be attributed to the direct contact between the PU rubber and water, leading to a higher Young's modulus of the rubber at compressed state and a lower loss factor. Since this study does not specifically target pressure-immune performance, we did not perform further optimization to broaden the pressure range of the REUSBH sample. Future work will focus on designing structures with improved hydrostatic pressure resistance and obtaining precise measurements of the rubber's dynamic properties under varying pressure conditions.

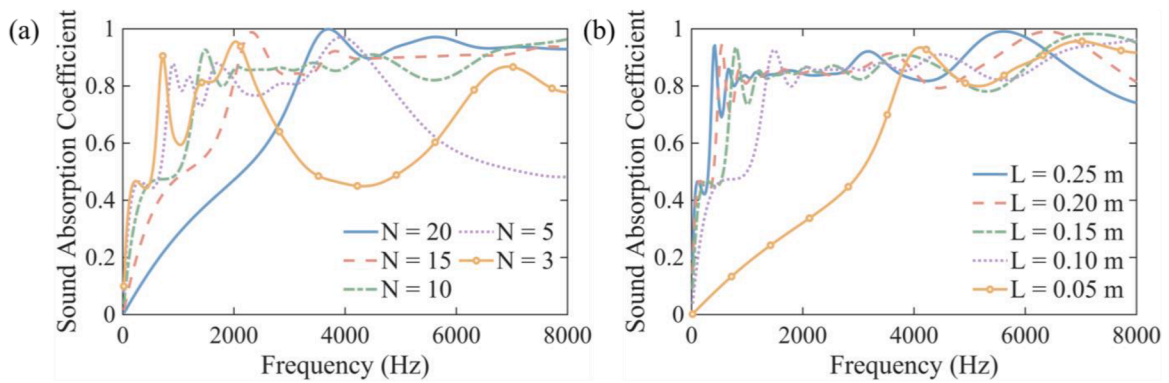


Fig. 15. Effect of geometric parameters on sound absorption coefficient for REUSBH. (a) Total thickness. (b) Ring number.

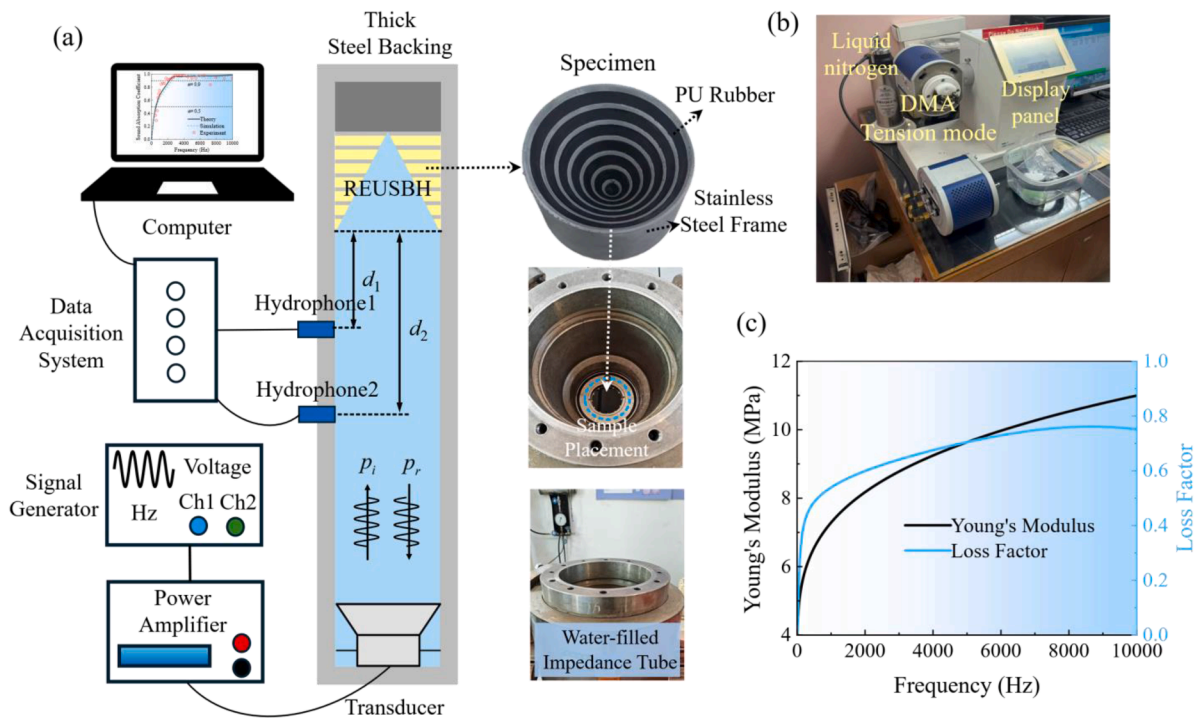


Fig. 16. Experimental setup, fabricated specimen, and dynamic mechanical characterization of the selected PU rubber. (a) Setup diagram of the water-filled impedance tube for underwater sound absorption test and specimen. (b) DMA test for the selected PU rubber. (c) Measured dynamic mechanical parameters (Young's modulus and loss factor) for the tested PU rubber at the reference temperature of 10 °C.

7. Conclusions

In this paper, an underwater sound absorber based on the Sonic Black Hole (SBH) principle is proposed and investigated. The Rubber Enhanced Underwater SBH (REUSBH) inherits the classical SBH configuration, which involves inserting steel rings with varying cross-sectional dimensions at regular intervals into a circular waveguide. The inner part is filled with water, and rubber materials are inserted into the side cavities formed by the steel rings. Rubber primarily provides two functions: First, it reduces the structural-acoustic coupling induced ring vibrations, thereby retaining the desired slow-sound effect and energy accumulation. Second, it greatly enhances the internal damping, thereby improving the energy dissipation. A theoretical model for predicting the acoustic performance of REUSBH is developed based on the transfer matrix method. The model assumes rigid steel rings and treats the rubber layers between steel rings as an equivalent fluid medium. The equivalent density and longitudinal wave modulus are derived, which allows for the subsequent calculation of surface impedance. A 2D axisymmetric FEM model is developed to validate the theoretical model, and the sound absorption coefficients are computed under various conditions. The theoretical results and FEM agree well, which validates the proposed model and theory.

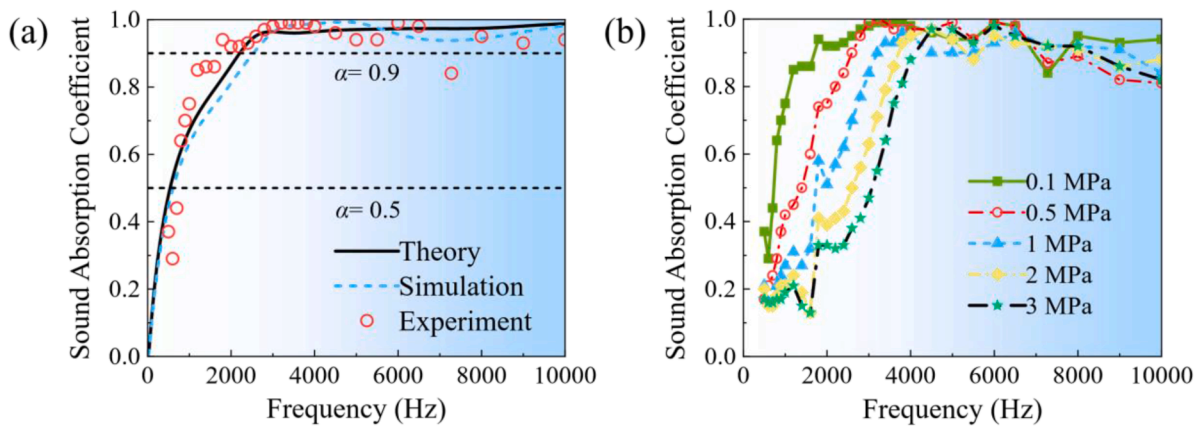


Fig. 17. Experimental validation of the REUSBH and its underwater sound absorption performance under hydrostatic pressure. (a) Comparison of the theoretically predicted, simulated, and measured sound absorption coefficients of the REUSBH under rigid backing. (b) measured sound absorption coefficients of the REUSBH under hydrostatic pressures of 0.1, 0.5, 1, 2, and 3 MPa.

The sound absorption mechanisms of the proposed REUSBH have been comprehensively analyzed. It is found that the rubber insertion mainly provides shear restoring force and damping to the steel rings, thereby suppressing their vibrations. The results affirm the rigidity assumption made for the steel rings in the theoretical formulation. In terms of sound absorption, REUSBH performs significantly better than a pure rubber layer with the same size and physical parameters, owing to the slow-wave-induced effects. The surface impedances, complex frequency representations and rubber deformation modes have been analyzed to elucidate the dominant energy dissipation mechanisms. It is shown that sound absorption exhibits a global-to-local transition from low to high frequencies, and the resonance modes and rubber deformations exhibit distinct behaviors. Energy dissipation is primarily through longitudinal movement of rubber layers and shear strain near the interface between the steel rings. The slow-wave effect leads to effective sound absorption at lower frequencies. The selection of rubber parameters significantly affects sound absorption performance. Among these, Young's modulus and loss factor have the greatest influence, while Poisson's ratio and density have a lesser effect. To achieve a high overall average sound absorption, a low modulus and a high loss factor are preferred. Finally, based on the parametric study, a suitable polyurethane (PU) rubber was selected, and its dynamic mechanical parameters were measured via dynamic mechanical analyzer (DMA). Test specimens were also prepared. The test results obtained from the water-filled impedance tube have verified its outstanding sound absorption performance and the accuracy of the proposed modeling method.

This work represents an initial effort to adapt the concept of SBH for underwater environments, introducing a novel strategy for developing underwater sound-absorbing materials and devices.

CRediT authorship contribution statement

Yuanze Li: Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Investigation, Conceptualization. **Linfeng Li:** Software, Resources, Methodology. **Li Cheng:** Writing – review & editing, Project administration, Methodology, Funding acquisition. **Xiang Yu:** Writing – review & editing, Writing – original draft, Supervision, Software, Project administration, Methodology, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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