

Sound radiation of nonlinear meta-plates: mechanism and mitigation through linear-nonlinear band control

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ABSTRACT

Mitigating structural sound radiation is crucial for numerous engineering applications, which requires simultaneous handling of structural vibration and sound radiation. Nonlinear acoustic metamaterial (NAM) plates have demonstrated remarkable broadband vibration suppression abilities, yet their sound radiation characteristics and the corresponding underlying physical mechanisms have not been fully explored. This issue is investigated in this paper using a Rayleigh integral model and spatial-domain analysis. It is shown that the introduction of nonlinearity mainly leads to fluctuations in both sound radiation efficiency and sound power near the bandgap, whereas sound radiation mitigation in other regions is primarily governed by vibration. Notably, the nonlinearity-induced third harmonics significantly enhance sound radiation, driven by energy pumping near the bandgap and the high radiation efficiency at mid-to-high frequencies. To mitigate this, linear-nonlinear band control strategies are proposed through eliminating harmonic interference near the bandgap, alongside a proper deployment of structural damping to suppress the third harmonic radiation in the mid-to-high frequency regime. By fine-tuning system parameters, we ultimately achieve broadband sound control, as evidenced by a systematic reduction of 8–30 dB at all resonant peaks below 600 Hz, as a result of optimized design with only a 10% mass penalty.

1. Introduction

Techniques for controlling structural vibration and sound radiation have garnered significant attention in engineering, particularly within aerospace [1–6] and mechanical/civil sectors [7–9]. Across a wide range of applications, there is a burgeoning demand for lightweight, low-frequency, and broadband solutions to achieve highly efficient noise [10–12] and vibration mitigation [13–16]. Among existing approaches, acoustic metamaterials offer transformative solutions through strategic wave manipulation [17–20]. In linear acoustic metamaterials (LAMs), bandgaps are typically realized through Bragg scattering and local resonance effects. Building upon these principles, LAMs can enable exotic properties, including negative effective mass density [21,22], negative modulus [23,24], and negative refractive index [25]. However, the performance of LAMs is often constrained by narrow operational bandwidths, heavily dependent on the attached mass ratio [26–28]. Furthermore, research on elastic wave propagation in LAMs overlooks the vast potential offered by nonlinear dynamic responses [29–31].

Conversely, nonlinear acoustic metamaterials (NAMs) give rise to behaviors that are inaccessible in their linear counterparts [28,32–34]. By leveraging nonlinear features—such as amplitude-dependent bandgaps [35–37], chaotic bands [28,38,39], and bridging-coupling bandgaps [40]—NAMs have demonstrated superior ability in delivering ultra-broadband and ultra-low-frequency vibration suppression. Integrating diverse mechanisms within NAMs can further enhance vibration control performance. For instance, Sheng *et al.* [14] systematically investigated the influence of system parameters on bandwidth and efficiency of a NAM beam's vibration reduction. An optimized system reduced the added mass from 56.2% to 28.1% while achieving a 15 dB suppression of resonance peaks within the 30–450 Hz range—a result with substantial implications for practical engineering applications. They then developed lightweight, high-stiffness honeycomb sandwich NAM plates, achieving an average reduction of 23.5 dB in vibration response with only a 17.7% mass penalty [13]. Despite these efforts in vibration suppression, the challenge of using NAM for sound radiation control remains largely unaddressed.

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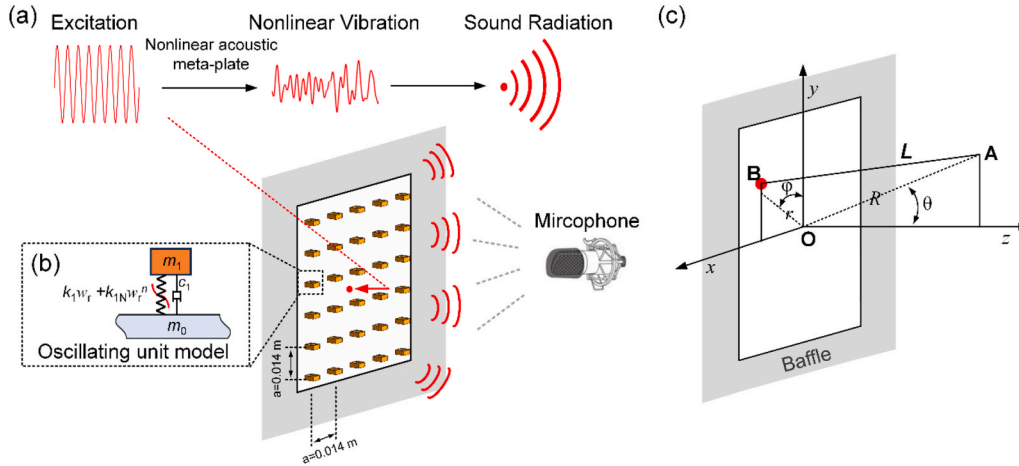


Fig. 1. Meta-plate model. (a) A finite plate equipped with periodically arranged nonlinear resonators in a semi-infinite acoustic field. (b) Equivalent mechanical model of the nonlinear resonators. (c) Computational model for the sound radiation of the *meta*-plate with an infinite and acoustically rigid baffle.

Some preliminary attempts have been conducted to explore the acoustic benefits of NAMs [41]. For instance, Fang *et al.* [42] investigated different mechanisms leading to wave suppression and sound radiation mitigation, including nonlinear resonance saturation, energy pumping into higher harmonics, and modal modulation. Zhang *et al.* [43] characterized nonlinear wave scattering in one-dimensional waveguides by explicitly accounting for nonlinear acoustic effects. However, the fundamental link between sound radiation and structural vibration has not received enough attention up to now.

Effectively minimizing sound radiation of a vibrating structure necessitates the simultaneous control of both structural vibration and its subsequent energy conversion into acoustic energy. Elucidating the underlying coupling mechanisms between structural waves and the surrounding acoustic medium is the key to acquiring a more comprehensive understanding on how NAMs would affect the entire energy conversion process. As shown before, the inherent nonlinearity of NAMs triggers the generation of higher-order harmonics and subharmonics [42]. Given that the human ear is more sensitive to high-frequency sound [44,45], the radiation of these higher-order harmonics would potentially be an issue to be considered during NAM designs. In a broader sense, the mechanisms governing sound radiation differ from those of structural vibration in a NAM plate. Moreover, the influence of various structural parameters on sound radiation in such a high-dimensional and strongly nonlinear system remains poorly understood.

The present study addresses the key issues of elucidating the potential benefit of NAM embodiment in a sound-radiating plate and understanding the sound-radiation specific physical mechanisms, as well as developing proper mitigation strategies to achieve broadband sound radiation mitigation. To this end, we examine high-dimensional meta-plates incorporating prototypical nonlinear local oscillators. A modeling framework is first developed to predict the sound radiation of the *meta*-plate, assisted by a dedicated wavenumber analysis. The ability of the NAM plates in dictating their sound radiation is systematically examined for both fundamental and harmonic components. Leveraging physical understanding acquired, we then propose four strategies to eliminate the harmonic effects near the bandgap through the creation of new band gaps, and to utilize structural damping to suppress the third harmonic radiation in the mid-to-high frequency range. Finally, under a 10% added mass and by optimizing the system parameters and targeting sound radiation below 600 Hz, we successfully achieve effective and multi-harmonic sound radiation control across different frequency bands. These findings lay a solid foundation for the realization of lightweight, broadband, and highly efficient NAM-based noise control in engineering applications.

2. Sound radiation modeling of nonlinear *meta*-plate

2.1. Modelling of the *meta*-plate

Fig. 1(a) shows a nonlinear *meta*-plate, comprising a rectangular thin plate on which is mounted an array of periodic oscillators. The plate, surrounded by an acoustically rigid baffle along its four edges, radiates sound into a semi-infinite free field. Each oscillator unit contains a mass m_1 , linear stiffness k_1 , viscous damping constant c_1 and a mass m_0 as shown in Fig. 1(b). m_0 denotes the mass at the interface between the oscillators and the plate, introduced by connection elements such as bolts or welds. The transverse displacement of the host plate and that of the oscillators are denoted by w_0 and w_1 , respectively, with relative motion $w_r = w_1 - w_0$. Each oscillator has an inherent natural frequency $\omega_1 = 2\pi f_1$; thus, the linear stiffness of the resonators is $k_1 = (\omega_1)^2 m_1$. The damping constant is taken as $c_1 = 0.01 k_1 / \omega_1$ [38]. Plate damping and geometric nonlinearity are neglected, so all nonlinearity originates from the local oscillators. Denoting the nonlinear stiffness of each oscillator by k_{1N} , the equation of motion for the s^{th} metacell ($s = 1, 2, \dots, 30$) can be written as:

$$\begin{cases} m_0 \ddot{w}(x_s, y_s, t) - k_1 w_{1r}^{(s)} - k_{1N} (w_{1r}^{(s)})^n = F_s(x_s, y_s) \\ m_1 (\ddot{w}(x_s, y_s, t) + \ddot{w}_{1r}^{(s)}) + k_1 w_{1r}^{(s)} + k_{1N} (w_{1r}^{(s)})^n = 0 \end{cases} \quad (1)$$

where n is a nonlinear factor that can be achieved through oscillating unit design [46]. Here, we adopt the cubic stiffness nonlinearity ($n = 3$) and the design parameters of the *meta*-plate in Ref. [33,36,37]. The dynamics of a *meta*-plate comprising 30 metacells is modelled via Assumed Modes Method (AMM) [38], which can be expressed in a matrix form as:

$$\mathbf{M}\ddot{\mathbf{Q}} + \mathbf{C}\dot{\mathbf{Q}} + \mathbf{K}\mathbf{Q} + \mathbf{K}_d\mathbf{Q}^3 = \mathbf{F} \quad (2)$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{K}_d$ are the mass, damping, stiffness and nonlinear stiffness matrices of the system, respectively; $\mathbf{Q}$ and $\mathbf{F}$ are the displacement and excitation force vectors of the system, respectively. Equation (2) can be solved via the harmonic balance method. Given the cubic nonlinearity, the 1/3-order sub-harmonic and the 3-order super-harmonic dominate the nonlinear spectrum. Accordingly, the steady-state response writes:

$$\begin{aligned} \mathbf{Q} = & \mathbf{A}_1 \sin\left(\frac{1}{3}\omega t\right) + \mathbf{B}_1 \cos\left(\frac{1}{3}\omega t\right) + \mathbf{C}_1 \sin(\omega t) + \mathbf{D}_1 \cos(\omega t) + \mathbf{E}_1 \sin(3\omega t) \\ & + \mathbf{F}_1 \cos(3\omega t) \end{aligned} \quad (3)$$

Substituting Eq. (3) into Eq. (2) and equating the coefficients of the sinusoidal terms, the unknowns A_1 , B_1 , C_1 , D_1 , E_1 and F_1 are obtained through Newton's iteration, with all initial values set to zero for each prescribed excitation frequency. The total amplitude of each harmonic can be obtained by $Q_{1/3}=(A_1^2 + B_1^2)^{(1/2)}$, $Q_1=(C_1^2 + D_1^2)^{(1/2)}$ and $Q_3=(E_1^2 + F_1^2)^{(1/2)}$, where the $^{(1/2)}$ and ^{2} are the Hadamard root and Hadamard power, respectively. Through modal superposition, the resulting coefficients then yield the displacement response of each harmonic at any location on the meta-plate. Detailed modeling and solution procedures were discussed in [38].

2.2. Sound radiation model

The sound radiation of the meta-plate is calculated using Rayleigh integral under baffled conditions, as shown in Fig. 1(c). The sound pressure at any point in the free field A (x_a, y_a, z_a) is calculated by [42]:

$$p(A) = \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \frac{i\omega\rho_0}{2\pi L_M} v(x_b, y_b) e^{i(\omega t - kL_M)} dx dy \quad (4)$$

where the air density is ρ_0 ; $v(x_b, y_b)$ is the vibration velocity at point B on the meta-plate; L is the distance between A and B.

The meta-plate is discretized into $m_x \times n_y$ grids, and a discrete summation is used to compute the sound pressure. Thus, the major components of the sound pressures, corresponding to the 1/3-order harmonic, fundamental and 3-order harmonics, $p_{1/3}$, p_1 and p_3 , can be computed as:

$$p_{order} = \sum_{m_x} \sum_{n_y} \frac{i\omega\rho_0}{2\pi L_M(x_{m_x}, y_{n_y})} v_{order}(x_{m_x}, y_{n_y}) e^{i \cdot order \cdot kL(x_{m_x}, y_{n_y})} l_{mx} l_{ny} (order) \\ = 1/3, 1, 3, \quad (5)$$

where l_{mx} and l_{ny} denote the size of the $(m_x, n_y)^{th}$ grid; v_{order} ($order = 1/3, 1, 3$) denotes the out-of-plane velocity of each harmonic at the center of each grid cell. The corresponding sound power, $W_{1/3}$, W_1 and W_3 , can then be obtained as:

$$W_{order} = 2\pi R_0^2 I_{order} = \int_0^{2\pi} \int_0^{\pi/2} \frac{|p_{order}(R_0, \theta, \phi)|^2}{2\rho_0 c_0} R_0^2 \sin\theta d\theta d\phi (order) \\ = 1/3, 1, 3, \quad (6)$$

where R_0 is the radius of the integrating hemispherical surface; I_{order} the sound pressure intensity; θ and ϕ the polar and azimuth angles as shown in Fig. 1(c). The integrating hemispherical surface is discretized into an $r_1 \times r_2$ grid, and a discrete summation is then used to compute the sound power:

$$W_{order} = \sum_{r_2} \sum_{r_1} S(\theta_{r_1}, \phi_{r_2}) I_{order} \\ = \sum_{r_2} \sum_{r_1} \frac{|p_{order}(R_0(\theta_{r_1}, \phi_{r_2}), \theta_{r_1}, \phi_{r_2})|^2}{2\rho_0 c_0} S(\theta_{r_1}, \phi_{r_2}) (order) \\ = 1/3, 1, 3, \quad (7)$$

where $S(\theta_{r_1}, \phi_{r_2})$ is the area of the discrete hemispherical surface at $(\theta_{r_1}, \phi_{r_2})$. For a given harmonic excitation force of amplitude F_0 , we define the generalized radiation sound power levels as:

$$L_{w_order} = 10\log_{10} \frac{W_{order}}{W_{ref} F_0} (order = 1/3, 1, 3). \quad (8)$$

The total sound power level resulting from the superposition of multiple harmonics, L_{w_total} , writes:

$$L_{w_total} = 10\log_{10} \frac{W_{1/3} + W_1 + W_3}{W_{ref} F_0} \quad (9)$$

with $W_{ref} = 1 \times 10^{-12}$ W. For analysis, the radiation efficiency of the meta-plate is scrutinized, defined as:

Table 1

Model parameters.

Symbol	Definition	Value	Symbol	Definition	Value
a	Lattice constant	0.014 m	m_0	Concentrated mass	0.2 kg
$L_p \times W_p$	Length $\times$ Width	0.7×0.84 m	m_1	Oscillator mass	0.4 kg
ρ	Thickness	0.006 m	f_1	Natural frequency	100 Hz
E	Density	2700 kg/m ³	k_{1N}	Nonlinear coefficient	1×10^{13} N/m ³
ν	Elastic modulus	67 GPa	η	Total loss factor	0
	Poisson's ratio	0.3			

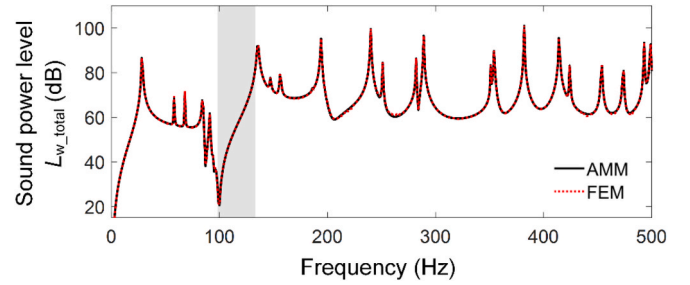


Fig. 2. Calculated sound power level L_{w_total} of the linear meta-plate by AMM and FEM.

$$\sigma_{order} = 10\log_{10} \frac{W_{order}}{\rho_0 c_0 S_v \langle |v|^2 \rangle} (order = 1/3, 1, 3), \quad (10)$$

where S_v is the surface area of the vibrating plate, and $\langle |v|^2 \rangle$ is the surface mean square velocity of the plate.

2.3. Wavenumber analysis

Wavenumber spectra describe the vibration field in terms of wavenumber components, with their amplitudes demarcating energy between subsonic and supersonic waves. These characteristics can thus be used to elucidate the sound radiation mechanisms of the NAM structures. To this end, the spatial velocity field $v(x, y, f)$ is mapped to the wavenumber domain via a two-dimensional Fourier transform as:

$$V(k_x, k_y, f) = \iint v(x, y, f) e^{-jk_x x} e^{-jk_y y} dx dy \quad (11)$$

where k_x and k_y denote the wavenumbers in the x - and y -directions, respectively. The amplitude of wavenumber spectra writes:

$$Amp_w = 20\log_{10} (|V(k_x, k_y, f)|^2) \quad (12)$$

2.4. Model validation

A linear meta-plate with a 5×6 array of periodic metacells is first examined to determine the number of grid elements to be used in the subsequent simulation and to validate the established model. The plate, simply-supported along its four edges, is subjected to a unit harmonic force excitation at $(x_c = 0.385 \text{ m}, y_c = 0.315)$, as shown in Fig. 1(b). The parameters used in the simulation are listed in Table 1 (while k_{1N} is set to zero for this linear case). The modal truncation of the AMM method follows the parameters in Ref. [38]. Fig. 2 shows the sound power level of the linear meta-plate using the AMM with $(m_x, n_y) = (100, 120)$ and $(r_1, r_2) = (10, 20)$, represented by the black curve. For comparison,

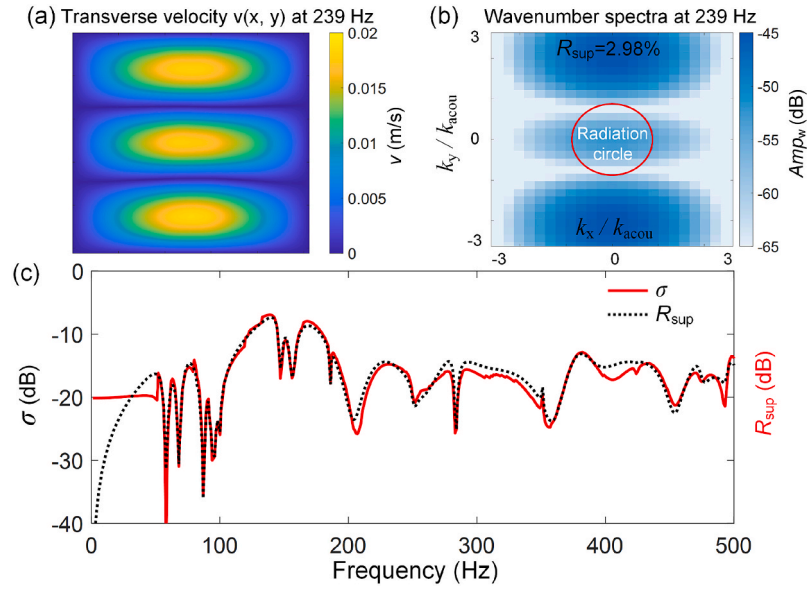


Fig. 3. Validation of the wavenumber analysis. (a) Velocity distribution at 239 Hz. (b) Normalized wavenumber spectra comparison at 239 Hz, with red solid line denoting the radiation circle. (c) Comparison between σ by Eq. (10) and R_{sup} by Eq. (14).

results from the finite element method (FEM) are also shown by the dotted line. Both the grid elements (m_x, n_y) and the FEM model have been verified through convergence analysis. It can be observed that FEM and AMM deliver consistent results that are in good agreement. Furthermore, the bandgap of the corresponding infinite plate is calculated from the Bloch–Floquet theorem and represented by the shaded region [33]. The valley of the sound power level occurs within the same range (99–133 Hz), thereby corroborating the reliability of the proposed model.

A frequency of 239 Hz is selected from Fig. 2 as a typical representative case to perform wavenumber analysis: Fig. 3(a) shows the distribution of the transverse velocity $v(x, y, f)$ across the meta-plate. Fig. 3(b), in which Amp_w represents the energy level in the wavenumber spectra, presents the corresponding wavenumber transformation of Fig. 3(a), normalized by the acoustic wavenumber $k_{\text{sound}} = 2\pi f/c_0$. A red circle, referred to as the radiation circle corresponding to $(k_x^2 + k_y^2)^{0.5}/$

$k_{\text{sound}} = 1$, is marked as a reference. Structural wave components within this circle are therefore supersonic and radiate sound efficiently into the far field. Accordingly, we define the supersonic energy, enclosed in the radiation circle, as:

$$E_{\text{sup}} = \iint_{\frac{(k_x^2 + k_y^2)^{0.5}}{k_{\text{sound}}} \leq 1} \text{Amp}_w(k_x, k_y) dk_x dk_y \quad (13)$$

Conversely, components outside the circle are subsonic and exhibit significantly weaker sound radiation. To quantify the radiation capability of a structure, we introduce a metric denoted as R_{sup} , defined as the ratio of supersonic wave energy to the total wave energy:

$$R_{\text{sup}} = \frac{E_{\text{sup}}}{\iint_{-\infty}^{+\infty} \text{Amp}_w(k_x, k_y) dk_x dk_y} \quad (14)$$

The validity of R_{sup} is corroborated by σ derived from Eq. (10), as

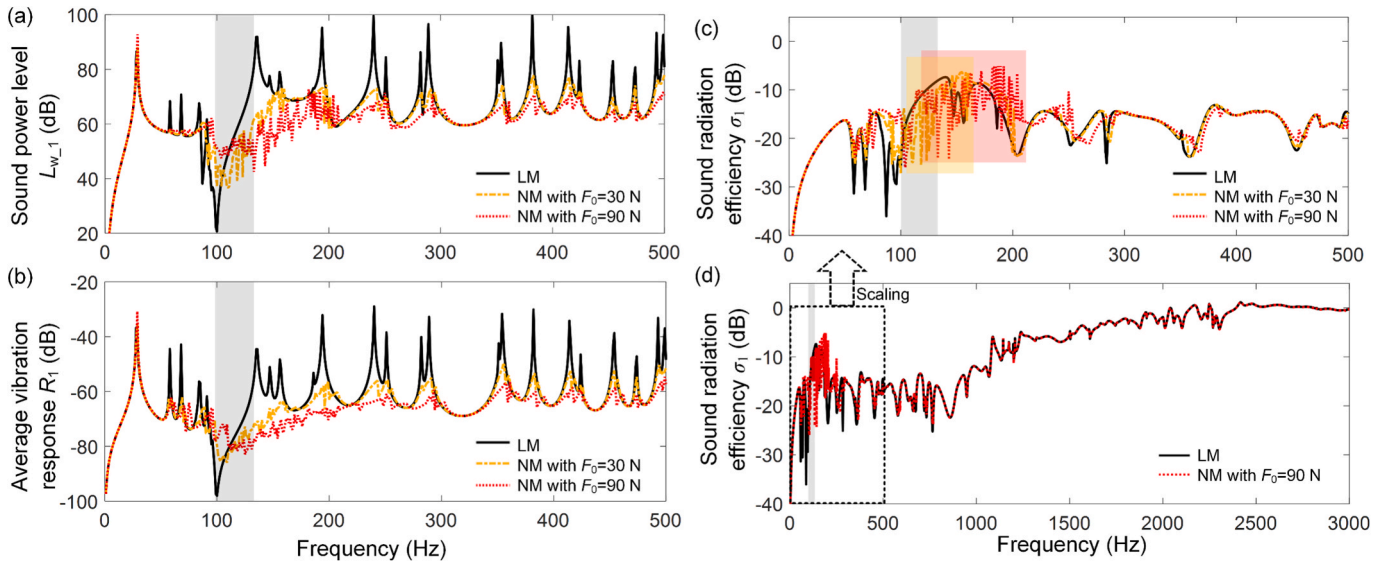


Fig. 4. Sound power level, average vibration response and sound radiation efficiency at the fundamental frequency (excitation frequency) for the linear and nonlinear meta-plate with increasing F_0 . (a) Sound power level $L_{w,1}$. (b) Average vibration response R_1 defined by Eq. (A1) [38] (see Appendix A.1). (c) Sound radiation efficiency σ_1 from 0 to 500 Hz. (d) Sound radiation efficiency σ_1 from 0 to 3000 Hz.

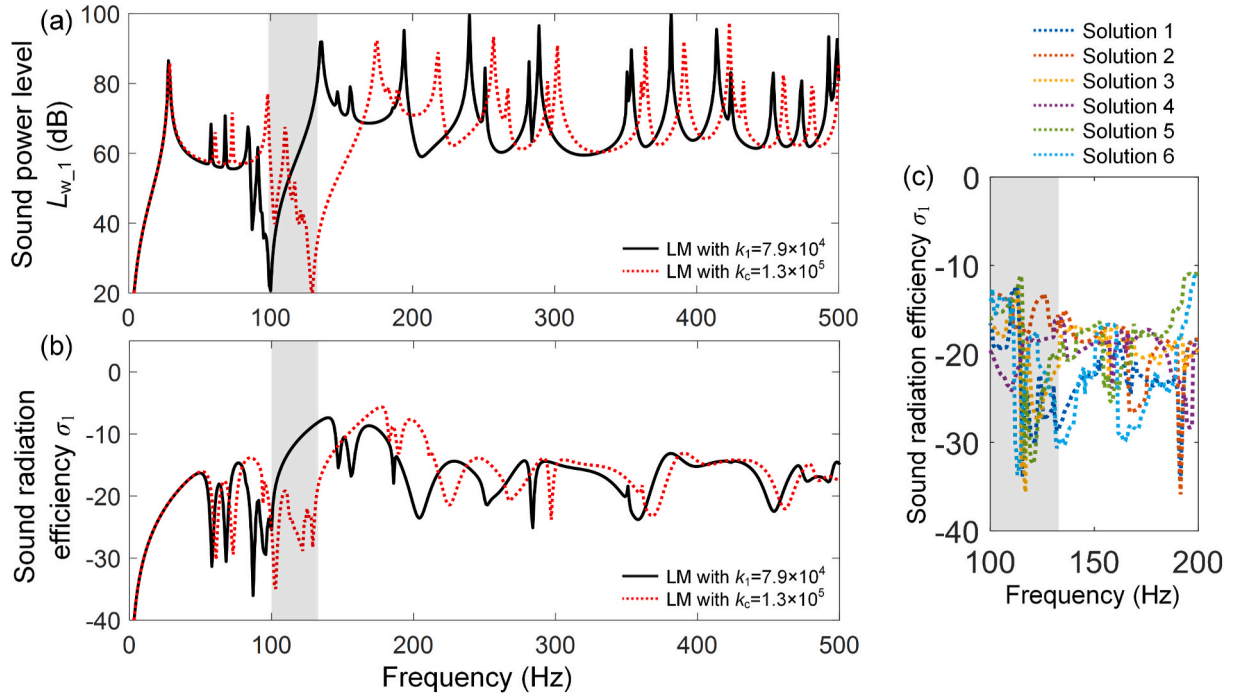


Fig. 5. Analysis of the fluctuation in σ_1 . (a) Sound power level $L_{w,1}$ of the equivalent linear system with a tunable stiffness k_c . (b) Sound radiation efficiency σ_1 of the equivalent linear system with a tunable stiffness k_c . (c) Solutions obtained via the continuation method using different initial values.

depicted in Fig. 3(c). The two metrics demonstrate excellent agreement, indicating that R_{sup} can indeed be used to characterize the sound radiation ability of a structure, like σ . The differences observed in the low-frequency range (0–50 Hz) are attributed to the insufficient resolution in the wavenumber space imposed by the finite physical dimensions, which hinders the precise identification of supersonic wave components.

3. Analysis of sound radiation

3.1. Sound radiation properties and mechanisms of nonlinear meta-plate

We now consider the afore-defined nonlinear meta-plate (NM) with simply-supported boundaries. Fig. 4(a) shows the sound power level ($L_{w,1}$) of the NM ($k_{1N} = 1 \times 10^{13} \text{ N/m}^3$) at the fundamental excitation frequency under different forcing levels of F_0 , in comparison with the corresponding linear meta-plate (LM). As a baseline, the LM case exhibits numerous $L_{w,1}$ peaks across the 0–500 Hz range, corresponding to the system's resonances. Notably, deploying NM leads to obvious sound radiation mitigation from these resonant peaks. This performance enhancement becomes more pronounced as the level of nonlinearity increases as F_0 increases from 30 N to 90 N. Except for the first resonance peak, the hardening nonlinearity of the NM influences the passbands surrounding and beyond the bandgap [29,38]. Conversely, $L_{w,1}$ within the bandgap (red region) shows an upward trend: as F_0 increases, $L_{w,1}$ valleys are filled; the minimum $L_{w,1}$ for the NM increases from 20 dB to 52 dB. These observations suggest that the introduction of nonlinearity significantly alters the sound radiation, mainly at resonant peaks and within the bandgap region. Fig. 4(b) shows the corresponding changes in the averaged vibration response, R_1 , exhibiting a very similar variation trend to the sound power. The phenomena observed on vibration curves have been discussed in [38], which are primarily attributed to band degeneration and chaotic effects arising from the nonlinearity of the system.

The strong similarity between R_1 and $L_{w,1}$ under both LM and NM suggests that nonlinearity plays a minimal role in influencing sound radiation efficiency. To verify this, Fig. 4(c) shows the variation of the corresponding sound radiation efficiency, σ_1 , derived from Eq. (10),

from 0 to 3000 Hz in Fig. 4(d). Overall, σ_1 remains virtually unaltered despite the introduction of nonlinearity. Furthermore, σ_1 generally increases with frequency, consistent with the common understanding that high-frequency vibrations radiate sound more efficiently. However, a closer inspection near the bandgap of Fig. 4(c) reveals a distinct deviation from this general trend: With $F_0 = 30 \text{ N}$, σ_1 fluctuates significantly to the right of the bandgap (yellow region). As F_0 increases to 90 N, this fluctuation zone still exists but shifts further to the right, transitioning to the red region. Alterations in σ_1 do not alter the overall trend of the $L_{w,1}$ near the bandgap, but induce fluctuations in the $L_{w,1}$, as shown in Fig. 4(a).

The observed shift in the fluctuation region of σ_1 is primarily driven by the resonance frequency shift of the oscillators. As F_0 increases, the nonlinearity level increases, leading to an increase in the equivalent stiffness of the oscillators. We can approximate this effect as an equivalent linear system with a tunable stiffness, k_c . As illustrated in Fig. 5(a), with the increase of k_c , the corresponding resonance peaks of the entire system shift towards higher frequencies. Consequently, as shown in Fig. 5(b), the σ_1 curve also shifts towards higher frequencies, which accounts for the rightward shift of the sound radiation fluctuation region.

The observed fluctuations in σ_1 are owing to the multiple solution branches. The continuation method with different initial values is utilized to get the response in the 100–200 Hz range, revealing numerous solution branches, as shown in Fig. 5(c) [42]. This indicates that the nonlinear solutions are highly sensitive to initial conditions within this frequency range. A detailed investigation into these complex dynamics is omitted here, as it is beyond the scope of this paper and does not inherently affect our main conclusions.

In summary, the introduction of nonlinearity impairs sound radiation, while the mitigation in sound radiation is primarily attributable to the vibration suppression enabled by nonlinearity. Near the bandgap, the existence of multiple solution branches causes σ_1 to fluctuate, which in turn leads to fluctuations in $L_{w,1}$.

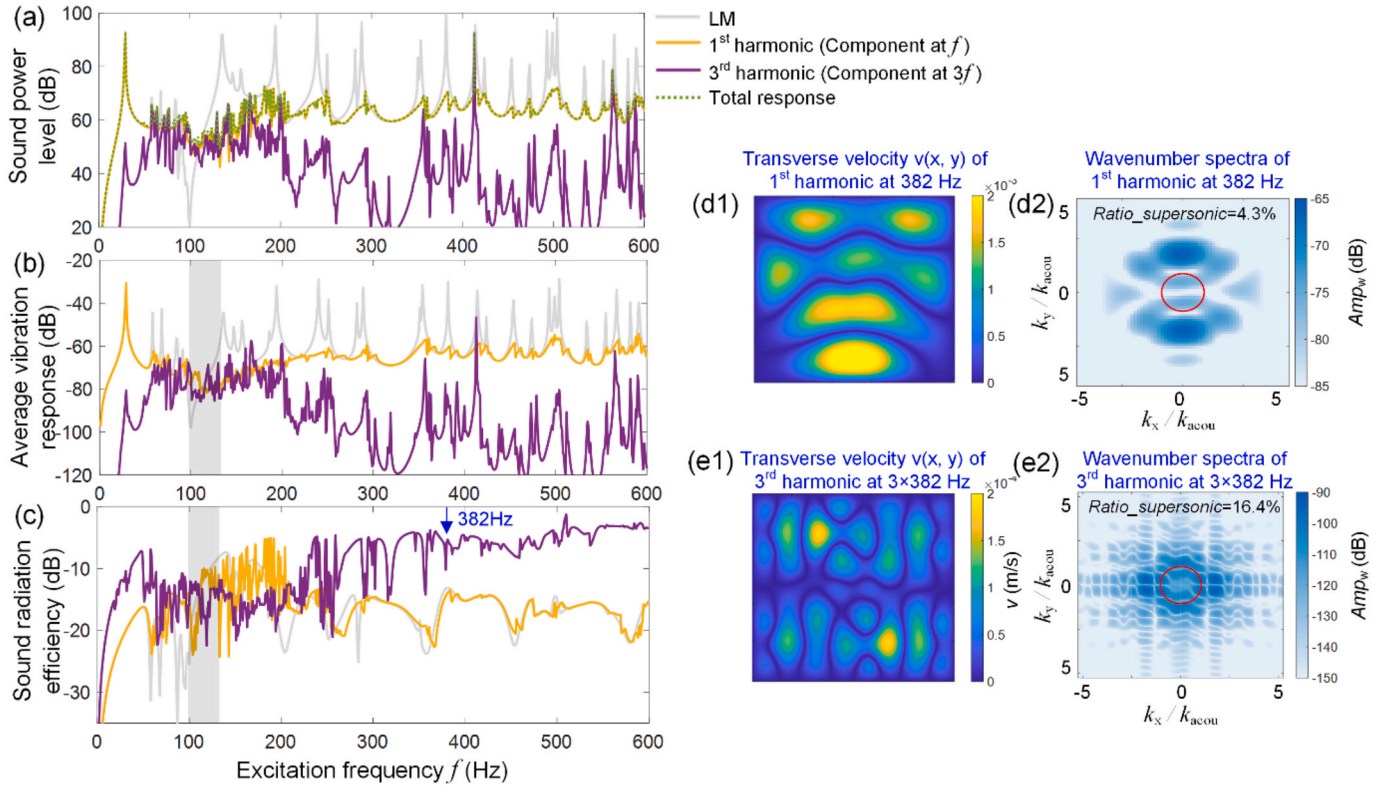


Fig. 6. Sound power levels $L_{w,1/3}$, $L_{w,1}$ and $L_{w,3}$, average vibration transmission $R_{1/3}$, R_1 and R_3 and sound radiation efficiencies $\sigma_{1/3}$, σ_1 and σ_3 for the LM and NM under $F_0 = 90$ N. (a) Sound power levels. (b) Average vibration transmission. (c) Sound radiation efficiencies. (d1, e1): Velocity distribution for NM at 382 Hz. (d2, e2): Normalized wavenumber spectra for NM at 382 Hz, with red solid line denoting the radiation circle. Results at fundamental frequency (d1, d2) and third harmonic frequency (e1, e2).

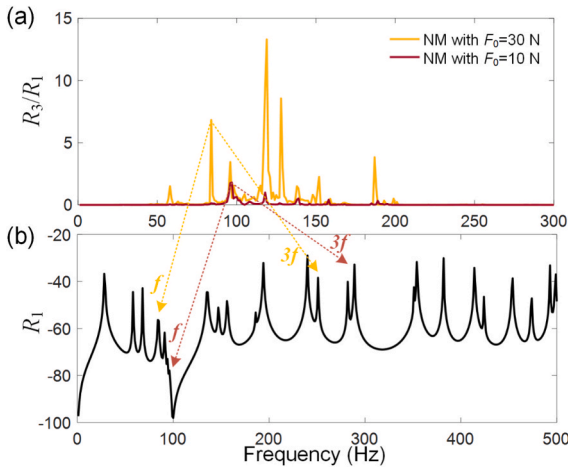


Fig. 7. Average vibration response to show energy pumping. (a) Ratio of the average vibration response at third harmonic frequency R_3 to that at fundamental frequency R_1 . (b) Average vibration response at fundamental frequency R_1 .

3.2. Sound radiation from sub- and super-harmonics

We further investigate the sound power levels at other harmonic frequencies. Assuming an excitation frequency f , the resulting sound power level response comprises $1/3f$, f , and $3f$ components (denoted as $L_{w,1/3}$, $L_{w,1}$ and $L_{w,3}$, respectively), along with their total sound power level ($L_{w,total}$ calculated via Eq. (9)) for the NM under $F_0 = 90$ N, as shown in Fig. 6. Taking $f = 382$ Hz as an example, the $1/3$ -order sub-harmonic, fundamental, and third harmonic responses denote the

respective components at $1/3f$ ($1/3 \times 382$ Hz), f (382 Hz), and $3f$ (3×382 Hz). Since $L_{w,1/3}$ is negligible (remaining below 20 dB), it is excluded from the following discussion. Across different frequency ranges, $L_{w,1}$ and $L_{w,3}$ serve as the predominant contributors to the total sound power level, $L_{w,total}$. Specifically, near the bandgap, $L_{w,3}$ dominates the radiation, with its magnitude comparable to $L_{w,total}$. In other frequency ranges, $L_{w,1}$ remains the primary sound radiation contributor, dictating the overall profile of the $L_{w,total}$ curve. To identify the cause of the excessive $L_{w,3}$, we scrutinize the structural vibration (Fig. 6(b)) and sound radiation efficiency (Fig. 6(c)).

As illustrated in Fig. 6(b), the profile of the average vibration response at the third harmonic frequency (R_3) mirrors that of $L_{w,3}$, demonstrating that the elevated $L_{w,3}$ near the bandgap is vibration-induced. This heightened R_3 is attributed to the energy pumping effect [14,28,42]. As shown in Fig. 7(a), R_3/R_1 exhibits numerous peaks for NM, indicating that the fundamental wave energy is pumped to the third harmonic frequency as a natural nonlinear process. These peaks in R_3/R_1 occur when the excitation frequency and its third harmonic frequency both coincide with the system's natural frequencies, as shown in Fig. 7 (b). This is a typical energy-pumping effect triggered by 1:3 internal resonance: while the third harmonics are typically suppressed by structural impedance, the exact match of $3f$ with a resonance frequency creates a low-impedance state. Consequently, energy from the fundamental frequency efficiently transports into the third harmonic frequency, triggering a pronounced third harmonic response.

Fig. 6(c) illustrates the acoustic radiation efficiency at the third harmonic frequency (σ_3). Near the bandgap, σ_3 fluctuates around -15 dB without any abrupt changes, confirming that the elevated $L_{w,3}$ in this region is primarily vibration-induced. Notably, beyond 300 Hz, σ_3 begins to increase abruptly, even exceeding the fundamental radiation efficiency (σ_1). This phenomenon is attributed to a higher proportion of supersonic components at the third harmonic ($R_{sup} = 16.4\%$ in Fig. 6

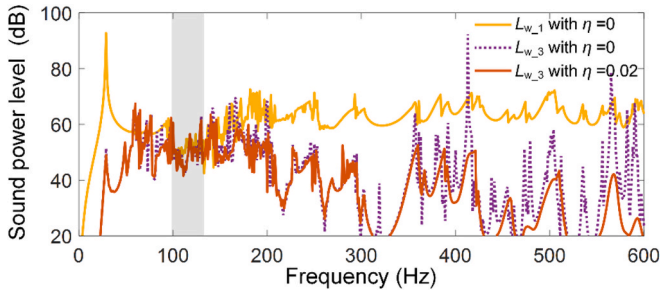


Fig. 8. Sound power levels $L_{w,1}$ and $L_{w,3}$ with structural damping for the NM under $F_0 = 90$ N.

(e2)) compared to the fundamental wave ($R_{\text{sup}} = 4.3\%$ in Fig. 6(d2)). Consequently, this elevated σ_3 in the mid-to-high frequency range drives excessive $L_{w,3}$ radiation. As depicted in Fig. 6(a), $L_{w,3}$ even exceeds $L_{w,1}$ at specific frequencies around 400 Hz and 600 Hz.

To suppress this third-harmonic radiation in the mid-high frequency range, introducing structural damping is found to be an effective approach. Herein, a loss factor η is introduced to characterize the damping effect of the plate, and its equivalent complex stiffness is defined as $E_{\text{modify}} = E(1 + i\eta)$ [47,48]. As shown in Fig. 8, the application of structural damping significantly attenuates $L_{w,3}$ over the 200–600 Hz, effectively suppressing $L_{w,3}$ below the $L_{w,1}$ baseline. However, the effect at the low-frequency bandgap (grey region) remains limited. This suggests that relying solely on structural damping is insufficient to mitigate

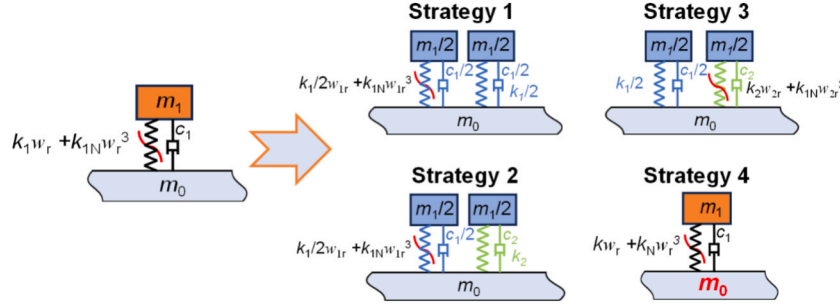


Fig. 9. Equivalent mechanical model of the nonlinear resonators in four strategies.

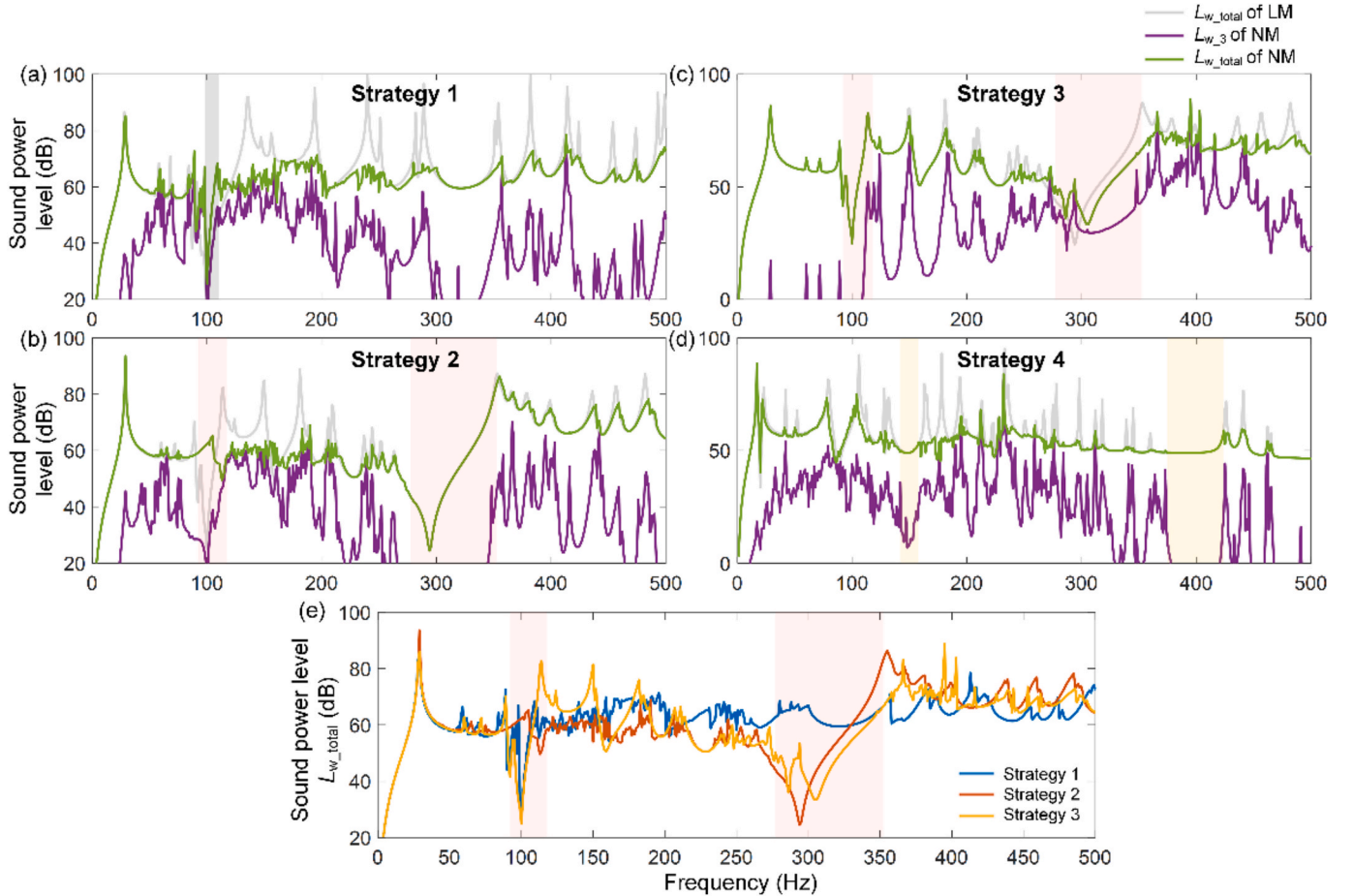


Fig. 10. Sound power levels $L_{w,1}$, $L_{w,3}$ and $L_{w,\text{total}}$ for four strategies under $F_0 = 90$ N. Sound power levels of (a) Strategy 1; (b) Strategy 2; (c) Strategy 3; (d) Strategy 4. (e) Comparison of total sound power level $L_{w,\text{total}}$ among them.

excessive third-harmonic response near the low-frequency bandgap, thereby underscoring the need to explore targeted strategies to address this issue in NAMs.

3.3. Strategies for suppressing higher-order sound radiation

To tackle the non-negligible sound radiation from the third harmonic component near the bandgap, we strategically decompose the configuration of a single-degree-of-freedom oscillator into the one having two degrees of freedom (oscillators put in parallel with different mechanical properties). The transverse displacements of the two oscillators are denoted by w_{1r} and w_{2r} , respectively. By modulating the combination between the two oscillators, four strategies are formulated, as depicted in Fig. 9.

Strategy 1: By introducing a new linear oscillator array with parameters identical to the linear components of the original nonlinear array (Strategy 1 in Fig. 9), a linear bandgap (grey region) is created to inhibit the generation of the third harmonic component, as shown in Fig. 10(a). This strategy allows both the fundamental and third-harmonic components within the bandgap to form distinct “valleys” for $L_{w,1}$ and $L_{w,3}$, while simultaneously achieving an ultra-wide chaotic band beyond the bandgap. The underlying mechanism relies on suppressing the fundamental component within the bandgap, which reduces the nonlinear intensity and subsequently limits the generation of the third harmonic component. Note that the effective range is confined to the bandgap region.

Strategy 2: Inspired by Ref. [49], we leverage the principle that bandgaps can simultaneously influence the fundamental and higher-order harmonics. By tuning the mass and the stiffness of the additional oscillator to satisfy $\omega_2 = 3\omega_1$, where $\omega_i = (m_i/k_i)^{0.5}$, $i = 1, 2$ (Strategy 2 in Fig. 9), two separate bandgaps are created (red regions in Fig. 10(b)). This allows the third harmonic component, initially located near the first bandgap, to fall into the newly formed second bandgap. Consequently, a valley in $L_{w,3}$ is formed near the first bandgap. Due to the inherent properties of the local resonance, the same added mass creates a wider bandgap at higher frequencies [38], resulting in a broader mitigation band for $L_{w,3}$ compared to Strategy 1. However, Strategy 2 sacrifices the bandgap since the nonlinearity is integrated into k_1 .

Strategy 3: We introduce nonlinearity into k_2 , while maintaining k_1 as a linear component as shown in Fig. 9. Strategy 3 similarly forms distinct “valleys” for $L_{w,1}$ and $L_{w,3}$ within the first bandgap, as shown in Fig. 10(c). Compared with Strategy 1, Strategy 3 achieves deeper valleys for $L_{w,3}$ by synergizing the effects of the first and second bandgaps: the first bandgap suppresses the fundamental component, which further reduces the nonlinearity strength. Simultaneously, the third harmonic component, initially located near the first bandgap, falls within the newly formed second bandgap. However, this strategy compromises a portion of the mitigation region for $L_{w,1}$ (between two red regions), typically induced by the chaotic band [28,38,39].

Strategy 4: A Bragg bandgap can be introduced by adjusting the concentrated mass m_0 , as shown in Fig. 9. As shown in Fig. 10(d), the third-harmonic component (first yellow region) falls within the Bragg bandgap (second yellow region). Owing to periodic scattering and destructive interference, waves entering the Bragg bandgap region transition from propagating waves to evanescent waves, thereby attenuating the third-harmonic component (first yellow region). The primary advantage of this strategy is that it obviates the need for additional oscillators. However, to shift the Bragg bandgap toward the target frequency around 100 Hz, one must either enlarge the lattice constant or reduce the plate thickness. Furthermore, introducing a concentrated mass m_0 to widen the bandgap inevitably imposes an additional weight penalty on the structure (see Appendix A.2). Consequently, the adoption of Strategy 4 requires a trade-off between acoustic performance and structural weight constraints.

Since Strategy 4 circumvents the need for additional oscillators, its specific design considerations are deferred in Appendix A.2. Here, we

Table 2
Properties of four strategies.

	Strategy 1	Strategy 2	Strategy 3	Strategy 4
Natural frequencies	$\omega_1 = \omega_2$	$3\omega_1 = \omega_2$	$3\omega_1 = \omega_2$	Only ω_1
Nonlinear stiffness	$k_{1N} = 0$, $k_{2N} = 1 \times 10^{13} \text{N/m}^3$	$k_{1N} = 1 \times 10^{13}$, $k_{2N} = 0 \text{N/m}^3$	$k_{1N} = 0$, $k_{2N} = 1 \times 10^{13} \text{N/m}^3$	Only $k_{1N} = 1 \times 10^{13} \text{N/m}^3$
Mitigation Range for Fundamental	99–112 Hz	Bandgap degeneration	99–112 Hz	Bandgap degeneration
Mitigation Range for Third Harmonics	99–112 Hz	92–117 Hz	0–277 Hz	Bandwidth is proportional to m_0 .
Chaotic bands	Above 133 Hz	Above 133 Hz	Above 352 Hz	Above 133 Hz

Table 3
Key performance indicators of four strategies.

	Strategy 1	Strategy 2	Strategy 3	Strategy 4
Maximum sound power reduction	35.2 dB	31.8 dB	20.3 dB	37.5 dB
Effective mitigation bandwidth within 0–120 Hz (below 50 dB)	9 Hz	NA	12 Hz	6 Hz
Effective mitigation bandwidth within 0–500 Hz (below 50 dB)	9 Hz	44 Hz	59 Hz	57 Hz
Structural complexity	High	High	High	Moderate

compare the total sound power levels $L_{w,\text{total}}$ of Strategies 1, 2 and 3, which share a similar structural framework (see Fig. 10(e)). Unlike Strategy 2, both Strategies 1 and 3 prioritize the retention of the first bandgap. Thus, in scenarios in which maintaining the first bandgap is necessary, Strategy 1 produces a lower $L_{w,\text{total}}$ due to the chaotic effect between 100 and 300 Hz (between two red regions). Conversely, Strategy 3 introduces an additional bandgap near 300 Hz. However, in cases where preserving the first bandgap is not the primary objective, Strategy 2 demonstrates superior overall sound radiation mitigation performance. Overall, each strategy offers unique benefits and drawbacks that should be balanced to accommodate different applications. A comprehensive comparison of the data and key performance indicators are presented in Tables 2 and 3.

Table 2 details the specific parameters relating to the four strategies, including the natural frequency of the oscillator, nonlinear stiffness, mitigation ranges for the fundamental and third harmonics, and range of the chaotic band. These parameters reflect the performance advantages and disadvantages of the four strategies from a quantitative perspective, to support the preceding analyses.

To further highlight the features of each strategy, several key indicators are extracted for characterization, as summarized in Table 3:

Maximum sound power reduction: to represent the maximum mitigation value of the overall sound power level, reflecting the peak mitigation capability of the strategy.

Effective mitigation bandwidth within 0–120 Hz: to quantify the bandwidth where the overall sound power level remains below 50 dB within the 0–120 Hz range, reflecting the low-frequency mitigation performance.

Effective mitigation bandwidth within 0–500 Hz: to denote the bandwidth where the overall sound power level stays below 50 dB within the 0–500 Hz range, indicating the overall mitigation bandwidth.

Structural complexity: to reflect the intricacy of the configuration. Since the unit cells of the first three strategies involve one more oscillator than that of the fourth strategy, their structural complexity is higher.

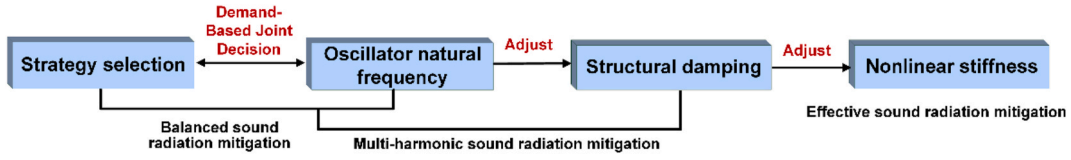


Fig. 11. Schematic of the parameter optimization workflow, comprising strategy selection, tuning of oscillator resonance frequencies, structural damping adjustment, and nonlinear stiffness regulation to achieve multi-harmonic, balanced, and effective sound radiation control.

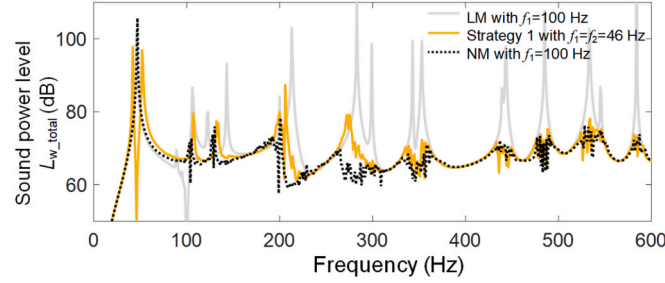


Fig. 12. Sound power level L_{w_total} for LM with $f_1 = 100$ Hz, strategy 1 with $f_1 = f_2 = 46$ Hz and NM with $f_1 = 100$ Hz under a 10% added mass.

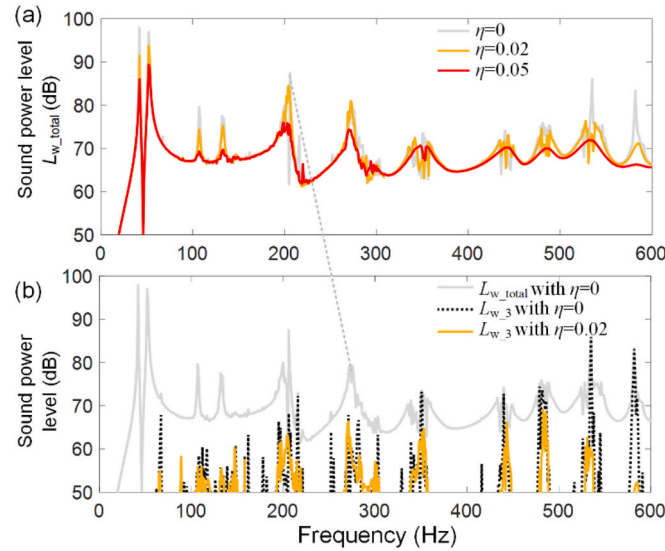


Fig. 13. Sound power levels with different loss factor η . (a) L_{w_total} with increasing η . (b) L_{w_3} with increasing η .

Evaluation of these metrics lead to the following selection guidelines. Strategy 1 or 4 should be favored if the primary concern is the maximum mitigation level; if low-frequency mitigation bandwidth is prioritized, Strategy 3 is recommended; if overall mitigation bandwidth is desired, Strategy 2 is preferred; and for lower structural simplicity, Strategy 4 is a better choice.

4. Parameter optimization for broadband sound radiation mitigation

While the four proposed strategies provide a robust framework for general NAM systems design, further optimization must account for critical engineering parameters. Effectively controlling both the fundamental and third-harmonic radiation through parameter tuning remains a formidable challenge. NAM designs often exhibit a frequency-dependent imbalance, where superior radiation mitigation at high

frequencies is offset by insufficient performance in the low-frequency regime. In practical applications, excessive added mass to an existing structure is a concern that should be alleviated [13,14]. Moving forward, we now aim to bridge these gaps through parameter tuning, striving to achieve multi-harmonic, balanced and effective sound radiation control at a given added mass level (10% is adopted as a compromise).

Here, we will select one strategy as an example to perform the parameter optimization. First, achieving effective sound radiation control requires tuning the oscillator's natural frequency and selecting an appropriate strategy. Secondly, implementing structural damping is a well-established engineering approach for vibration and noise reduction, especially targeted at the third harmonic. However, its application necessitates a balance with the structural load limits. Thirdly, nonlinear stiffness is another crucial parameter, as either excessive or insufficient nonlinear intensity can affect the overall mitigation effectiveness.

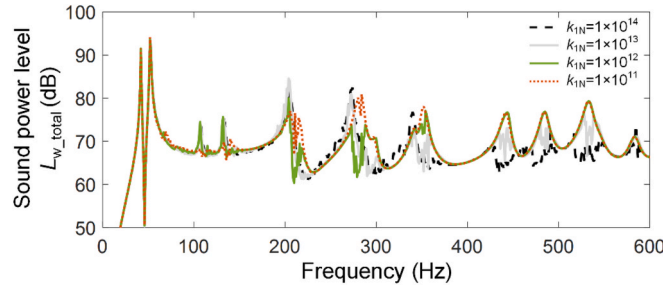


Fig. 14. Sound power level L_{w_total} for LM and NM with different nonlinear stiffness k_{1N} .

Following the logic flow illustrated in Fig. 11, the effects of these parameters are systematically investigated in the following sections.

4.1. Parameter tuning

As shown in Fig. 4(a), using the initial model configuration ($f_1 = 100$ Hz), the chaotic bands effectively suppress L_{w_total} only at frequencies around and beyond 100 Hz. Our previous study [38] suggests that nonlinearity should be introduced into oscillators with the minimum feasible stiffness to maximize the chaotic bandwidth. Furthermore, this initial model configuration ($f_1 = 100$ Hz) leaves the system's first-order resonance peak with a relatively high L_{w_total} . Consequently, Strategy 1 can be adopted, wherein f_1 is strategically adjusted to coincide with the system's primary resonance frequency by modifying the linear stiffness k_1 . This alignment serves a dual purpose: directly mitigating the prominent first-order resonance peak through the dynamic absorber principle (both at fundamental and third harmonic frequencies) while simultaneously broadening the effective bandwidth of the chaotic band for enhanced broadband performance. As shown in Fig. 12, the first resonance peak of the system is mitigated, and broadband mitigation is still achieved beyond this peak due to the chaotic band.

Constrained damping layers, asphalt, and other viscoelastic materials are commonly employed to mitigate vibration and sound radiation of structures in the mid-to-high frequency range [50,51]. Here, we use the same method in Section 3.3 to simulate the damping effect of the plate. Based on the above optimization, Fig. 13(a) shows the variation in L_{w_total} with increasing η under $F_0 = 90$ N. It is observed that when $\eta = 0$, the L_{w_total} exhibits multiple distinct radiation peaks in the frequency range of 300 Hz to 600 Hz. When we extract the L_{w_total} and L_{w_3} with $\eta = 0$ separately in Fig. 13(b), these radiation peaks are primarily dominated by excessive third harmonic components. Increasing η to 0.02—a level that can be achieved by spraying a very thin layer of damping paste onto the plate (almost no additional weight)—the peak values of L_{w_total} are suppressed by 3–5 dB, as shown in Fig. 13(a). Most importantly, the third harmonic components are suppressed, falling well below L_{w_total} with $\eta = 0$, as shown in Fig. 13(b), evidencing an effective mitigation of

the third harmonic radiation in the mid-to-high frequency range. As expected, if η can be further increased to 0.05 (or higher), the mitigation of sound radiation would be even more significant; however, this would inevitably introduce additional mass. Weighing the trade-off between the noise reduction and the requirement for a lightweight design, $\eta = 0.02$ is selected as the target parameter in the subsequent optimization.

The nonlinearity strength is governed by the synergistic contribution of the nonlinear stiffness k_{1N} and the excitation force F_0 . Here, we maintain $F_0 = 90$ N constant and vary k_{1N} to investigate its specific influence on the radiated sound power level L_{w_total} . As shown in Fig. 14, based on the damping optimization strategy, the parameter k_{1N} is adjusted from its reference value of 1×10^{13} N/m³ (gray line) to 1×10^{11} , 1×10^{12} , and 1×10^{14} N/m³, respectively. The results indicate that when k_{1N} is increased to 1×10^{14} N/m³, although the low-

Table 4
Optimized model parameters.

Symbol	Definition	Value	Symbol	Definition	Value
a	Lattice constant	0.014 m	m_0	Lumped mass	0
$L_p \times W_p \times H_p$	Length $\times$ Width $\times$ Thickness	$0.7 \times 0.84 \times 0.006$ m	m_1	Oscillator mass	0.016 kg $\times 2$
ρ	Density	2700 kg/m ³	f_1, f_2	Natural frequency	46 Hz
E	Elastic modulus	67 GPa	k_{1N}	Nonlinear coefficient	1×10^{11} N/m ³ (low) 1×10^{12} N/m ³ (mid) 1×10^{14} N/m ³ (high)
ν	Poisson's ratio	0.3	η	Total loss factor	0.02

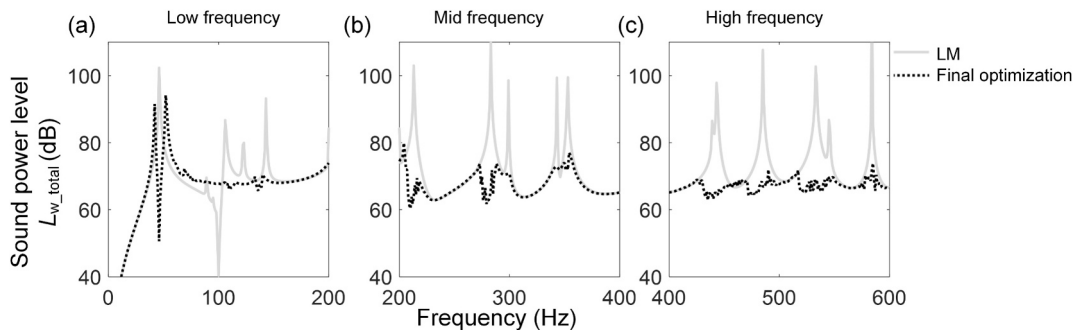


Fig. 15. Sound power levels L_{w_total} . (a) Final optimization for low frequency ($k_{1N} = 1 \times 10^{11}$ N/m³); (b) Final optimization for mid frequency ($k_{1N} = 1 \times 10^{12}$ N/m³); (c) Final optimization for high frequency ($k_{1N} = 1 \times 10^{14}$ N/m³).

frequency response remains unchanged, the total sound power level $L_{w, \text{total}}$ is significantly reduced in the frequency range of 350–600 Hz. Conversely, when k_{1N} is decreased to $1 \times 10^{12} \text{ N/m}^3$, the mitigation of $L_{w, \text{total}}$ shifts to the mid-frequency range of 200–300 Hz. If k_{1N} is further reduced to $1 \times 10^{11} \text{ N/m}^3$, the $L_{w, \text{total}}$ in the low-frequency range below 200 Hz is suppressed to the maximum extent, at the expense of compromising the control performance at higher frequencies.

4.2. Final design

By tuning k_{1N} , we demonstrate that a proper reconfiguration of k_{1N} enables tailored acoustic radiation control targeting specific frequency ranges. Therefore, targeting the low- (0–200 Hz), mid- (200–400 Hz), and high-frequency (400–600 Hz) ranges, respectively, the final optimized designs are obtained by incorporating the selected Strategy 1, optimal oscillator resonant frequencies and structural damping, as illustrated in Fig. 15. The specific optimized parameters are tabulated in Table 4. The results indicate that in the low-frequency range in Fig. 15(a), compared to the LM, the first resonance peak is suppressed by 8 dB, while the reduction of all other resonance peaks exceeds 13 dB. For the mid-frequency range optimization in Fig. 15(b), the mitigation of each resonance peak reaches at least 20 dB; whereas in the high-frequency range in Fig. 15(c), the attenuation of all resonance peaks exceeds 30 dB. It can thus be seen that under the proposed optimization philosophy, a higher target frequency range yields a more significant sound radiation mitigation. Furthermore, owing to Strategy 1 and the inherent damping treatment of the plate, the third harmonic radiation near the bandgap as well as in the mid-to-high ranges is effectively suppressed (see Appendix A.3).

In summary, the optimized design allows the resonance peaks to be suppressed by 8–30 dB with a mere 10% added mass ratio. It successfully achieves highly efficient, broadband, and multi-harmonic sound radiation control across various target frequency ranges. These findings highlight the efficacy and practicality of the proposed approach, offering a promising and lightweight solution for vibration and noise reduction in demanding engineering applications.

5. Conclusions

This study systematically examines the sound radiation characteristics of typical NAMs, including both fundamental and harmonic frequency components. It is revealed that, within the bandgap, increased nonlinearity level triggers bandgap degeneration in structural vibration, causing the NAMs to lose their local resonance effect; consequently, the sound power level rises significantly relative to their linear counterparts. Furthermore, the existence of multiple solution branches causes the sound radiation efficiency to fluctuate, which in turn leads to

fluctuations in the sound power level. Outside the bandgap, radiation efficiency remains rather independent of the specific NAM design, with sound radiation mitigation primarily governed by vibration suppression. Crucially, the acoustic power level at the third-harmonic frequency is found to be pronouncedly elevated, driven by energy pumping near the bandgap and high radiation efficiency in the mid-to-high frequency regime, respectively.

Structural damping can effectively suppress the third-harmonic component in the mid-to-high frequency range with very limited success near the low-frequency bandgap. To overcome this limitation, we propose four distinct strategies based on linear-nonlinear band control, thereby offering versatile and effective solutions tailored to diverse engineering scenarios. Alongside the proposed band-based control strategies, further optimization is showcased through system parameter tuning. By fine-tuning these parameters (resonance frequency of the oscillators, structural damping and nonlinearity strength, ...), in conjunction with the proposed strategy 1 as an illustrative example, the optimized design entails the reduction of resonance responses by 8–30 dB with a mere 10% added mass. These findings underscore that the proposed NAM design can successfully achieve highly efficient, broadband, and multi-harmonic sound radiation control across various target frequency ranges. The proposed approach proves highly viable for a wide range of practical engineering applications, particularly where lightweight and high-performance noise reduction solutions are needed.

CRediT authorship contribution statement

Chen Gong: Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Xin Fang:** Writing – review & editing, Supervision, Resources, Project administration, Funding acquisition, Formal analysis, Conceptualization. **Li Cheng:** Writing – review & editing, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A1. . Average vibration response R

The average vibration response writes:

$$R_{\text{order}}(f) = 20 \log_{10} \left(\sqrt{\frac{\sum_{i=1}^{30} (V_{s, \text{order}}(f))^2 S_i}{S_{\text{total}}}} \right) \quad (\text{order} = 1/3, 1, 3), \quad (\text{A1})$$
 in which $V_{s, \text{order}}$ represents the velocity of the order -harmonic ($\text{order} = 1/3, 1, 3$) component at each metacell location (x_s, y_s) ($s = 1, 2, \dots, 30$), S_s and S_{total} are the area of the s^{th} metacell and that of the entire *meta*-plate, respectively.

Appendix A2. Influence of Bragg bandgap with lumped mass m_0

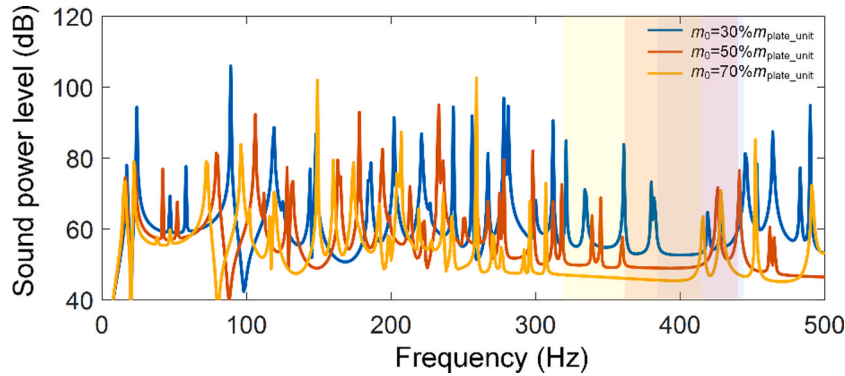


Fig. A2. Sound power level L_{w_total} for LM with different concentrated mass m_0 .

In plate-like metamaterials, incorporating concentrated masses m_0 into the plate can induce a significant Bragg bandgap. To generate a Bragg bandgap below 400 Hz, two primary approaches are available: reducing the plate thickness or increasing the lattice constant. Utilizing a lattice constant of $a = 0.2$ m as a representative case, we explore the relationship between m_0 and the Bragg bandgap characteristics, as shown in Fig. A2. The yellow, orange, and blue regions correspond to the Bragg bandgaps produced by masses m_0 equaling 30%, 50%, and 70% of the unit plate's mass, respectively.

As the added mass increases, the Bragg bandgap width gradually expands. However, m_0 must reach at least 70% of the unit plate's mass to establish a substantial and effective bandgap. In practical engineering applications, such a significant weight penalty is often prohibitive due to structural integrity concerns and strict weight constraints. When m_0 is restricted to a more feasible 30% of the unit plate's mass, it only suppresses a highly localized range of $L_{w,3}$, as shown in Fig.10(d). This narrow mitigation bandwidth is insufficient to fulfill the design objective of achieving broadband sound radiation control. Consequently, given these inherent physical and engineering constraints, the Bragg bandgap strategy is ultimately abandoned in the final optimization.

Appendix A3. Mitigation of third harmonic radiation via final optimization

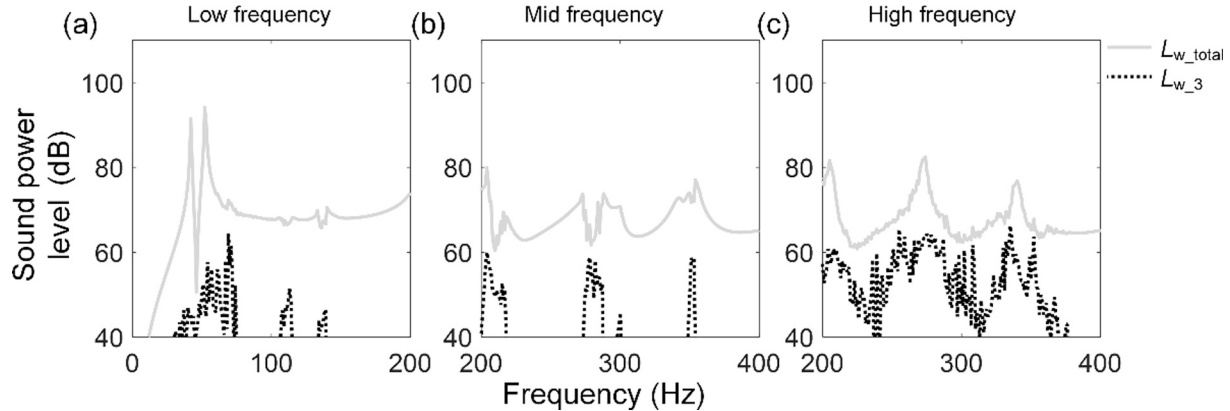


Fig. A3. Sound power levels L_{w_total} and $L_{w,3}$ (a) Final optimization for low frequency ($k_{1N} = 1 \times 10^{11}$ N/m³); (b) Final optimization for mid frequency ($k_{1N} = 1 \times 10^{12}$ N/m³); (c) Final optimization for high frequency ($k_{1N} = 1 \times 10^{14}$ N/m³).

Fig. A3 shows the final optimized total sound power level L_{w_total} and its corresponding sound power level at the third harmonic frequency $L_{w,3}$. The results indicate that, owing to the synergistic effect of Strategy 1 and the inherent damping of the plate, the L_{w_total} near the bandgap as well as in the mid- to high-frequency bands is effectively suppressed below the L_{w_total} .

Data availability

Data will be made available on request.

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