Rotational Compliance Measurements of a Flexible Plane Structure Using an Attached Beam-like Tip, Part 1: Analysis and Numerical Simulation

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The present paper describes a simple, indirect technique for measuring the rotational compliance of an attached thin-walled plane structure. The technique, called "Tip Excitation Technique (TET)," uses an L-shaped beam tip fixed at one point of the original structure where the rotational compliance is to be evaluated. The tip acts as a mechanical converter transforming an exciting force applied at the tip into an excitation moment applied to the original structure and reciprocally, to convert a rotational response into a translational one. The rotational compliance is evaluated using simple analytical relations that partially eliminate the effect of tip inertia on the structure. Finite element simulations show that the technique can be applied to the study of a typical plate over a frequency range involving relatively strong modal behavior. The low sensitivity of the technique to variations of different physical factors of the tip-plate configuration demonstrates its potential for experimental purposes.

1 Introduction

Compliance or mobility is widely used in investigations of mechanical structures since it characterizes their intrinsic dynamic behavior. Mobility is used more frequently than compliance in the literature, but essentially, related by a simple relation, both provide the same information. In engineering practice, the compliance, or mobility, is used in various ways. For example, it is usually used to model the effect of supporting structures (Goyder and White 1980; Petersson and Plunt 1982; Pinnington and Pearce 1990) in different domains such as aeronautics, nuclear energy and machine-tool applications. Instead of assuming an ideal support, the support mobility is used to describe its dynamic behavior. The mobility concept can also be used to describe the mechanical characteristics at the joint of interconnected components of complex structures (Cuschieri 1990; Petersson 1993; Goyder 1980). In such applications, however, each component may have a high modal density, making it difficult to obtain good estimates of the mobility. The majority of work reported in the literature only uses translational mobility, since this can be easily measured, although it is generally admitted that rotational mobility also plays an important role in the analysis of flexible structures; this is usually the case for dynamic and sound radiation applications (Ewins 1991; Cheng 1994). Recognizing the broad application of rotational mobility, or compliance, in structural analysis, several researchers have deplored the lack of experimental techniques available to measure it (Goyder 1980; Ewins 1991; Naji 1993). Moreover, numerous engineering applications involve complex structures whose mobility can only really be obtained experimentally. This is particularly true for certain experimental-analytical hybrid methods treating mechanically coupled structures (Goyder 1980; Cheng and Richard 1995). Hence, reliable experimental techniques must be developed.

Generally speaking, mobility, or compliance is a tensor relating a generalized response (translation or rotation) to a generalized excitation (force or moment). Translational mobility can be properly measured using a number of techniques that are well documented in the literature. A good review on the subject can be found in the papers of Ewins (1981; 1986) which summarize various approaches used in Europe. Similar techniques are also used worldwide. Measurement of rotational mobility or compliance, however, is still a major problem as pointed out by Ewins (1984). The first problem is that it is more difficult to measure rotation than translation. The use of a pair of accelerometers, fixed a short distance apart on the structure, has been evaluated in the past (Ewins 1984). Using this technique, great care must be taken in matching the two accelerometers if accurate measurements are required. The newly developed laser measurement techniques seem to provide a potential solution to the problem (Eastwood and Halliwell, 1985). A second difficulty in estimating rotational mobility is how to produce the excitation torque. In fact, a point torque, although easy to define mathematically, is experimentally rather difficult to implement (Goyder 1980; Ewins 1972). This problem is even more crucial when one deals with light thin-walled structures, since any measuring device physically connected to the system will tend to affect the dynamics of the original structure. Available excitation methods using a single exciter at multiple positions and the use of special fixtures are described by ANSI S.2.34 (1984). Also of interest is the work by Petersson (1987) where giant magnetostrictive devices were used to produce a moment excitation.

The two present papers introduce a simple and practical technique for measuring the rotational compliance of flexible structures. The technique was first tested using a cantilever rectangular plate, as shown in Figs. 1(*a*) and 1(*b*). A beam-like tip was rigidly attached to a point on the plate where the rotational

Contributed by the Technical Committee on Vibration and Sound for publication in the JOURNAL OF VIBRATION AND ACOUSTICS. Manuscript received Sept. 1994; revised Sept. 1995. Associate Technical Editor: J. Mottershead.

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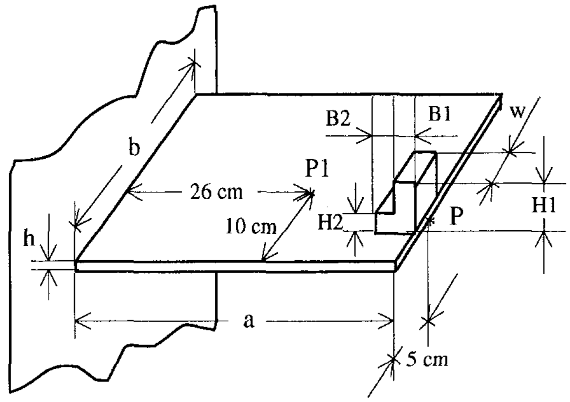

Fig. 1a Geometrical configuration

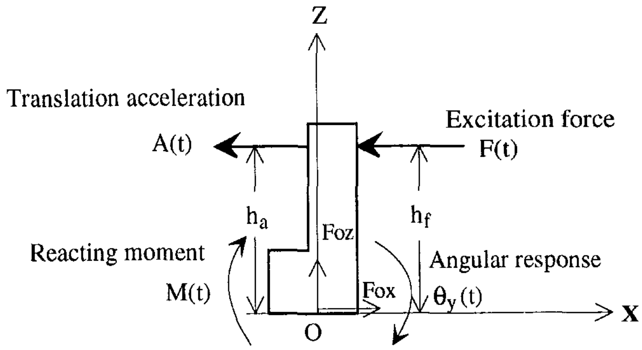

Fig. 1b Dynamic model of the beam-like tip and the local coordinate system

compliance had to be evaluated (a point located on the free edge is chosen to illustrate the technique). By applying a driving force $F(t)$ at the tip, an excitation moment $M(t)$ was produced at the plate edge. Based on simultaneous measurements of the exciting force and the resulting acceleration $A(t)$, we derived the rotational compliance of the plate $\theta(t)/M(t)$, where $\theta(t)$ is the rotation angle of the plate edge. A rigid body dynamic analysis of the tip was performed, taking into account the coupling at the intersection of the plate and the tip, resulting in analytical formulas for rotational compliance. In order to evaluate both the advantages and limitations of the technique, a systematic numerical analysis was also carried out using finite element methods. The various hypotheses made in developing the model were analyzed, as well as the sensitivity of the measuring technique to variations in different parameters of the system. Other factors that cannot be numerically simulated are presented in a companion paper experimentally assessing the technique (Qu, Cheng and Rancourt, 1997).

The proposed technique is called "Tip Excitation Technique (TET)". It can be regarded as a modification and simplification of the technique presented by Ewins (1984), by considering the test structure as a light-weighted plane structure. The main advantage of the TET is that one can use a single accelerometer and excite the structure in the in-plane direction, thus minimizing the unwanted transverse motion which results from a transverse excitation. In addition, TET has two major appealing features: (1) it converts a rotational measuring problem to a translational one, requiring only standard equipment such as an accelerometer, a shaker or a hammer; and (2) analytical expressions can partially eliminate the additional tip inertia effects and thereby minimize the interference of the tip with the original plate.

2 Derivation of the Rotational Compliance Equations

For a typical plane structure, a local Cartesian coordinate system (O-XYZ) is employed as illustrated in Fig. 1(*b*). The rotational compliance about the Y axis will be studied. Generally speaking, the tip will undergo a motion involving three translational movements and three rotations. Two consequences follow: (1) the angular response of the plate is a result of not only the direct exciting force, but also all the inertia forces and moments transmitted by the tip; (2) as some coupling exists between these quantities, to obtain accurate estimates of the excitation moment on the plate, this must be taken into account. In order to make the technique practical, assuming that the exciting force is applied in the in-plane direction of the plate, we propose a number of hypotheses and simplifications regarding the tip-plate model. As to the first consequence mentioned above, hypotheses are made concerning the relative importance of the different inertia terms. A detailed check is then made in Sec. 3 using numerical simulations. Regarding the second one, a complete dynamic of the tip is performed considering all possible inertia terms.

The following assumptions are made:

(1) the tip is properly dimensioned so that it can be regarded as a rigid body with respect to the test structure, i.e., the stiffness of the tip is such that its natural frequencies are much higher than the frequency range of interest;
(2) the tip is rigidly connected to the plate to ensure continuity. It is assumed that the internal damping of the tip, and losses due to friction at the tip-plate interface, are negligible;
(3) the resulting amplitudes are small so that one can apply linear theory;
(4) the interaction effects between the structure and the tip are represented by a force passing through the origin O, and a resulting moment whose component about Y-axis is $M(t)$. Due to the excitation force applied to the tip, the resulting moment $M(t)$ is assumed to dominate the other five reactive inertia forces and moments.
(5) the excitation force $F(t)$ and the response acceleration $A(t)$ are parallel to the X axis.
(6) the governing wavelength is large compared with the tip dimensions.

On the basis of these assumptions, a dynamic analysis of the tip motion is performed to evaluate the reacting moment $M(t)$. By isolating the tip and applying the Euler equation for rotational movement about the Y axis, one obtains the following equation:

$$m_t\cdot(a_{ox}\bar{z} - a_{oz}\bar{x}) + I_{yy}\cdot\ddot{\theta}_y + (I_x - I_z)\cdot\dot{\theta}_x\cdot\dot{\theta}_z + I_{yz}\cdot(\dot{\theta}_y\cdot\dot{\theta}_x - \ddot{\theta}_z) - I_{xy}\cdot(\dot{\theta}_y\cdot\dot{\theta}_z + \ddot{\theta}_x) + I_{xz}\cdot(\dot{\theta}_x^2 - \dot{\theta}_z^2) = M(t) - h_f\cdot F(t) \quad (1)$$

where I_{yy}, I_{yz}, I_{xy}, I_{xz}, are the moments of inertia of the tip; θ_x, θ_y, θ_z the angular displacements about the X, Y, Z axes respectively; $\bar{x}$, $\bar{z}$ the coordinates of the mass center of the tip in the XOZ plane; a_{ox}, a_{oz} the two acceleration components of the tip along the X and Z axes; and m_t the mass of the tip and h_f the vertical distance between the applied force $F(t)$ and the X axis.

The first two terms on the left-hand side of the equation represent the effects of the inertia forces of the tip. Since the Z axis is set to pass through the mass center of the tip, $\bar{x} = 0$. Moreover, a_{ox} is the in-plane acceleration component. Considering the high stiffness of the thin-walled structure in this direction (in-plane motion), a_{ox}, θ_z, $\dot{\theta}_z$ and $\ddot{\theta}_z$ become negligible. Additionally, $I_{xy} = I_{yz} = 0$ due to the symmetric geometry of the tip about the XOZ plane. With all these factors taken into account, Eq. (1) becomes:

$$I_{yy}\ddot{\theta}_y + I_{xz}\cdot\dot{\theta}_x^2 = M(t) - F(t)\cdot h_f \quad (2)$$

Note that the second term on the left-hand side of the Eq.

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(2) represents the coupling between the two rotational motions about the Y and X axes. Assuming small amplitudes, Eq. (2) can be linearized and, neglecting higher order terms, reduces to:

$$I_{yy}\ddot{\theta}_y = M(t) - F(t)\cdot h_f \tag{3}$$

The excitation is assumed to be harmonic with the form:

$$F(t) = F_o\cdot e^{j\omega t}, \tag{4}$$

where F_o is the force amplitude and ω, the angular frequency.

The steady-state solution is written as

$$\theta_y(t) = \theta_o\cdot e^{j(\omega t+\phi_\theta)} \tag{5}$$

where θ_o is the amplitude of the angular displacement response and ϕ_θ, the phase angle. Similarly, the translational acceleration also takes a sinusoidal form:

$$A(t) = A_o\cdot e^{j(\omega t+\phi_a)} \tag{6}$$

where A_o is the amplitude of the linear acceleration response and $\phi_a = \phi_\theta$ is the phase angle.

Substituting Eqs. (4) and (6) into Eq. (3) gives:

$$M(t) = (h_f\cdot F_o - I_{yy}\cdot\theta_o\cdot\omega^2 e^{j\phi_\theta})e^{j\omega t} \tag{8}$$

According to the rigid body hypothesis concerning the tip, the following relation exists between the angular displacement and linear acceleration amplitudes;

$$\theta_o\cdot\omega^2 = \frac{A_o}{h_a} \tag{9}$$

where h_a is the distance between the X axis and the point where the linear acceleration is measured.

Using the above expression, Eq. (8) becomes

$$M(t) = \left(h_f\cdot F_o - I_{yy}\cdot\frac{A_o}{h_a}e^{j\phi_a}\right)\cdot e^{j\omega t} \tag{10}$$

According to the definition of the rotational compliance CR and the sign conventions illustrated in Fig. 1(b),

$$CR = -\theta_y(t)/M(t) \tag{11}$$

Eqs. (10) and (5) are used together with Eq. (7) to calculate CR. After simplifying the resulting expressions, one obtains CR in the complex form:

$$CR = \frac{1}{\omega^2}\bigg/\left(I_{yy} - \frac{K}{T}\cdot e^{-j\phi_a}\right) \tag{12}$$

where $K = h_f\cdot h_a$ is a geometric parameter, and $T = A_o/F_o$ is the amplitude of the translational accelerance.

From Eq. (12), the compliance can be expressed in terms of the amplitude A_{cr} and the phase angle ϕ_{cr} as:

$$A_{cr} = \frac{1}{\omega^2}\bigg/\sqrt{\left(\frac{K}{T}\right)^2 - 2\cdot I_{yy}\cdot\frac{K}{T}\cdot\cos\phi_a + I_{yy}^2}$$

$$tg\phi_{cr} = \frac{-I_{yy}\sin\phi_a}{I_{yy} - \frac{K}{T}\cdot\cos\phi_a} \tag{13}$$

Several remarks are worth mentioning concerning the above expressions. Apart from the geometrical properties of the tip, the rotational compliance is expressed as a function of one single measurable quantity: the translational accelerance (amplitude T and phase angle ϕ_a) at a given point on the tip along the X-axis. Furthermore, since the above expressions include the motion of the tip, the inertia effects of the tip can be partially eliminated. Except for the assumption of a plane structure, no other information about the measured structure was used to obtain these expressions. Consequently, the obtained compliance expressions apply, in principle, to any type of thin-walled plane structure.

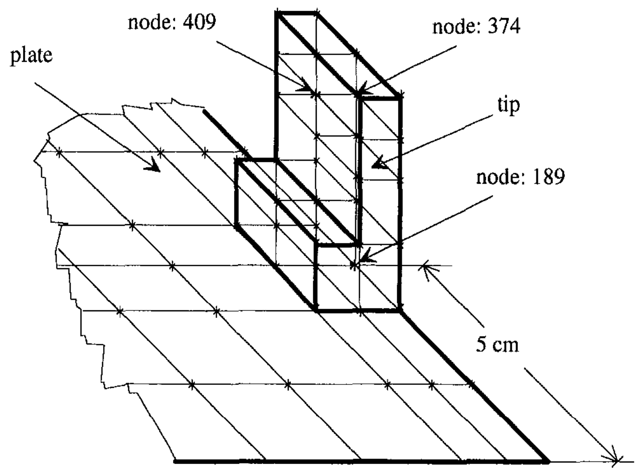

Fig. 2 Finite element mesh of the structure

3 Numerical Simulations

Numerical simulations were used to assess the reliability of the proposed technique and to evaluate the tip effects on the compliance measurements. Since no analytical solutions exist for the plate-tip combined system, a finite element analysis (FEA) was performed on a cantilever rectangular plate (c.f. Fig. 1(a)). The FEA was used to simulate two systems. The first one is referred to as the *reference system*, i.e. a single plate with a pure moment excitation applied at the edge. The results obtained from this system will be used as a reference basis. The second system simulated by FEA is referred to as the *simulated system*, i.e. the plate-tip combination, where the tip is also considered as a flexible component. For this model, a driving force was applied to a given point on the tip.

The FEA was carried out with I-DEAS. The mesh of the structure is given in Figure 2. The plate was discretized into 352 linear quadrilateral thin shell elements and the tip was divided into 10 linear solid brick elements. Nine common nodes were used to simulate a rigid plate-tip joint. For the *reference system* model, a unit moment was applied at node 189, as shown in Figure 2. The angular displacement at the same node was calculated to derive the *expected results* of the rotational compliance. For the *simulated system*, a unit force excitation parallel to the X-axis was applied at node 374, which has the same X, Y coordinates as node 189. The translational acceleration was calculated at node 409. The rotational compliance was then calculated using the formula derived in the previous section. The results obtained are referred to as the *simulated TET results*.

A basic configuration was chosen with a steel plate having the following characteristics: length a = 400 mm, width b = 300 mm, thickness h = 3 mm, density ρ_p = 7820.0 kg/m^3, Young's modulus E_p = 2.068E + 11 N/m^2 and Poisson ratio μ_p = 0.30. An L-shaped aluminum tip was fixed at the edge of the plate as shown in Fig. 1a. The tip dimensions are given in Table 1. The tip geometry was dimensioned based on two main criteria: firstly, the tip should be stiff enough to minimize dynamic coupling with the plate and properly transmit a moment excitation to the plate; secondly, it should be small enough to reduce the inertia effect on the plate dynamics. The aluminum tip had the following material properties: density 2768 kg/m^3, Young's modulus E_p = 7.2E + 10 N/m^2, and Poisson ratio of 0.3. A damping ratio of 0.01 was introduced for both the plate

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Table 1 Dimensions of the tip for the basic configuration

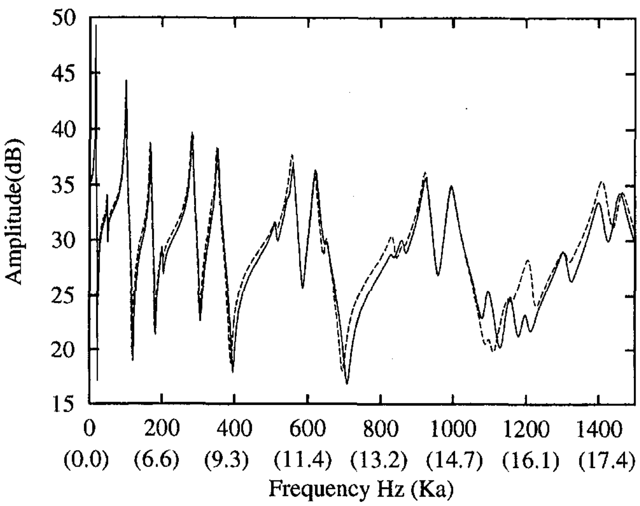
	B=B1+B2	H1	B1	H2	B2	W
Dimension (mm)	10	30	5	10	5	20
Dimensions normalized to B	1	3	0.5	1	0.5	2

and the tip. The rotational compliance of the plate was evaluated at the point P shown in Figure 1a.

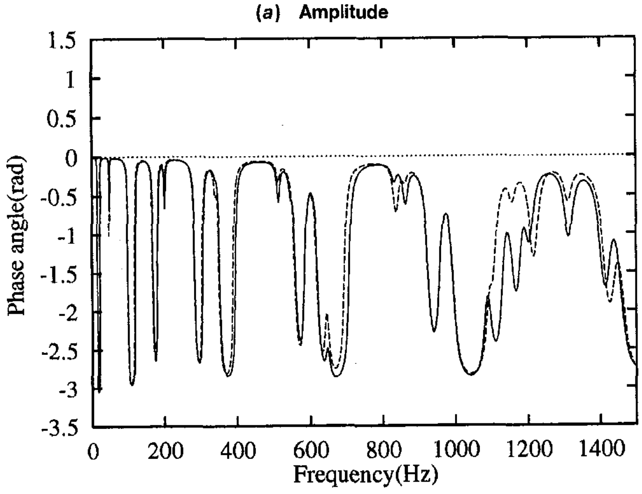
To demonstrate the overall performance of the technique, a comparison was made between the *expected results* and the *simulated TET results.* The rotational compliance is presented in terms of amplitude (Fig. 3(a)) and phase angle (Fig. 3(b)). The amplitude of the compliance is expressed in dB referenced to 1.0E−6 rad/N·m (10 log (A_{cr}/1.0E−6)), and the phase angle in radians. The calculations were performed for a frequency range of 0–1500 Hz. A secondary scale is given in parallel with the frequency scale, using a non-dimensional parameter Ka, where K is the bending wavenumber of the plate ($K = \sqrt[4]{\rho_p h\omega^2/D}$; D being the flexural rigidity) and a the length of the plate.

Generally speaking, there is good agreement between the two sets of results for the investigated structure, especially in the low to middle frequency ranges (<1000 Hz). In fact, the amplitude difference is within 1.5 dB for this range and the phase difference is also reasonably small. These results support several assumptions that we made previously. They show that a moment excitation can be effectively generated at the plate-tip interface. Furthermore, the angular response measured with the tip characterizes the angular response of the plate very well. The tip has its first resonant frequency around 7600 Hz, which is sufficiently far above the frequency range considered in this analysis. Consequently, for the present plate-tip configuration, it can be approximated as a rigid body. Finally, it has been well demonstrated in the literature that the addition of a local mass may significantly decrease the natural frequencies of a structure (Robert and John 1990). The good coincidence of the resonant peaks in Fig. 3(a) indicates that the analytical compliance expressions which were developed, partially suppress the rotational inertia effect introduced by the tip. The tip effect becomes more significant as the frequency increases, as one can observe beyond 1000 Hz. In order to verify these remarks, a series of tests were carried out, changing the tip location and other geometrical parameters of the system. Due to the restriction on the length of the paper, results of only one other point, *i.e.* point $P1$ in Fig. 1(a), are presented in Fig. 4. Similar results were obtained for other cases.

Among all the hypotheses used to develop the method, the fourth one, regarding the inertia quantities, constitutes the major simplification from reality. In order to demonstrate the effects of other tip inertia quantities on the response, that have been neglected by the technique, additional calculations were performed using FEA. Under the force excitation, tip accelerations were first calculated by FEA. All six inertia forces and moments of the tip were then calculated. These forces and moments were applied independently to the plate, without the tip, to calculate the resulting angular response in the XOZ plane over the frequency range (0–1000 Hz). Figure 5 shows the rotational contribution (in the XOZ plane) of $M(t)$, as well as that of the inertia force in the OZ direction and the inertia moment in the YOZ plane. Other inertia forces and moments due to the in-plane motion of the plate were shown to be much smaller than the three quantities presented in Fig. 5. It can be seen from Fig. 5 that, compared to $M(t)$, the other two inertia quantities have negligible effects on the system for most frequencies. However,



(*a*) Amplitude



(*b*) Phase angle

Fig. 3 Comparison of the rotational compliance based on the basic configuration. expected value: (——); simulated TET value: (---).

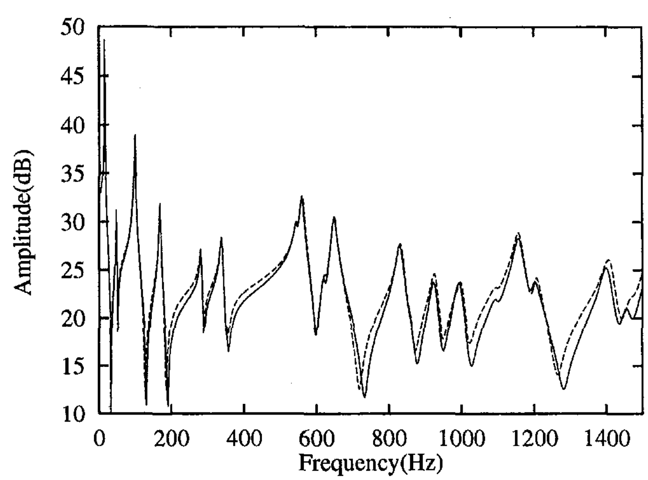


Fig. 4 Comparison of the rotational compliance amplitude at point P1 of the plate.expected value: (——); simulated TET value: (---).

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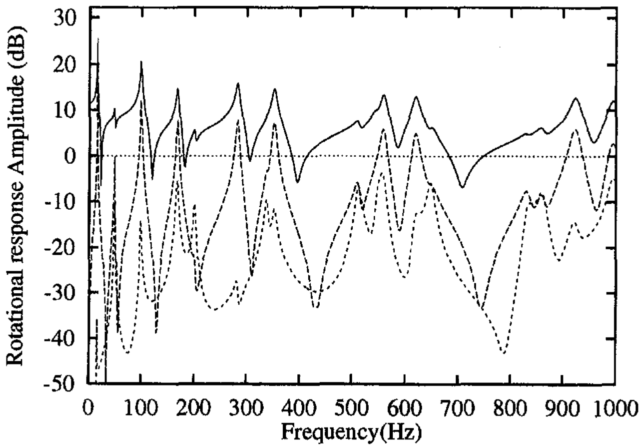
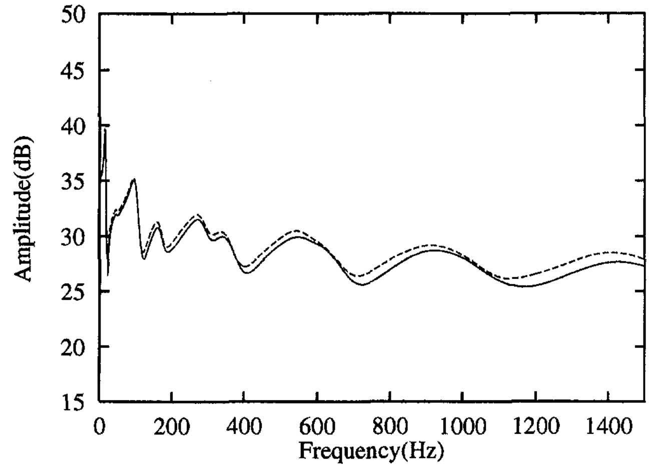


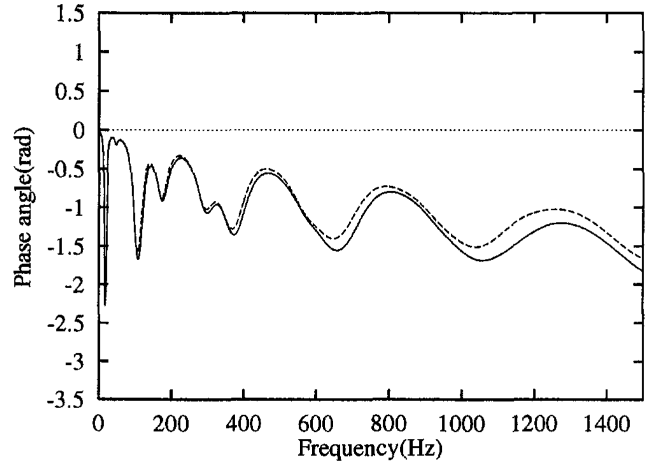
Fig. 5 Rotational response due to inertia forces and moments: moment $M(t)$ in XOZ plane: (——); force in OZ direction: (– – –); moment in YOZ plane: (- - -).

the transverse motion (in the OZ direction) seems to have a more noticeable influence on the measurements around the resonant values of the system. This observation suggests that the accuracy of the technique may be enhanced under certain circumstances, at the price of adding another accelerometer in the transverse direction to include the transverse inertia force into the compliance expression. For the present configuration, however, this seems to be unnecessary, since even at resonance, the transverse inertia force effects are still 8 to 9 dB weaker than the moment excitation effects. Considering the definition of the dB used in this paper, the transverse inertia effects are only 12 to 16% of the moment excitation effects. These contributions can therefore be neglected for the structure investigated here, particularly at frequencies far from resonance.

Influence of the System Damping. The damping effect that was neglected in the analytical treatment was analyzed using FEA. Identical damping values were assigned to the plate and the tip. Figs. 6(*a*) and 6(*b*) respectively present the amplitude and phase angle for a highly damped structure with a damping ratio of 0.1. Again, one notices the good agreement between the two sets of results, indicating that the model developed without damping is still applicable to a highly damped system. This is understandable since a tip of such dimensions behaves as a rigid body, so that the internal damping of the tip has little noticeable effect on the measurements. The differences between Figs. 3 and 6 illustrate the influence of the damping characteristics. It can be clearly observed that the amplitude, as well as the phase angle, depends strongly on the damping value of the plate over the full frequency range. Nevertheless, from the point of view of compliance measurements using TET, the presence of damping should not affect the accuracy of the measurements.



(*a*) Amplitude



(*b*) Phase angle

Fig. 6 Compliance results with high internal damping (η = 0.1): expected value: (——); simulated TET value: (- - -)

Influence of the Material Properties. Both the mass and the stiffness of the tip can affect the measuring technique. An appropriate choice of tip material properties (Young's modulus and density) is therefore necessary. In order to demonstrate the influence of the material properties, Figure 7 compares the compliance *expected results* with the *simulated TET results* using three typical tip materials: aluminum, steel and epoxy resin. The physical properties of each material are summarized in Table 2.

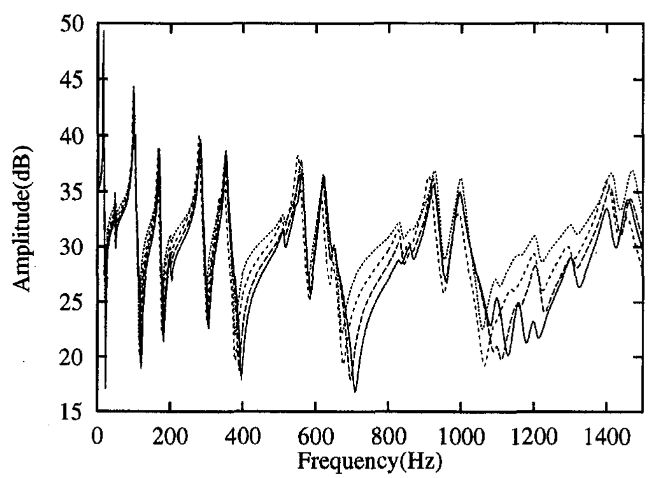


Fig. 7 Compliance amplitude comparison using three typical tip materials: expected value: (——); simulated TET value: (1) aluminum (– – –); (2) steel (- - -); (3) epoxy resin (· · ·).

In all cases, a steel plate was used. A comparison of the three curves with respect to the *expected results* illustrates several interesting points for the present configuration. Firstly, for the three materials used, the aluminum tip provided the closest agreement. This configuration, which had been analyzed previously, satisfied the rigid body hypothesis whilst limiting the inertia tip effects. With the steel tip, the resulting curve follows a similar pattern to that of the aluminum tip case, however with more obvious deviations at the higher frequencies. Compared with the aluminum tip, any further increase in the Young's

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Table 2 Tip material properties

	Aluminum	Steel		Epoxy	
	Value	Value	Ratio *vs.* Alum.	Value	Ratio *vs.* Alum.
Density (kg/m^3)	2768	7820	2.825	1120	0.405
Young's modulus (N/m^2)	7.20E+10	2.07E+11	2.872	4.43E+09	0.062
Poisson's ratio	0.33	0.3		0.4	

modulus does not result in any significant improvements. On the other hand, the steel tip is heavier than the aluminum one, so that the influence of the mass is amplified. The use of the epoxy resin, which is lighter and softer, has just the opposite effect. In fact, the low stiffness of the material violates the rigid body hypothesis which is fundamental for the validity of the method. Consequently, for all three materials tested, aluminum is a good compromise for balancing the mass-stiffness effects.

Tip Geometry Effects. The tip dimensions were changed, while maintaining similar geometry. Both a 50 percent increase and reduction in size were investigated and the results are compared with the simulated results in Fig. 8. It can be seen that a 50 percent increase in size perturbs the results more than a corresponding decrease. From the practical point of view, the size should be reduced as much as possible whilst ensuring that the following two criteria are satisfied: firstly, the tip should be stiff compared to the basic structure over the frequency range of interest; secondly, the tip should be sufficiently high so that a moment can be transmitted effectively to the structure with a force applied at the upper portion of the tip. From our point of view, the proposed basic configuration seems to be a good compromise.

Influence of Misalignments of the Excitation. In this section, we examine the effects of possible deviations of the exciting force on the rotational compliance estimates. Deviations of both force position and orientation were introduced in the basic configuration.

Figure 9 compares the rotational compliance amplitude in the ideal case with two different configurations: the first one corresponding to the case where the force was displaced 5 mm downwards in the negative Z-direction, and the second, 10 mm in the Y-direction. One notices that the *simulated TET results* are not very sensitive to a lateral shift in the Y-direction. This can be easily understood since a force shifted laterally may introduce a tangential excitation force which excites the in-plane bending vibration more efficiently in the plane of the plate, only at higher frequencies. On the other hand, Figure 9 shows that a vertical shift in the Z-direction has a greater influence on the estimated compliance amplitude. An investigation of the first expression of Eq. (13) shows that, with the tip moment of inertia I_{yy} usually being small, the rotational compliance amplitude is approximately proportional to the amplitude of the translational accelerance. As a result, the resulting error is approximately the same order of magnitude as $\Delta h_f / h_f$, where h_f is the distance between the force and the X-axis and Δh_f, the position shift.

A similar analysis was performed on orientation effects. Deviations of 5 deg. and 10 deg. of the input force were introduced respectively in two planes. Firstly, angular deviations were introduced in a plane parallel to the XY plane, corresponding to a laterally oblique excitation. In this case, amplitude data presented in Fig. 10(a) shows that the resulting errors due to a laterally oblique excitation are negligible. Secondly, the same angular deviations were introduced in the XZ plane direction, resulting in a vertically oblique excitation. The corresponding results in Fig. 10(b) indicate somewhat larger deviations from the ideal case. It is obvious that a transverse exciting force is applied to the plate in this case, and consequently, the system is disturbed in a more significant manner. However, in both cases, misalignment of up to 10 deg can still provide reasonably good results. Similar results were obtained for the case of the phase angle estimates.

4 Concluding Remarks

A method called TET, "Tip Excitation Technique," has been proposed for measuring the rotational compliance of an attached flexible plane structure. An L-shaped beam tip attached to the structure under investigation produces an excitation moment and the resulting angular response was then estimated. The technique converts the rotational compliance problem into a translational compliance problem which is much easier to study

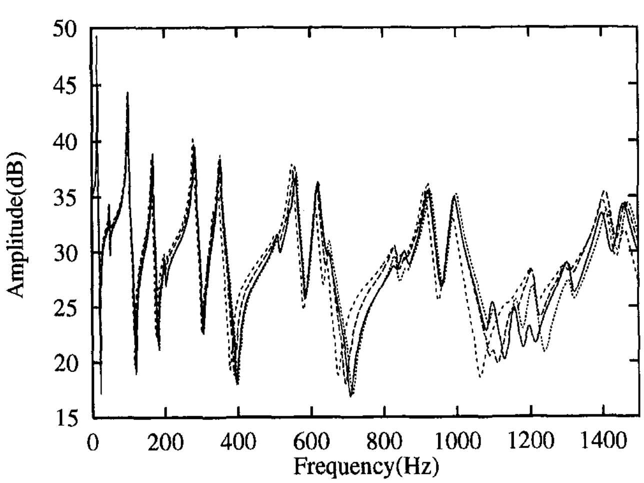

Fig. 8 Effects of the tip dimension on the compliance amplitude. expected value: (——); simulated TET value: (1) basic configuration (– – –); (2) +50 percent (- - -); (3). −50 percent (· · ·).

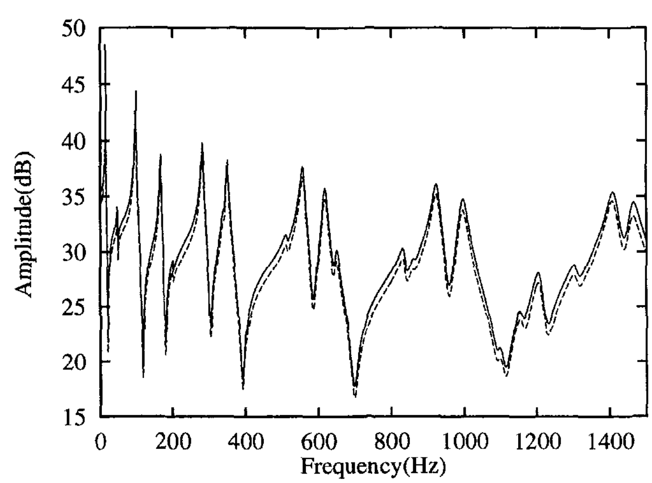

Fig. 9 Effects of the excitation position on the compliance amplitude (1) ideal case (——); (2) 5 mm down (– – –); (3) 10 mm aside (- - -).

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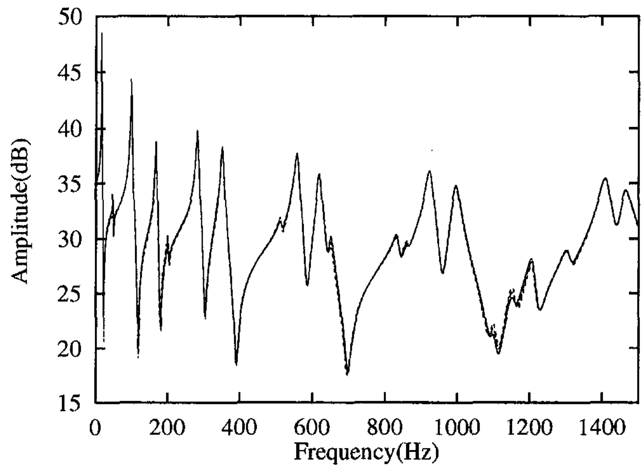
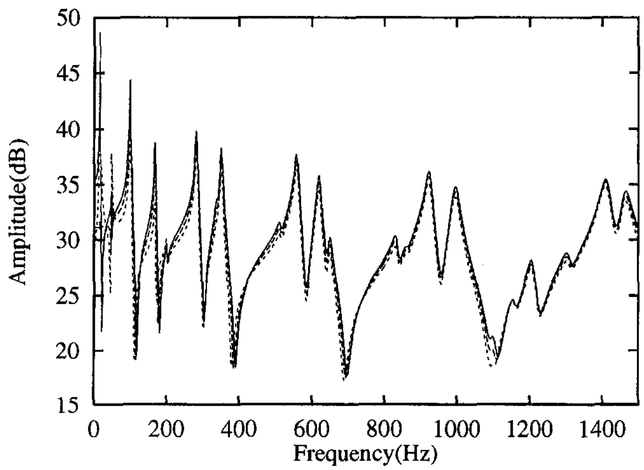


(*a*) lateral deviation (in XY plane)



(*b*) vertical deviation (in XZ plane)

Fig. 10 Effect of the excitation orientation on the compliance amplitude: (1) ideal case (——); (2) angle derivation 5 deg (- - -); (3) angle derivation 10 deg (---).

experimentally. Compliance relationships have been developed to estimate the rotational compliance while eliminating the most significant mass (inertia) effects introduced by the tip on the original structure.

This paper, which is the first of two companion papers, provides a preliminary assessment of the technique using numerical simulations. It has been demonstrated that the TET can be used for low to middle frequency range compliance estimates of attached plane structures. The parametric study simulated the effects of different factors and it provides guidelines for the experimental assessment of the technique. It has been shown that:

(1) the hypotheses made concerning the moment production and angular response estimation are reasonably well respected. As a result, the rotational compliance can be derived using simple analytical expressions;

(2) the inevitable presence of damping in the system should not affect the accuracy of the technique;

(3) regarding the influence of the added mass and the rigidity of the tip, the ideal choice for the tip material should be one providing low density and high stiffness. Based on the numerical simulations, a steel plate with an aluminum tip seems to be an acceptable choice;

(4) a vertical shift of the exciting force has a stronger influence on the accuracy of the compliance estimation than a lateral one. Estimates is worse in the presence of a vertically oblique driving force than a laterally oblique force.

It should be pointed out that all the previous conclusions are based on numerical simulations on several typical configurations, and should be subjected to experimental verification. Furthermore, experiments are needed to study other important factors that cannot be numerically simulated such as plate-tip attachment, excitation method, as well as other factors encountered during practical measurements. All these issues are the subjects of the companion paper.

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