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ScienceDirect
Composite Structures 78 (2007) 402–409
COMPOSITE STRUCTURES
www.elsevier.com/locate/compstruct

Application of eigenvalue perturbation theory for detecting small structural damage using dynamic responses

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Available online 20 December 2005

Abstract

Current methods for structural damage identification, such as genetic algorithms and artificial neural networks, are often implemented based on a few measured data and a large number of simulation data. The tremendous time-consuming computational work needed for calculating the response data to establish the dynamic model of damaged structures is an important issue for dynamic damage detection. In this paper through using the advanced modeling method of element stiffness matrix modification, the order of the global stiffness matrix can be kept invariable in establishing the model of intact and damaged structures. Then, eigenvalue perturbation theory is introduced to obtain the eigenvalues and eigenvectors of the damaged structure for reducing the computation load. Two artificial neural networks (ANN) are trained based on the response data simulated using finite element method (FEM) and perturbation theory enhanced finite element method (PFEM), respectively. The damage identification capability of these two ANN's are compared. Results show that the PFEM using the first order eigenvalue perturbation theory provides enough precision for detecting small structural damage and the computational requirement is greatly reduced. Typically, the eigensolution computational time for obtaining the train sample data using PFEM is only 1% of that using the traditional FEM.

Keywords: Eigenvalue perturbation theory; Damage detection; Computational time

1. Introduction

Online detection of the location and severity of structural damages is of great significance for ensuring the safety and reliability of in-service structures in many important fields of engineering. Structural damage detection using changes in dynamic characteristics has received much attention in recent years [1]. Various damage indices have been used to characterize the change in dynamic characteristics caused by damage, such as natural frequencies [2], frequency response function [3], flexibility matrix [4], mode shapes [5] and structure dynamic responses [6]. In the study of non-linear mapping relationships between the structural damage indices and various damage statuses, soft computing techniques, such as the neural networks and genetic algorithm (GA), have been increasingly utilized owing to their excellent pattern recognition capability [7–9].

However, most of the vibration based methods rely on a few measured data and a large number of simulation data, usually obtained using finite element method. For instance, the GA with a population of 100 used by Chou and Ghaboussi [8], needs to perform up to 2000 generations to accomplish damage detection for a truss structure with 26 elements. In fact, in order to get enough precise damage information, much more elements should usually be used for large complex structure when FEM is adopted, and it is well known that for a dynamic problem the required computation load increases exponentially with the number of elements. Then, for large complex structures, even one analysis cycle is already very time-consuming. For damage detection purpose, heavy dynamic analyses are usually required. For this reason, most of the vibration methods

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doi:10.1016/j.compstruct.2005.11.007

reported in the literature can only be applied to simple beam-like or truss structures. There is an obvious lack of work on the identification of damage in large complex structures.

The FEM dynamic structural analysis can be divided into four steps: (1) meshing of the structure, (2) calculating the element stiffness and mass matrices for each element and then assembling them to form the global stiffness and mass matrices, (3) calculating the eigenvalues and eigenvectors of the global stiffness and mass matrices, and (4) performing post-processing to obtain the required dynamic characteristics. Among these four steps the main computational workload is the calculation of eigensolution. When analysing the intact structure, the classic eigensolution algorithms are used to find the eigenvalues and eigenvectors. However, when the damaged structure is to be subsequently analyzed, most of the published work for vibration based damage detection uses the same algorithms again, which turns out to be very time-consuming. In fact, a better method can be used for this purpose. If the eigenvalues and eigenvectors of a matrix are known, then after this matrix is changed, the new eigenvalues and eigenvectors of the changed matrix can be quickly calculated based on the pre-calculated eigenvalues and eigenvectors of the original matrix. This method, called eigenvalue perturbation theory, was developed over the past few decades, and has been successfully applied in many areas. In holographic vibration analysis, the first-order eigenvalue perturbation theory was used in the re-analysis method to quickly calculate the new mode shapes and natural frequencies from the known vibration mode of a structure [10]. In the area of structure dynamic modification, the eigenvalue perturbation theory has been proved to be highly efficient [11]. In the area of damage detection, although an iterative method based on the general-order perturbation theory and optimization method for multiple structural damage detection has been developed by Wong et al. [12], the most valuable advantage of eigenvalue perturbation theory, i.e. its time-saving feature, has not been paid enough attention to. The damage detection methods presented in this paper clearly show the benefit of using eigenvalue perturbation theory.

The basic requirement for applying eigenvalue perturbation theory is that the order of matrix must be kept invariable. There are two methods for simulating crack in a FEM model, i.e. direct mesh and modification of local structural material elastic coefficients. The former changes the order of the global stiffness and mass matrices, while the latter will not. Therefore, the latter allows the application of eigenvalue perturbation theory. Our previous work showed that if a structure with small damage is directly meshed to depict the geometry of the small damage for establishing structural dynamics model, excessive mesh will be required. Then, it is time-consuming for subsequent calculations, and different meshings required by different damage sizes will cause significant numerical error. Such error may even cover the effect produced by small structural damage on structural dynamic characteristics, such as natural frequencies. Therefore, the direct mesh method is not suitable for simulating crack in damage detection [13]. In fact, local damage in a structure always causes a decrease in structural local stiffness, and these variations can be reflected by changes in the local structural material elastic coefficients [14]. Hence, dynamic model of a damaged structure can be established using the modified material elastic coefficients at the damage location. Thus, it is possible to avoid problems caused by direct mesh of structures and keep the degree of freedom (DOF) of the FEM invariable for the application of eigenvalue perturbation theory.

2. First order eigenvalue perturbation theory

The free vibration eigenvalue problem for an n-DOF undamped dynamic structure is given by

$$[K_0]\{U_0\} = \lambda_0[M_0]\{U_0\} \tag{1}$$

where $[K_0]$ and $[M_0]$ are the $n \times n$ global stiffness and mass matrices, respectively. Classic eigensolution algorithms, such as Lanczos algorithm, can be used to obtain the eigenvalue λ_0 and eigenvector $\{U_0\}$ of the intact structure from Eq. (1).

When damage is introduced, the free vibration eigenvalue problem for the damaged structure becomes

$$[K_\mathrm{d}]\{U_\mathrm{d}\} = \lambda_\mathrm{d}[M_\mathrm{d}]\{U_\mathrm{d}\} \tag{2}$$

where $[K_\mathrm{d}]$ and $[M_\mathrm{d}]$ are the global stiffness and mass matrices of the damaged structure, respectively, which can be expressed as:

$$[K_\mathrm{d}] = [K_0] + [K_1] \tag{3}$$

$$[M_\mathrm{d}] = [M_0] + [M_1] \tag{4}$$

where $[K_1]$ and $[M_1]$ are the perturbation of global stiffness and mass matrices, respectively caused by the modification of local structural material elastic coefficients.

The eigenvalue λ_d and eigenvector $\{U_\mathrm{d}\}$ of the damaged structure can be solved from Eq. (2) using the classic eigensolution algorithms. Besides, the approximate eigenvalue $\lambda_\mathrm{d}^\mathrm{P}$ and eigenvector $\{U_\mathrm{d}^\mathrm{P}\}$ of the damaged structure can be obtained using the eigenvalue perturbation theory. It has been pointed out that when the change of structural parameter is more than 15%, the second order perturbation should be taken into account [15]. In practice, the variation of local elastic modulus caused by small damage is usually far below 15%, so only the first order perturbation is used in this study.

According to the eigenvalue perturbation theory, for structures with distinct eigenvalues, the first order perturbation of eigenvalue and eigenvector can be respectively obtained as follows:

$$\lambda_{1i} = \{U_{0i}\}^\mathrm{T}[K_1]\{U_{0i}\} - \lambda_{0i}\{U_{0i}\}^\mathrm{T}[M_1]\{U_{0i}\} \tag{5}$$

$$\{U_{1i}\} = \sum_{s=1\, s\neq i}^{N} \frac{1}{\lambda_{0i}-\lambda_{0s}}(\{U_{0s}\}^{\mathrm{T}}[K_1]\{U_{0i}\} - \lambda_{0i}\{U_{0s}\}^{\mathrm{T}}[M_1]\{U_{0i}\})\{U_{0s}\} - \frac{1}{2}\{U_{0i}\}^{\mathrm{T}}[M_1]\{U_{0i}\}\{U_{0i}\} \tag{6}$$

where λ_{0i} is the eigenvalue of order i for the intact structure; λ_{1i} the first order perturbation of λ_{0i}; $\{U_{0i}\}$ the eigenvector of order i for the intact structure and $\{U_{1i}\}$ the first order perturbation of $\{U_{0i}\}$. Then, $\lambda_{\mathrm{d}}^{\mathrm{P}}$ and $\{U_{\mathrm{d}}^{\mathrm{P}}\}$ can be obtained as:

$$\lambda_{\mathrm{d}}^{\mathrm{P}} = \lambda_0 + \lambda_1 \tag{7}$$

$$\{U_{\mathrm{d}}^{\mathrm{P}}\} = \{U_0\} + \{U_1\} \tag{8}$$

Compared with the amount of computation needed for solving Eq. (2), the computational time for obtaining the approximate eigenvalue and eigenvector can be greatly reduced by using perturbation method. For instance, for a 20-bar truss, when one bar's cross-section is changed, the reanalysis time of using Eqs. (5)–(8) is only 4.5% of that by the classic eigensolution algorithms [11].

For dynamic damage detection, the damage-induced change in mass distribution is negligible. Then, $[M_1]=[0]$, Eqs. (5) and (6) can be reduced to

$$\lambda_{1i} = \{U_{0i}\}^{\mathrm{T}}[K_1]\{U_{0i}\} \tag{9}$$

$$\{U_{1i}\} = \sum_{s=1\, s\neq i}^{N} \frac{1}{\lambda_{0i}-\lambda_{0s}}(\{U_{0s}\}^{\mathrm{T}}[K_1]\{U_{0i}\})\{U_{0s}\} \tag{10}$$

This can reduce the computational load of Eqs. (5) and (6). Besides, Eqs. (9) and (10) are also the simplified form of improved precise perturbation method by William [16] in the absence of mass changes.

Because both $[K_0]$ and $[K_{\mathrm{d}}]$ are diagonal and symmetric matrices, and the modifications of structural material elastic coefficients for simulating crack take place locally, only a few elements in the stiffness matrix undergo changes. Therefore, $[K_1]$ is a diagonal, symmetric and sparse matrix as follows:

$$[K_1] = \begin{bmatrix} 0 & & & & & \\ & \ddots & & & & \\ & & k_{kk} & \cdots & k_{kj} & & \\ & & \vdots & \ddots & & & \\ & & k_{jk} & & k_{jj} & & \\ & & & & & \ddots & \\ & & & & & & 0 \end{bmatrix} \tag{11}$$

This characteristic of $[K_1]$ can be further employed to reduce the computational load, because the locations of non-zero elements in $[K_1]$ can be determined based on the location of damage in the structure before the use of Eq. (9). The non-zero elements of $[K_1]$ and the corresponding elements in $\{U_{0i}\}$ can form a new low-order matrix and a new vector. Then the calculation of eigenvalue and eigenvector for damaged structure using Eq. (9) can be reduced to low-order matrix multiplication as follows.

$$\lambda_{1i} = \{U_{0i}\}^{\mathrm{T}}[K_1]\{U_{0i}\} = \{u_k \ \cdots \ u_j\}_{0i} \begin{bmatrix} k_{kk} & \cdots & k_{kj} \\ \vdots & \ddots & \vdots \\ k_{jk} & \cdots & k_{jj} \end{bmatrix} \begin{Bmatrix} u_k \\ \vdots \\ u_j \end{Bmatrix}_{0i} \tag{12}$$

A similar reduction can be performed for Eq. (10). So without change in mass matrix and with only small local change in stiffness matrix, the application of perturbation method in dynamic damage detection will be more efficient than in other areas, such as structure dynamic modification. Furthermore, the workload for the classic eigensolution algorithms increases exponentially with n, while it only increases linearly with n in the present case. When n is very large for large complex structure problem, the reduction in workload will be very significant.

3. Numerical verifications

When the perturbation method is used to calculate the dynamic characteristics for other applications, such as structure dynamic modification, an error of about 1% is acceptable. In damage detection applications, however, the change in dynamic characteristics caused by small damage is usually very small, for instance, the change in natural frequencies is generally less than 1%. So whether the error of perturbation method caused by neglecting the high order parts is acceptable in damage detection should be verified.

The origin of errors indicates that the smaller the change of matrix is, the more precise the perturbation method will be. The change in stiffness matrix caused by small damage is very small, so the perturbation method can have enough precision in damage detection. A cantilever composite plate is used as an example to support this point of view.

3.1. Composite plate specimen

A square cantilever plate, shown in Fig. 1 is used as an example. The side length of the plate is 550 mm and its thickness is 10 mm. It is made of resin glass fibre with orthogonal layer $(-45°/45°)_{10}$. Material parameters of the sample are $E_1 = 47.518$ GPa, $E_2 = 4.588$ GPa, $G_{12} = 2.201$ GPa, $\mu_{12} = 0.0419$, $\mu_{21} = 0.434$, and $\rho = 1850$ kg/m^3. The plate is divided into 10×10 elements in the FEM model. Two identical piezoelectric patches (one as the actuator, and the other as the sensor) embedded in the composite plate are used to provide the excitation and sensing system for plate vibration. The size of the patch is $30\times27.49\times0.25$ mm^3, and its piezoelectric strain coefficients are $d_{33} = 285\times10^{-12}$ C/N and $d_{31} = 170\times10^{-12}$ C/N. The elastic parameters of the piezoelectric material are $E_{\mathrm{p}} = 65$ GPa, $G_{\mathrm{p}} = 25$ GPa and $\mu_{\mathrm{p}} = 0.3$. A square wave signal with a fundamental frequency of 4 Hz

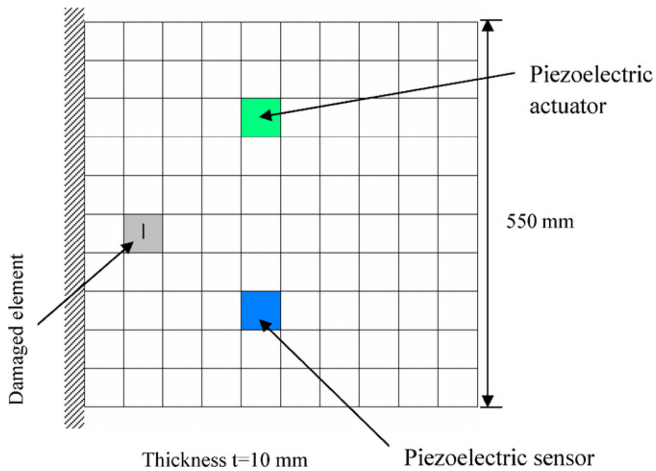

Fig. 1. Model of a laminated square composite plate.

and magnitude of 100 V is fed into the piezoelectric actuator embedded in the composite plate.

3.2. Crack simulation in FEM model

For a composite structure with crack damage, the modified local elastic modulus can be calculated using Eq. (13) as follows [14]

$$\left.\begin{aligned} E_1^{\mathrm{d}} &= E_1 + 2\omega_3(C_3 + C_6(\mu_{12})^2 - C_{12}\mu_{12}) \\ E_2^{\mathrm{d}} &= E_2 + 2\omega_3(C_6 + C_3(\mu_{21})^2 - C_{12}\mu_{21}) \\ \mu_{12}^{\mathrm{d}} &= \mu_{12} + \omega_3 \frac{1-\mu_{12}\mu_{21}}{E_2}(C_{12} - 2C_6\mu_{12}) \\ \mu_{21}^{\mathrm{d}} &= \frac{E_2^{\mathrm{d}}}{E_1^{\mathrm{d}}}\mu_{12}^{\mathrm{d}}; \quad \mu_{12}^{\mathrm{d}} \gg \mu_{21}^{\mathrm{d}} \\ G_{12}^{\mathrm{d}} &= \frac{E_2^{\mathrm{d}}}{2(1+\mu_{12}^{\mathrm{d}})}, \quad G_{23}^{\mathrm{d}} = G_{13}^{\mathrm{d}} = G_{12}^{\mathrm{d}} \end{aligned}\right\} \tag{13}$$

where E_1^{d}, E_2^{d}, μ_{12}^{d} and G_{12}^{d} are the elastic moduli, Poisson's ratio and shear modulus of the thin composite plate with crack damage, respectively. E_1, E_2, μ_{12} and G_{12} are the elastic moduli, Poisson's ratio and shear modulus of the intact composite plate, respectively. C_1 to C_{12} are the material coefficients independent of strains and damage, but dependent on the composite configuration, i.e. fiber geometry and orientations, fiber volume fraction and ply stacking sequence, etc. These parameters can be determined by measuring the specimen made of the same composite materials [14]. ω_3 is a variable representing the crack damage status, and it is related to the number, length and width of the crack. The expression of crack damage variable ω_3 can be written as

$$\omega_3 = \phi_{\mathrm{c}}\bar{a}_{\mathrm{c}}\bar{b}_{\mathrm{c}}\bar{f}_{\mathrm{c}} \tag{14}$$

where ϕ_{c} is the crack density, which is defined as the crack number in a unit area, $\bar{a}_{\mathrm{c}}$ and $\bar{b}_{\mathrm{c}}$ are the average length and width of the crack, respectively, and $\bar{f}_{\mathrm{c}}$ is an adjustment coefficient.

For other types of crack in a structure made of resin glass fibre, the modified local elastic modulus can also be calculated using Eq. (13) with its corresponding material coefficients. For other materials, the modified local elastic modulus can be calculated using a general method developed in our previous work [13].

The plate has a crack with length $\bar{a}_{\mathrm{c}} = 3$ mm and width $\bar{b}_{\mathrm{c}} = 0.1$ mm in the location shown in Fig. 1. Using Eqs. (13) and (14) and the experimentally determined data of crack damage parameters C_1 to C_{12} [14], the modified local elastic moduli can be calculated as:

$$E_1^{\mathrm{d}} = 47.47\ \text{GPa}, \quad E_2^{\mathrm{d}} = 4.5853\ \text{GPa},$$
$$G_{12}^{\mathrm{d}} = 2.192\ \text{GPa} \quad \text{and} \quad \mu_{12}^{\mathrm{d}} = 0.04164$$

Then the variations of local elastic moduli can be easily obtained as:

$$\Delta E_1 = -0.048\ \text{GPa}, \quad \Delta E_2 = -0.0027\ \text{GPa},$$
$$\Delta G_{12} = -0.009\ \text{GPa} \quad \text{and} \quad \Delta\mu_{12} = -0.00026$$

Obviously, the variations of local elastic moduli in this paper is far below 15%.

3.3. Natural frequencies and dynamic responses of structures

The natural frequencies and dynamic responses of the composite plate with damage are calculated to verify the precision of perturbation method.

For the intact structure, the natural frequencies are obtained by solving Eq. (1), then the structural dynamic response data are obtained using modal analysis.

Assume that the structural vibration amplitude is small, so that the structure is a linear vibration system. The equation of motion of a structure with damage can be written as

$$[M]\{\ddot{q}(t)\} + [C]\{\dot{q}(t)\} + [K]\{q(t)\} = \{F(t)\} \tag{15}$$

where $\{\ddot{q}\}$, $\{\dot{q}\}$ and $\{q\}$ are the vectors of structural nodal acceleration, velocity and displacement, respectively, $\{F(t)\}$ is the excitation force vector exerted on the structure, which is produced by piezoelectric patch actuators embedded in or bonded on structures and excited by external electric field. The detailed process of calculating the actuate force from applied voltage on the piezoelectric actuator is discussed in Ref. [6]. $[C]$ is the global proportional damping matrix.

Owing to the orthogonality of eigenvectors, Eq. (15), which is a set of n simultaneous equations can be reduced to a set of m independent equations of motion of a viscously damped single DOF system. Usually only the lowest m mode vectors are used in this process. In this paper the lowest 30 mode vectors are used to uncouple the 600 independent equations of Eq. (15). So the workload of solving the set of m independent equations is much less than that of solving the original n simultaneous equations.

After the dynamic analysis using FEM for the intact structure, the FEM scheme is modified to reduce the computational workload of dynamic analysis for damaged structure, leading to the so-called perturbation theory enhanced FEM (PFEM), which consists of the following four major steps:

1. Maintaining the same meshing as the intact structure, since the damage is simulated by modification of structural material elastic coefficients.
2. Establishing the global stiffness and mass matrices. Suppose that the material elastic coefficients are denoted by e, and the element stiffness matrix is denoted by $[\overline{K}]$. The change of material elastic coefficients is denoted by Δe. Then, $\Delta[\overline{K}]$ can be directly calculated by replacing e_i with Δe_i in the process of calculating $[\overline{K}]$, because $[\overline{K}]$ is a linear function of e_i. The perturbation of global stiffness matrix $[K_1]$ can be obtained by assembling $\Delta[\overline{K}]$.
3. Calculating the eigensolutions of the structure. The intact structure is considered as the unperturbed system and the damaged structure as the perturbed system. The eigensolutions of the damaged structure are obtained using Eqs. (9) and (10).
4. Calculating structural dynamic responses using modal analysis. The decoupling of Eq. (15) relys on the orthogonality of eigenvectors. However, since the eigenvectors $\{U_d^P\}$ are not exactly the same as eigenvectors $\{U_d\}$ of the damaged structure, so whether $\{U_d^P\}$ is orthogonal with respect to both the mass matrix and the stiffness matrix of the damaged structure should first be investigated.

Consider that there is no change in mass matrix caused by damage, substituting Eq. (8) into $\{U_d^P\}^T[M_d]\{U_d^P\}$ yields:

$$\begin{aligned}\{U_d^P\}^T[M_d]\{U_d^P\} &= (\{U_0\}+\{U_1\})^T[M_0](\{U_0\}+\{U_1\}) \\ &= \{U_0\}^T[M_0]\{U_0\}+\{U_0\}^T[M_0]\{U_1\} \\ &\quad +\{U_1\}^T[M_0]\{U_0\}+\{U_1\}^T[M_0]\{U_1\}\end{aligned} \tag{16}$$

According to Eq. (10), $\{U_1\}$ is a linear combination of $\{U_0\}$, therefore, $\{U_1\}$ is also the eigenvectors of Eq. (1) and is orthogonal with respect to $[K_0]$ and $[M_0]$. So the four terms at the right-hand side of Eq. (16) are all diagonal matrices. Hence, $\{U_d^P\}^T[M_d]\{U_d^P\}$ is still a diagonal matrix.

Substituting Eqs. (3) and (8) into $\{U_d^P\}^T[K_d]\{U_d^P\}$ yields:

$$\begin{aligned}\{U_d^P\}^T[K_d]\{U_d^P\} &= (\{U_0\}+\{U_1\})^T([K_0]+[K_1]) \\ &\quad \times(\{U_0\}+\{U_1\}) \\ &= \{U_0\}^T[K_0]\{U_0\}+\{U_1\}^T[K_0]\{U_0\} \\ &\quad +\{U_0\}^T[K_1]\{U_0\}+\{U_1\}^T[K_1]\{U_0\} \\ &\quad +\{U_0\}^T[K_0]\{U_1\}+\{U_1\}^T[K_0]\{U_1\} \\ &\quad +\{U_0\}^T[K_1]\{U_1\}+\{U_1\}^T[K_1]\{U_1\}\end{aligned} \tag{17}$$

Because $\{U_1\}$ and $[K_1]$ are the first order small quantity of $\{U_0\}$ and $[K_0]$, respectively; so in the right-hand side of Eq. (17), $\{U_0\}^T[K_0]\{U_0\}$ is the main part, and the other seven terms are the first order to the third order small quantities of $\{U_0\}^T[K_0]\{U_0\}$ as indicated below:

$\{U_1\}^T[K_0]\{U_0\}$, $\{U_0\}^T[K_1]\{U_0\}$ and $\{U_0\}^T[K_0]\{U_1\}$ are the first order small quantity,
$\{U_1\}^T[K_1]\{U_0\}$, $\{U_1\}^T[K_0]\{U_1\}$ and $\{U_0\}^T[K_1]\{U_1\}$ are the second order small quantity, and
$\{U_1\}^T[K_1]\{U_1\}$ is the third order small quantity.

Considering that $\{U_1\}$ is orthogonal with respect to $[K_0]$, the non-diagonal terms in matrix $\{U_d^P\}^T[K_d]\{U_d^P\}$ are only:

$\{U_0\}^T[K_1]\{U_0\}$: the first order small quantity,
$\{U_1\}^T[K_1]\{U_0\}$ and $\{U_0\}^T[K_1]\{U_1\}$: the second order small quantities,
$\{U_1\}^T[K_1]\{U_1\}$: the third order small quantity.

So, $\{U_d^P\}^T[K_d]\{U_d^P\}$ is not a diagonal matrix but its non-diagonal elements are high order small quantities of the diagonal elements. If $[K_1]$ is small enough, neglecting the non-diagonal elements in $\{U_d^P\}^T[K_d]\{U_d^P\}$ is reasonable.

When the eigenvalue and eigenvector of the damaged structure are calculated using the perturbation method, the response data of the damaged structure can be calculated using modal analysis in post-processing of the PFEM by neglecting the non-diagonal elements in matrix $\{U_d^P\}^T[K_d]\{U_d^P\}$.

The non-diagonal elements of $\{U_d^P\}^T[K_d]\{U_d^P\}$ can be discussed in another form. $\{U_d^P\}$ can be expressed as:

$$\{U_d^P\} = \{U_d\}+\{U_e\} \tag{18}$$

where $\{U_e\}$ is the error of $\{U_d^P\}$.

Substituting Eq. (18) into $\{U_d^P\}^T[K_d]\{U_d^P\}$ yields:

$$\begin{aligned}\{U_d^P\}^T[K_d]\{U_d^P\} &= (\{U_d\}+\{U_e\})^T[K_d](\{U_d\}+\{U_e\}) \\ &= \{U_d\}^T[K_d]\{U_d\}+\{U_d\}^T[K_d]\{U_e\} \\ &\quad +\{U_e\}^T[K_d]\{U_d\}+\{U_e\}^T[K_d]\{U_e\}\end{aligned} \tag{19}$$

Consider that $\{U_d\}$ is orthogonal with respect to $[K_d]$, the non-diagonal matrices in the right-hand side of Eq. (19) are $\{U_d\}^T[K_d]\{U_e\}$, $\{U_e\}^T[K_d]\{U_d\}$ and $\{U_e\}^T[K_d]\{U_e\}$. The non-diagonal elements in these non-diagonal matrices, which are caused by $\{U_e\}$ are neglected in post-processing of PFEM. This is the main origin of the error for obtaining response data using PFEM. So, the error of the previously obtained modal vectors will result in the error of response data. Then, if the response data are verified to be accurate enough, it is reasonable to believe that the modal vectors are also accurate enough.

3.4. Precision of natural frequencies and responses obtained using PFEM

In Table 1, f_i^0 are the natural frequencies of the intact structure, f_i^1 and f_i^2 are the natural frequencies of the

Table 1
Natural frequencies of the intact and damaged plate

	f_i^0 (Hz)	f_i^1 (Hz)	f_i^2 (Hz)	$\left\lvert\frac{f_i^2-f_i^1}{f_i^0}\right\rvert$ (%)	$\left\lvert\frac{f_i^2-f_i^1}{f_i^1-f_i^0}\right\rvert$ (%)
1	14.52	14.51418	14.51394	0.001653	4.123711
2	49.123	49.10763	49.10791	0.000570	1.821731
3	83.744	83.71622	83.71669	0.000561	1.691865
4	132.63	132.594	132.5951	0.000829	3.079911
5	162.09	162.0535	162.0549	0.000864	3.701673
6	259.01	258.986	258.9868	0.000309	3.333333
7	277.76	277.6123	277.6188	0.002340	4.400812
8	298.41	298.296	298.3006	0.001542	4.035088
9	350.16	350.0729	350.0749	0.000571	2.296211
10	474.04	473.6888	473.7044	0.003291	4.441913
11	498.78	498.6968	498.6986	0.000361	2.163462
12	502.87	502.8234	502.8257	0.000457	4.935622
13	512.82	512.6668	512.6735	0.001307	4.373368
14	559.29	559.2048	559.2076	0.000501	3.286385
15	597.44	597.2214	597.2316	0.001707	4.666057
16	714.26	713.5953	713.6185	0.003248	3.490296
17	751.53	751.2887	751.293	0.000572	1.782014
18	814.7	814.4225	814.4338	0.001387	4.072072
19	852.44	852.217	852.2241	0.000833	3.183857
20	860.73	860.2308	860.2565	0.002986	5.148237
Average				0.001295	3.5

damaged structure calculated using FEM and PFEM, respectively. Table 1 shows that the average error of PFEM, is only 0.001295% for the first 20 order natural frequencies. Although the change in structural natural frequencies due to damage is also very small, the maximum error of PFEM is less than 5.15% of the change due to damage, and the average error is only 3.5%. In fact this precision is much higher than the resolution of natural frequencies in most of the experiments. Hence, the perturbation method can be used to calculate the change in natural frequencies due to small structural damage with enough precision for damage detection.

As shown in Fig. 2, the difference between the response data of the intact structure (r_{i}) and the damaged structure is very small, no mater which method is used for obtaining the response data of damaged structure (r_{p} and r_{c}) (perturbation method or classic method). So a sensitive index needs to be found to better reveal the small changes in the response data caused by damage. It has been proved that the energy spectrum of the decomposed wavelet signal from structural vibration response can indicate the structural damage status with high sensitivity. It was shown that even when the ratio of damage size to total structural size is as small as 0.01–0.1%, the change in response can still be detected using the energy spectrum variation obtained by wavelet analysis [17]. Therefore, in this paper, the variation of the energy spectrum of decomposed wavelet signal is used to check the precision of the perturbation method.

Let the crack mentioned above be damage case A. The change in wavelet energy spectrum caused by another crack (damage case B) is also calculated. For damage case B there is a crack at the same location and with the same width as damage case A, but the crack length $\bar{a}_{\mathrm{c}} = 5$ mm.

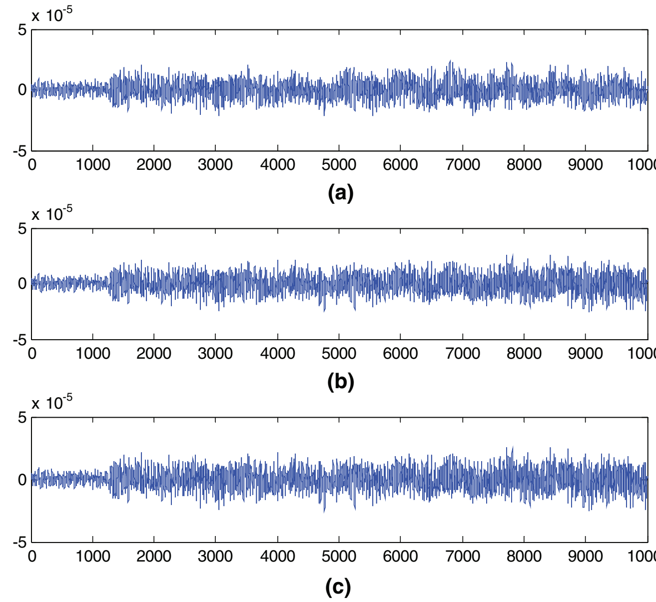

Fig. 2. Dynamic response data of the structure. Response data of (a) intact structure (r_{i}), (b) damage stucture (r_{c}), (c) damage structure calculated by perturbation method (r_{p}).

Fig. 3(a) shows the change of wavelet energy spectrum between the response data r_{i} and r_{c}. The change of wavelet energy spectrum between the response data r_{i} and r_{p} is shown in Fig. 3(b). $W_{\mathrm{A}}^{\mathrm{FEM}}$ and $W_{\mathrm{A}}^{\mathrm{PFEM}}$ are used to denote the damage indexes obtained using FEM and PFEM for damage case A, respectively. The average values of $W_{\mathrm{A}}^{\mathrm{FEM}}$ and $W_{\mathrm{A}}^{\mathrm{PFEM}}$ are 16.63 and 15.36, respectively; and the average value of $W_{\mathrm{A}}^{\mathrm{FEM}} - W_{\mathrm{A}}^{\mathrm{PFEM}}$, which is the error of perturbation method shown in damage index is only 1.34, which is only 8.06% of $W_{\mathrm{A}}^{\mathrm{FEM}}$. So the difference between $W_{\mathrm{A}}^{\mathrm{FEM}}$ and $W_{\mathrm{A}}^{\mathrm{PFEM}}$ is very small. This means that the error of using the eigenvalue perturbation method is far less than the change

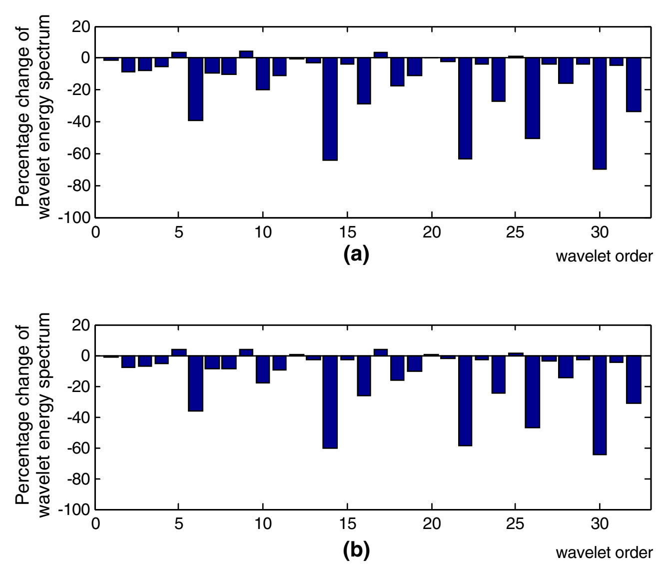

Fig. 3. Change of wavelet energy spectrum for damage case A. Change of wavelet energy spectrum between the response (a) r_{i} and r_{c}, (b) r_{i} and r_{p}.

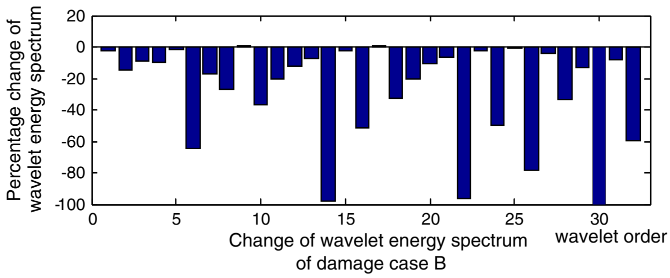

Fig. 4. Change of wavelet energy spectrum for damage case B.

caused by damage and would not be a problem to detect whether the structure is damaged.

The damage index for damage case B obtained using FEM is shown in Fig. 4 and denoted as W_B^{FEM}. Although damage case B is similar to damage case A, Fig. 4 is obviously different from Fig. 3(a) or (b). The average value of $W_B^{FEM} - W_A^{FEM}$, which is the difference of damage index for different damage cases is 13.54. This shows that the error of using the perturbation method would not cause problem for indication of different damage cases.

4. Identification of structural damage status using artificial neural networks

The precision of response data obtained using PFEM is finally verified by ANN identification of structural damage status, and the change of wavelet energy spectrum of structure dynamic responses is taken as the damage index. The vibration responses of 81 different cases are numerically simulated using FEM and PFEM, respectively. These 81 cases include the intact plate, plates with crack damage at four different locations and of 20 different crack lengths (1–8% of the plate width) at each crack location. Two BP neural networks (ANN1 and ANN2) both with 32 inputs and 2 outputs are designed. The inputs are the 32 elements of the damage index, and the outputs are the location area number and the length of crack (expressed in percentage of the plate width). The ANN1 are trained using samples numerically simulated using FEM, and ANN2 are trained using samples numerically simulated using PFEM. The results of crack identification for 6 sets of verification samples numerically simulated by FEM and PFEM are listed in Table 2. The data in Table 2 show that although the crack status identified by ANN1 is a little bit more precise than ANN2, the crack identification using ANN1 and ANN2 are both close to the actual crack status. So the perturbation method can be used to calculate the change of response with enough precision for detecting small structural damage. In this paper, the computational time for obtaining eigensolutions to simulating the train sample data using PFEM is only 1% of the computational time required by the traditional FEM.

5. Conclusions

In this paper, the eigenvalue perturbation theory is introduced to obtain the dynamic characteristics of damaged structures for damage detection. The precision of PFEM for calculating natural frequencies and response data is verified. Results show that in the calculation of natural frequencies, the average error caused by the use of perturbation method is only 3.5% of the change caused by damage; and the average error in energy spectrum of the decomposed wavelet signal is only 8% of the parameter change due to damage. An ANN is trained using the PFEM simulated data and is proved to be able to successfully identify the crack simulated using FEM for the same case. In addition, the computational time required by PFEM is reduced to only 1% of the time needed by FEM. Therefore, the use of eigenvalue perturbation theory can significantly reduce the tremendous time-consuming computation work needed by existing damage detection methods. The technique provides enough precision for damage detection with the deployment of the first order perturbation.

It should be mentioned that the perturbation method is only applied to cantilever composite plate with single crack in the present work. Based on the micro-mechanics theory of damage and the matrix perturbation theory for repeated or close eigenvalue problems, further research can be carried out to extend the application of this method to different types of structures, boundary conditions, materials, damage types and number of damage.

Acknowledgments

The authors would like to thank for the support by the Research Grants Council of Hong Kong Special Administrative Region of China under the Project No. PolyU 5313/03E; and Natural Science Foundation of China under the grant 50375123. The second author wishes to acknowledge the support from a special fund for new Chair Professors given by The Hong Kong Polytechnic University.

Table 2
Results of identifying crack damage using the trained ANN (crack location and length)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Crack location area number (real)	1.000	2.000	2.000	4.000	3.000	3.000
Crack location area number detected by ANN1	1.073	2.089	2.004	4.028	3.062	2.713
Crack location area number detected by ANN2	1.103	1.937	2.150	4.157	2.939	2.891
Crack length value (real) (%)	2.000	7.000	8.000	3.000	6.000	8.000
Crack length value detected by ANN1 (%)	1.916	6.893	8.201	3.020	5.998	8.233
Crack length value detected by ANN2 (%)	1.908	6.828	7.887	2.765	5.885	7.997

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