Nondestructive Detection of Internal Delamination by Vibration-based Method for Composite Plates

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(Received November 17, 2003)
(Revised March 13, 2004)

ABSTRACT: A method for locating internal delamination in multilayer composite plates is presented. A finite element method for composite plates is developed to investigate the relationship between delamination and change of modal parameters. The mechanism of the mode-dependent energy dissipation of composite plates is revealed by numerical analysis of delamination-induced variations of modal parameters. Experiments are carried out using piezoelectric actuator and accelerometer to measure the dynamic response to sinusoidal sweep excitation for several cantilever composite plates. The results show that the region of internal delamination in composite plates can be predicted according to the measured acceleration response and the computed modal strain energy.

KEY WORDS: internal delamination, modal parameters, multilayer composites, vibration measurement.

INTRODUCTION

THE DEMAND FOR high-performance materials has increased dramatically in the past decades. This demand has promoted the development of a wide variety of materials including a series of composites, which have superior strength, low density, and so on. With this occurrence of new materials some entirely new types of flaw or damage have appeared as a consequence of imperfections introduced during the manufacturing process or as a result induced by external loads during the operation or service of the composites. Composite materials introduce a completely different kind of difficulty into the nondestructive testing (NDT) due to their inherently complex structure. The widely used fiber-reinforced composites exhibit four main types of damage, i.e. delamination, matrix

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0021-9983/04/24 2183–16 $10.00/0 DOI: 10.1177/0021998304045590

cracking, fiber breakage, and interfacial debonding. Generally, failure is due to a combination of these kinds of damage. Early detection of initial damage can prevent a catastrophic failure or structural deterioration beyond repair. It is, therefore, important to detect such damage for practical composites or structures.

Vibration-based NDT has been developed in order to detect the damage at the early stages. Doebling et al. [1] gave a complete review in this area of research. The basic idea in vibration-based damage detection is that the modal parameters depend on the physical properties of the structure to be inspected. Therefore, changes in physical properties of a structure due to damage can result in detectable variations in its modal parameters, such as natural frequencies, mode-shapes, and modal damping. A number of studies have been carried out to show the effectiveness of dynamic response measurements for NDT of composites. Adams et al. [2] used frequency changes in longitudinal vibration modes to detect and locate damage in glass-epoxy tubes, which was equivalent to a 1% reduction in the cross-sectional area. Cawley and Adams [3,4] later extended this work to include two-dimensional carbon fiber-reinforced plates. Using a finite element model of the plate and a perturbation analysis of the stiffness and eigenvalue matrices, they were able to locate and roughly quantify the damage, such as holes, sawcuts, and localized crushing in cross-ply and angle-ply laminates. Tracy et al. [5] used vibration analysis to quantify impact damage in composite plates with various ply angles. A large number of vibration modes were measured before and after impact loading. Most of the work done was to relate changes in stiffness due to damage to changes in structural natural frequencies. Mantena et al. [6], however, related changes in damping to degradation and degree of matrix cracking in various composites. They found that damping changes were much higher than changes in either static or dynamic modulus as crack density increased. The underlying physics of the damage-induced variation of damping waits to be explained.

Laminated fibrous composite is the most frequently used type of composite. Delamination is a major concern for in-service laminae. Numerous researchers have studied various aspects of the delamination process including changes in dynamic response [7]. The introduction of damage into a material generally results in an increase of damping, which is related to energy dissipation during dynamic excitation [8–11]. This paper, therefore, focuses on the practical localization method of internal delamination based on the relationship between damage-induced variations of modal parameters and delamination location for multilayer fiber-reinforced composite plates. At first, a finite element model is established to compute the modal parameters such as natural frequency, mode-shape, and modal strain energy for vibrational composite plate samples. The relationship between the modal parameters and delamination location is analyzed using the numerical results. Then, the experiment is carried out to measure the modal damping for the whole plate. Finally, based on the numerical analysis and the fact that delamination at different locations can induce different increases of modal damping for different modes of a vibration plate, the delamination location is estimated by an investigation into the measured modal damping change combined with numerical results of strain energy distribution. The estimated result appears in good agreement with the actual one. The contribution of this paper is to develop a practical and reliable method for location prediction of damage in composites by means of measured modal damping and computed modal strain energy distribution. The attraction of the proposed method lies on its requirement of simple and economic equipment, convenience, and usage of mature technique.

FINITE ELEMENT MODEL

The finite element method (FEM) is used to compute modal parameters, such as natural frequency, mode-shape, and strain energy distribution of each mode for a laminated composite plate with or without delamination.

By introducing a Cartesian global coordinate system x, y, z to a rectangular plate with uniform thickness as shown in Figure 1, the displacements of a point (x, y, z) in the plate along x, y, and z axes are u, v, and w, respectively. Assume that the individual lamina is orthotropic with fiber orientation along x_1 axis of the Cartesian coordinate system x_1, x_2, x_3 (local material coordinates) as shown in Figure 1.

The finite element used in this paper is the eight-node rectangular thin plate element. For each node, there are three degrees of freedom, i.e., translations along the global coordinate axes of x, y, and z, respectively. The element thickness is assigned to be equal to that of the corresponding individual lamina, which may not be the same for all elements. The element coordinate system is arranged with the first axis being coincident with the fiber direction. All parameters throughout an element are assumed to be the same.

For an eight-node finite element with three degrees of freedom per node, the element stiffness matrix may be written as

$$[K^e] = \int_{V_e} [B]^T [A][C][A]^{-1}[B]dV \tag{1}$$

where $[C]$ is the elastic constant matrix of the material including the orthotropic elastic constants of the lamina: E_1, E_2, E_3, G_{12}, G_{13}, G_{23}, v_{12}, v_{13}, and v_{23}, $[A]$ is the coordinate transform matrix and $[B]$ represents the matrix related to the shape function of the element [12].

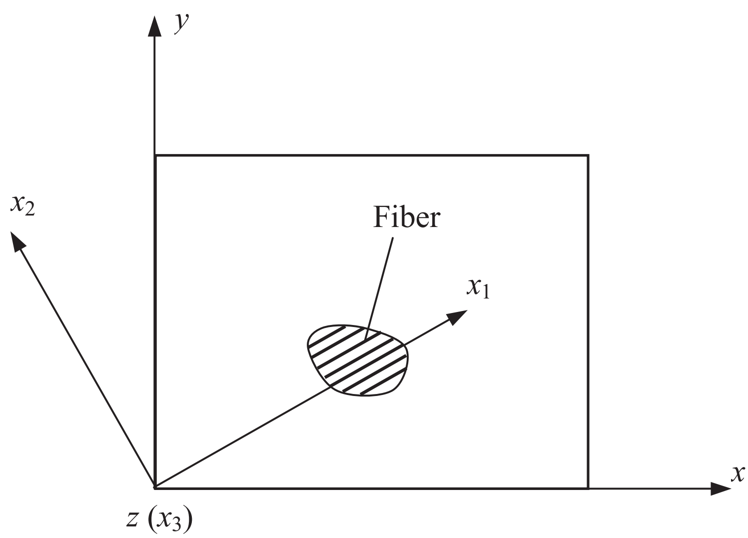


Figure 1. *The cross-ply laminated plate and the coordinate systems.*

After assembly of nodal displacements of all elements, the total strain energy of a multi-layer composite plate can be expressed as

$$U = \frac{1}{2}\{\delta\}^{\mathrm{T}}[K]\{\delta\} \tag{2}$$

where $\{\delta\}$ and $[K]$ are the global nodal displacement vector and stiffness matrix, respectively. Assuming that the composite plate experiences a harmonic motion with angular frequency ω, and using the Lagrange's principle, the equation of motion for free vibration of the composite plate is reduced to a standard eigenvalue problem as follows

$$([K] - \omega^2[M])\{\delta\} = 0 \tag{3}$$

where $[M]$ is the global mass matrix. The modal parameters of the plate, such as mode-shape, natural frequency, and modal strain, can be obtained by solving Equation (3).

For an arbitrary laminated plate, to ensure the material continuity, displacements, and their variations of each pair of coincident nodes on the upper and lower adjacent laminae have to be equal in the whole process of computation for the intact plate. When the plate is delaminated, each pair of the corresponding nodes just on the upper and lower surfaces within the delamination region is assumed to have no concrete connection with each other.

ENERGY DESCRIPTION OF DAMPING

The damping mechanism in composites differs entirely from that in conventional materials, such as metals and alloys. The dominant energy dissipation in laminated fibrous composites comes from not only the viscoelastic nature of matrix and/or fiber materials but also the interphase damping [13]. Besides, damage can induce damping increase due to energy dissipation in the damaged region and damping is more sensitive to damage than stiffness [14,15].

Associated with a given vibration mode-shape, there are displacements that yield the varying strain field in a vibrating object. Then, the maximum modal strain energy at each part of the object can be found. The modal strain energy gives a unique expression of the distribution of energy stored in the object for the mode-shape concerned.

From the concept of energy, the total damping in the material or structure is related to the damping of each element and the fraction of the total strain energy stored in that element. For a vibrating structure, the ratio between energy dissipated and the strain energy stored is usually defined as the specific damping capacity (loss factor). According to this definition, the loss factor of a vibrating structure is reported to be proportional to the damping ratio for a slightly damped system, and a highly dissipative material element has an effect on the loss factor only when it contributes significantly to the total stored energy of the entire structure [16–21]. Therefore, from each mode of the measured specimen, it should be possible to locate the delamination using a combination of modal strain energy distribution and damping measurements before and after delamination, provided that the delaminated region can be treated as the highly dissipative element.

NUMERICAL ANALYSIS ON DELAMINATION-INDUCED CHANGES OF MODAL PARAMETERS

Samples

The composite plates used for the numerical analysis are the same as those for the experiment in this study, and four samples of multilayer carbon fiber-reinforced epoxy plates are prepared. Each sample has an area of $300 \times 90\,\mathrm{mm}^2$ and consists of 16 layers in orientation of $[0/0/90/90/0/0/90/90]_s$. The laminate is fabricated using TC12K33/S-1 prepreg tapes with a thickness of 0.13 mm. Three of the samples (Plates A, B, and C, respectively) are delaminated at different positions by inserting Teflon films with a thickness of 0.015 mm and an area of $36 \times 54\,\mathrm{mm}^2$. Each Teflon film is inserted between the fourth and fifth layers counted from the top of the laminate when the laminate is fabricated. One of the short sides of each sample is fixed on a vibration-isolated table to form a cantilever plate, and the actual length of each plate is 240 mm. Each damaged plate has only one delamination, and their dimensions and locations are shown in Figure 2. The material constants are $E_1 = 125\,\mathrm{GPa}$, $E_2 = E_3 = 8.5\,\mathrm{GPa}$, $G_{12} = G_{13} = 4.5\,\mathrm{GPa}$, $G_{23} = 3.27\,\mathrm{GPa}$, $v_{12} = v_{13} = v_{23} = 0.3$, and $\rho = 2400\,\mathrm{kg\,m^{-3}}$.

Delamination-induced Variation of Natural Frequency

According to Equation (3), natural frequencies and mode-shapes are computed for the first five modes of the above-mentioned cantilever plates. Figure 3 shows the mode-shapes of the first five modes for the plate without delamination. Table 1 lists the natural frequencies and their percentage changes due to delaminations. It can be seen that the natural frequencies decrease slightly after the delamination is introduced, and the change of natural frequency varies with the position of delamination. The decrease of natural frequencies in Modes 4 and 5 are more significant than those in other modes for Plate A. However, Modes 3–5 undergo relatively large changes for Plate B and the decrease of natural frequency in Mode 4 is less than those in the other four modes for Plate C. Thus, delamination-induced variation of natural frequency is dependent on delamination location. This may give a hint of damage location, but it is difficult to realize in practice, because most of the frequency changes are too insignificant to be detected.

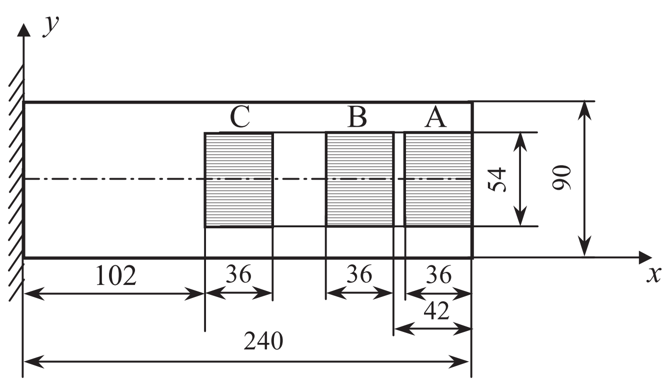


Figure 2. *Delamination locations and dimensions of the cantilever plates.*

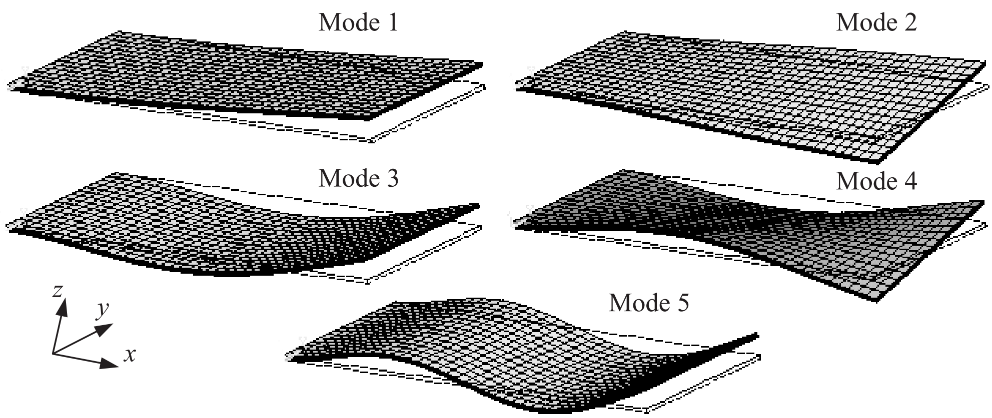


Figure 3. *The first five mode shapes of the intact cantilever plate.*

Table 1. Numerical results of percentage changes of natural frequencies for the damaged plates.

Mode	Natural frequency of the intact plate (Hz)	Percentage change of natural frequency (%)		
		Plate A	Plate B	Plate C
1	40.32	−0.00248	−0.01736	−0.04712
2	101.02	−0.34647	−0.0099	−0.06929
3	252.22	−0.23789	−0.17842	−0.02379
4	367.89	−0.89429	−0.03534	−0.00918
5	704.27	−2.39965	−0.18459	−1.00672

Delamination-induced Variation of Mode Shapes

From the above analysis it is known that the effect of delamination at a specific position on the plate is more significant in some modes than that at other positions. This may imply that the delamination region exerts specific effects on the related modes. Figure 4 shows the mode-shapes of the first five modes for Plate A, which shows an obvious variation of the fifth mode-shape.

As stated in the first section, energy dissipation will increase with the introduction of damage to a vibrating structure. When internal delamination exists in the vibrating plate, there may be shearing of one delamination surface with respect to the mating delamination surface or impact between the upper and lower surfaces within the delamination region. Thus, there is an increase in energy dissipation induced by delamination. For further investigation of the delamination position-dependent variations of modal parameters, the unit normalized local displacements of the plates with different delamination locations are analyzed.

ANALYSIS ON PLATE A

Figure 5 shows the displacements in the z-direction for ten pairs of points just on the upper and lower surfaces of the delamination region for Plate A. The locations of the selected ten pairs of points for the displacement plot are shown in Figure 6(a). Each pair of

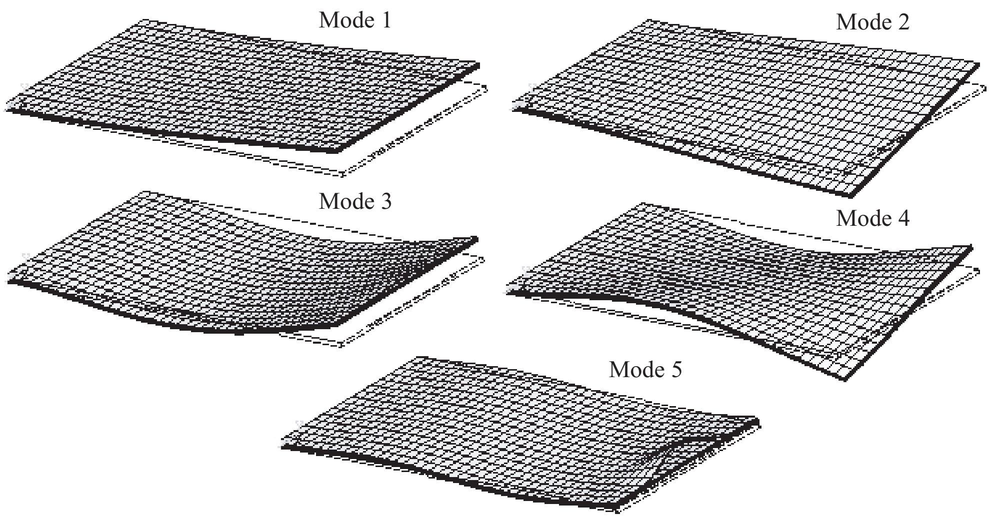


Figure 4. *The first five mode shapes of cantilever Plate A.*

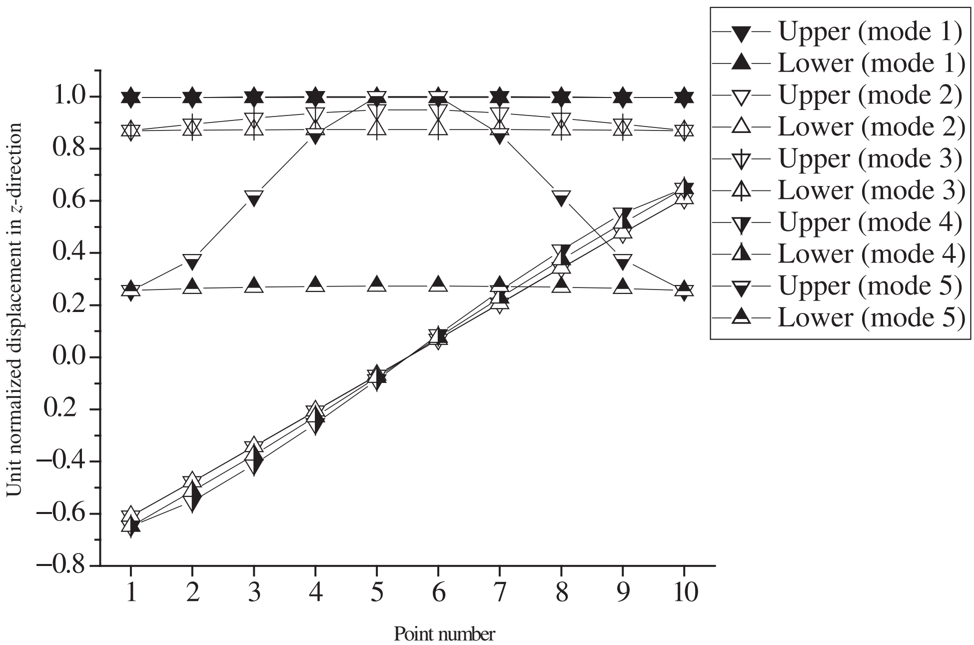


Figure 5. *Displacements along z-direction for points on the upper and lower surfaces within the delaminated region of Plate A.*

the upper and lower points is coincident with each other before motion. It can be seen that in the fifth mode the difference of displacements between the points just on the upper and lower surfaces of the delamination region is the largest, and apparent penetration occurs in the fourth mode, which is physically impossible. This indicates that relatively large

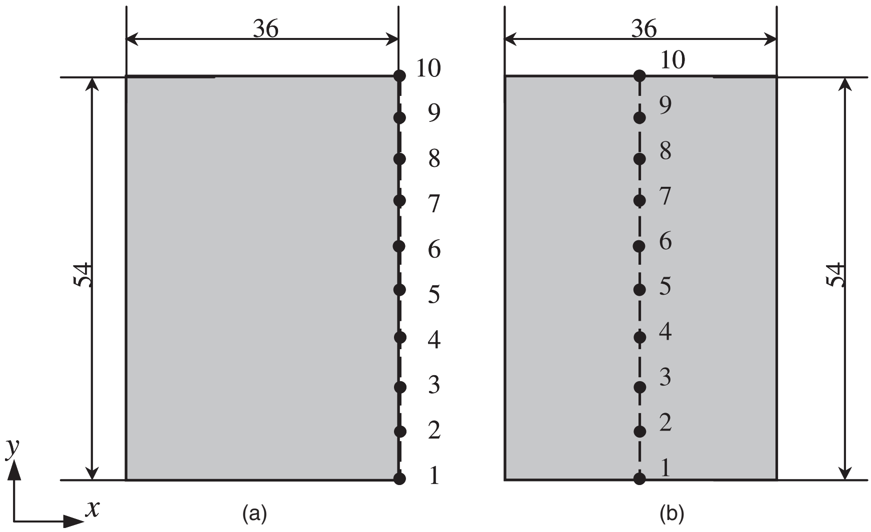


Figure 6. *Positions of points in the delamination region for displacement plot: (a) For Plate A and (b) For Plates B and C.*

Table 2. Experimental results of percentage changes of modal damping for the damaged plates.

Mode	Natural frequency of the intact plate (Hz)	Percentage change of modal damping (%)		
		Plate A	Plate B	Plate C
1	40.7	−3.28	5.19	15.41
2	104.3	0.44	3.30	18.51
3	249.3	−4.94	56.63	20.39
4	366.5	12.99	40.52	2.55
5	673.5	8.68	27.37	16.84
Average		2.78	26.60	14.74

interactive motion or impact occurs within the delamination region for the fourth and fifth modes of Plate A, i.e., the delamination at position A introduces more energy dissipation in motions of Modes 4 and 5 than that in other modes. These results are in accordance with those of frequency and damping change shown in Tables 1 and 2. Table 2 lists the measured results of modal damping. Thus, larger relative motion between two delaminated surfaces will lead to an increase of modal damping.

ANALYSIS ON PLATE B

As for Plate B the selected points in the delamination region for displacement plot are shown in Figure 6(b), and the displacements of these points along the z-direction are shown in Figure 7. It is noticed that an apparent penetration occurs in Mode 5 and the differences of displacements between the upper and lower points within the delamination region for Modes 3 and 5 are much larger than those for Modes 1, 2, and 4. Hence, the delamination at position B introduces relatively large energy dissipation in Modes 3 and 5.

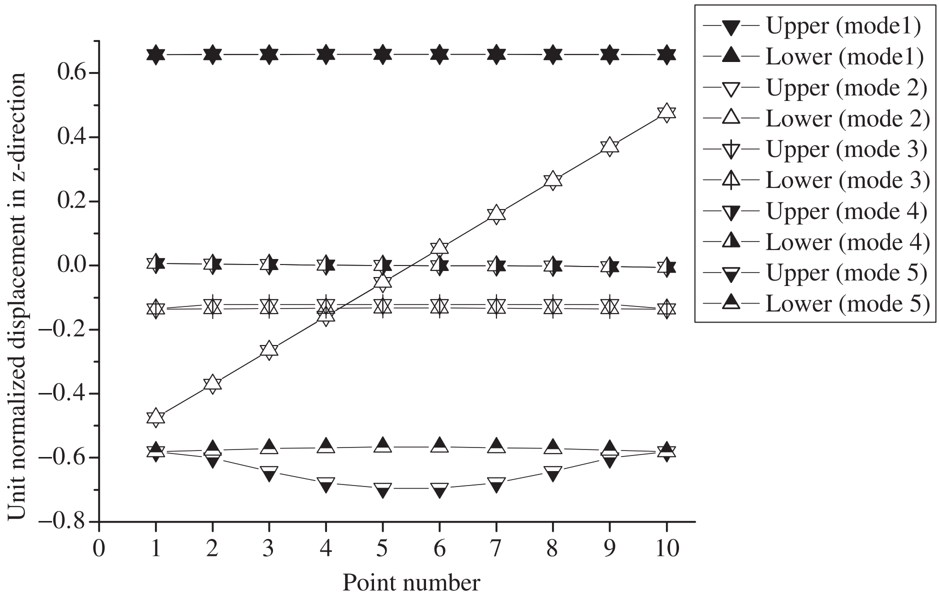


Figure 7. *Displacements along z-direction for points on the upper and lower surfaces within the delaminated region of Plate B.*

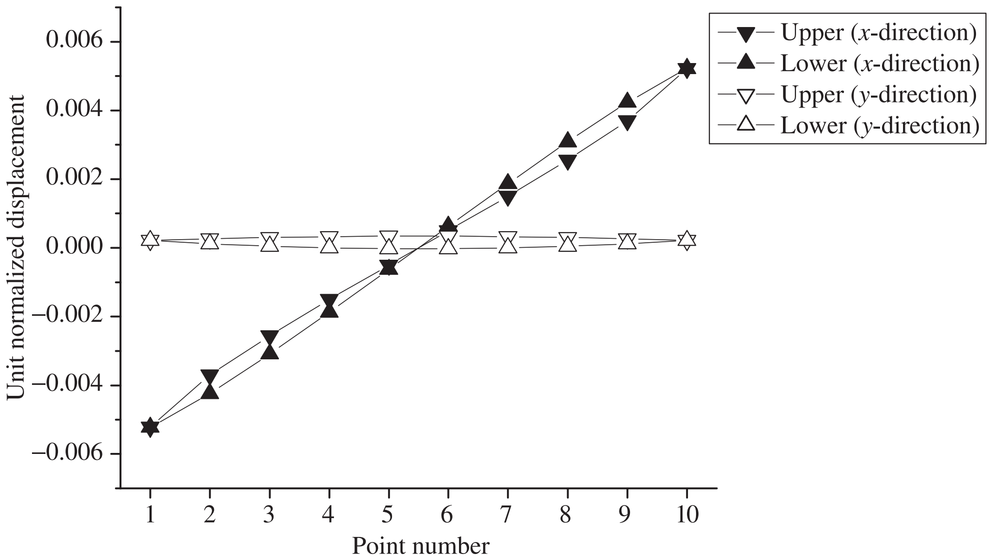


Figure 8. *Displacements along x- and y-directions for points on the upper and lower surfaces within the delaminated region of Plate B for mode 4.*

However, when the differences of displacements of these points in directions of x and y are considered, a significant interfacial slip across the delamination can be seen in Mode 4 for this sample as shown in Figure 8. Thus, in this mode relatively large energy dissipation may occur in the vibrating plate due to Coulomb friction. Therefore, the modal damping

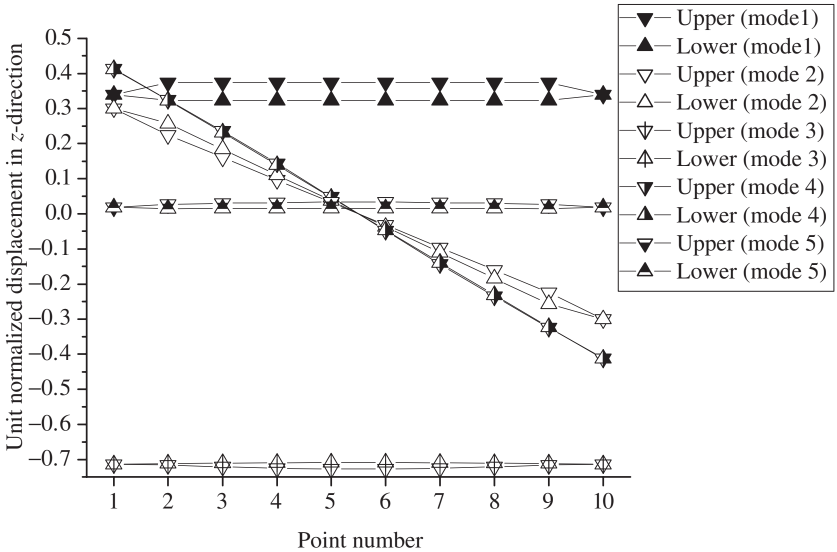


Figure 9. *Displacements along z-direction for points on the upper and lower surfaces within the delaminated region of Plate C.*

will increase significantly in Modes 3–5 than in Modes 1 and 2. It is also noticed that, from Tables 1 and 2, the decrease of natural frequencies and the increase of modal damping in Modes 3–5 are significant than those in Modes 1 and 2.

ANALYSIS ON PLATE C

When the delamination is at position C (Plate C), computational results show that penetration occurs within the delamination region in Modes 2 and 3 and the differences of displacements along the z-direction between the upper and lower surfaces are larger in Modes 1 and 5 than those in Mode 4 as shown in Figure 9. Thus, energy dissipation in Mode 4 is less than those in Modes 1–3 and 5. Therefore, the decrease of natural frequency and the increase of modal damping are more significant in Modes 1–3, and 5 than in Mode 4. These results are in good agreement with those shown in Tables 1 and 2.

Therefore, it can be deduced that if the delamination-induced changes of mode-shapes in Modes 4 and 5 are more significant than those in Modes 1–3, the delamination region is predicted near position A for the considered plates. In this case the energy dissipation in Modes 4 and 5 is larger than that in Modes 1–3. Similarly, more changes of mode-shapes in Modes 3–5 than those in Modes 1 and 2 predict a delamination at position B, and when the changes in Modes 1–3 and 5 are more than that in Mode 4, the delamination location is estimated at position C.

Although the above approach for delamination localization, which is based on the numerical analysis, cannot be put into practice, it is not only helpful to understand the intrinsic relationship between delamination-induced variations of modal parameters and delamination location in the vibrating plate but also can be a guidance to search the related practical method. Delamination location can be determined by change of modal damping combined with modal strain energy distribution.

EXPERIMENT

Experimental Setup and Procedures

Experimental modal analysis for the above-mentioned cantilever plates is conducted using the setup as illustrated in Figure 10. A piezoelectric patch with a thickness of 0.28 mm and an area of $15 \times 25\,\text{mm}^2$ is bonded on the top surface near the fixed end of each plate as an actuator. An accelerometer (Endevco 22) is used as the transducer. Excitation signal is generated by a waveform generator (TTi TGA1241). A power amplifier (TreK 603) and a charge amplifier (B&K 2635) are used to enhance signals from the generator and transducer, respectively. Both the excitation and response signals are recorded and analyzed by an FFT spectrum analyzer (B&K 3550).

Frequency response function (FRF) for the intact plate is obtained firstly using a sinusoidal sweep excitation in a frequency range from 1 to 1000 Hz. In order to increase the resolution of frequency, after the first five resonant frequencies are identified, sinusoidal sweep excitations in frequency spans of 6.25, 12.5, 25, 50, and 100 Hz are used for Modes 1–5, respectively to excite the resonant frequencies. To avoid missing of resonance region a sweep step of 0.05 Hz is adopted with a sweep speed of 1000 steps per second. Good resolution in the resonance region of the FRF is helpful for accurate determination of modal frequency and damping ratio. The average of five repeated measurements for each case is taken as the result to reduce the influence of noise.

After obtaining the FRF, damping coefficient of the ith mode is calculated from the real and imaginary parts of the FRF according to the equation

$$\xi_i = \frac{\delta f_i}{2 f_i} \tag{4}$$

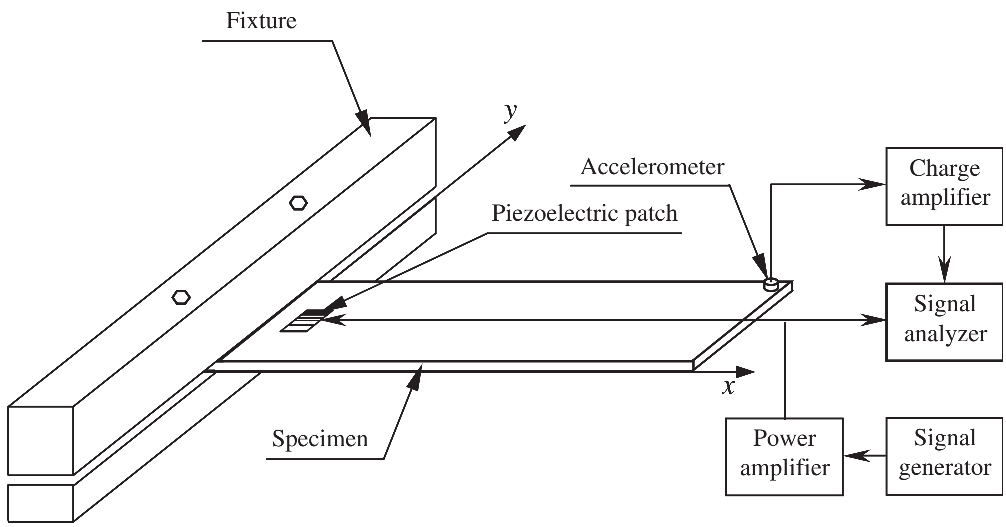


Figure 10. *The experimental setup for modal damping measurement of the composite plates.*

where f_i is the ith natural frequency of the plate identified according to the peak in the imaginary parts of the FRF, while δf_i is the frequency difference between the positive and negative peaks in the real part of the FRF.

Delamination-induced Change of Modal Damping

The percentage changes of modal damping obtained experimentally are listed in Table 2. It is obvious that the delamination-induced changes of modal damping are not only mode-dependent but also vary with the location of the delamination. It is also observed that the manner of damping change with mode order is similar to the results as illustrated by Table 1 and Figures 5, 7–9. Therefore, the values in Table 2 along with other modal parameters can be used to predict the location of delamination for each experimental specimen.

PREDICTION OF DELAMINATION LOCATION

Modal Strain Energy Distribution

According to Equation (3), corresponding to the maximum displacement state of each mode-shape, strains at all nodes and strain energy of each element are obtained for the cantilever plate specimen without delamination. Figure 11 shows the strain energy distribution on the top surface of the plate without delamination for the first five modes. The strain energy distribution in conjunction with the measured change of modal damping can provide the information for the location of delamination on the premise that modal damping should only go up when a highly dissipative element contributes significantly to the stored strain energy, i.e., the knowledge of strain energy in each mode can be used to predict the location of highly energy dissipative elements.

Localization of Delamination According to Modal Strain Energy and Damping

As stated in "Energy Description of Damping", if the delamination-induced increase of modal damping is more significant for a specific vibration mode, the delamination is assumed at the location of the elements, which contain relatively high percentage of modal strain energy, and vice versa. Therefore, the values in Table 2 along with Figure 11 can be used to predict the delamination location for each sample. Assume that the region with relatively small strain energy is defined as the place where the elements have strain energy less than 10% of the peak value, while the region with relatively large strain energy is the place where the elements possess strain energy larger than 10% of the peak value. Table 3 can illustrate the process to predict delamination region of each damaged plate. According to Table 3 the region of internal delamination, in Plate B for example, is estimated as follows:

1. Because the modal damping change of the first mode is 5.19%, which is much less than the mean value of 26.6%, the delamination locates at the place where the elements have relatively small modal strain energy. Thus the delamination locates at the shaded region as shown by the plot in the first row and the second column of Table 3.

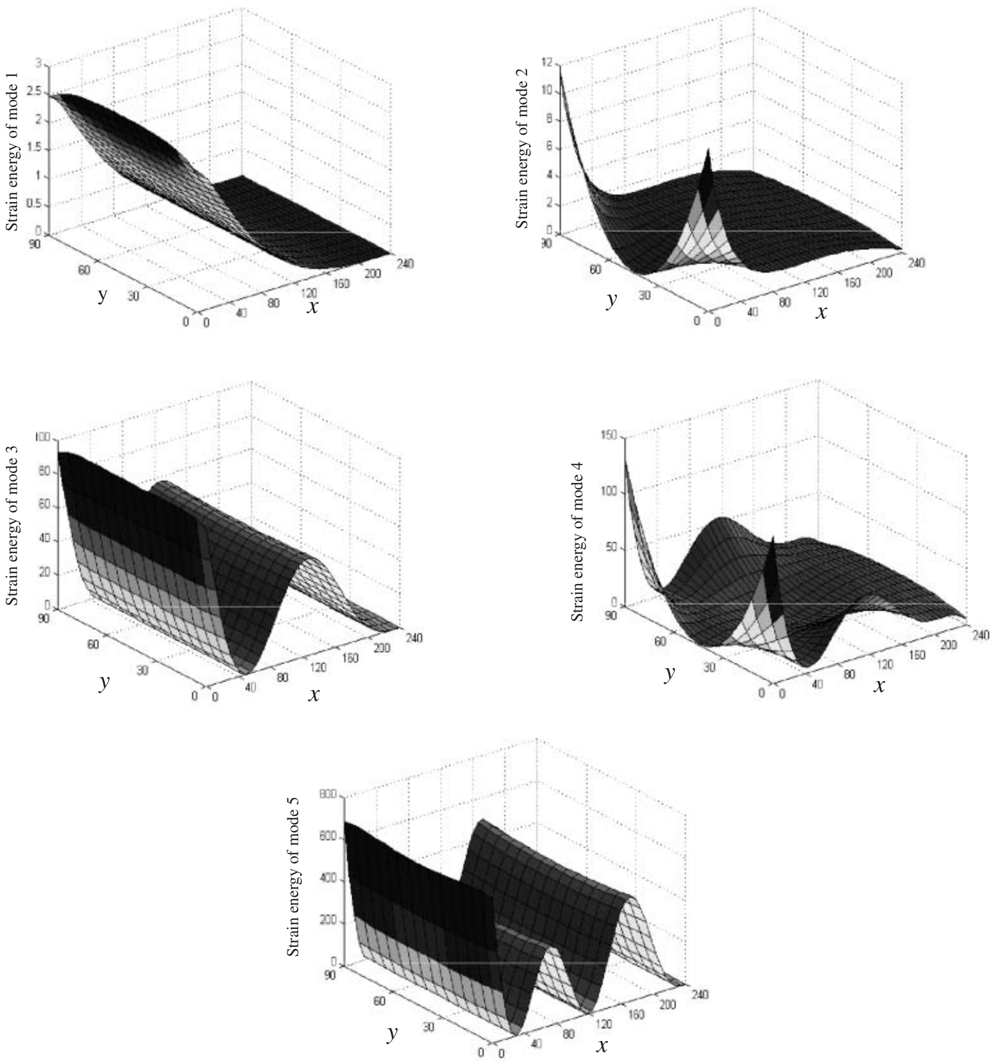
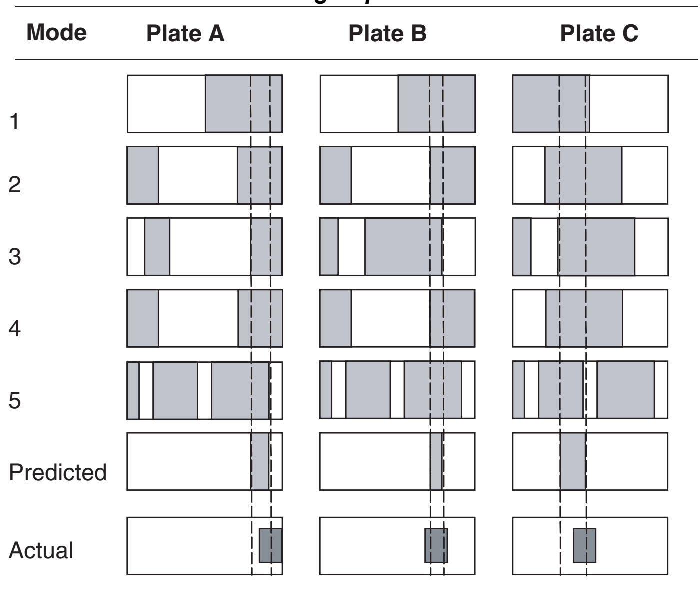


Figure 11. *Modal strain energy distribution for the first five modes of the intact cantilever plate.*

2. For Mode 2, a relatively small percentage change of modal damping (3.3%) is also observed. Thus, the delamination also associates with the elements of small quantity of modal strain energy. Therefore, the shaded region of the plot in the second row and the second column of Table 3 will be the predicted delamination area.
3. As the percentage change of modal damping for the third mode, as shown in Table 2, goes up to 56% due to the introduction of delamination, then, the prediction of the delaminated area from this information only is provided in the shaded area shown by the plot in the third row and second column of Table 3.
4. The percentage change of the fourth modal damping is also much larger than the average, so the delamination, which introduces damping increase, is associated with the elements that contain a relatively large percentage of modal strain energy. Thus, from this information only, the delamination region is estimated in the shaded region of the

Table 3. Delamination regions predicted using modal damping change combined with modal strain energy distribution for the damaged plates.



plot in the fourth row and the second column of Table 3. Similarly, the prediction of the delamination area is obtained from information of the fifth mode.

5. The overlap of all the five shaded areas gives the prediction of the delamination area for Plate B as shown by the plot in row 6 and column 2 of Table 3.

The above approach is also applied to Plates A and C, as illustrated by the first and third columns of Table 3. The last two rows of Table 3 show the comparison of predicted and actual delamination areas for the three plates, respectively. It is seen that the predicted areas have a reasonable agreement with the actual ones.

CONCLUSIONS

A new method for the prediction of delamination location in multilayer composite plate is presented in this paper. Numerical and experimental investigations into the delamination-induced changes of modal parameters have been carried out. Numerical analysis shows that an intrinsic connection exists between delamination location and the changes of modal parameters. The effect of delamination location on delamination-induced variation of mode-shape is consistent with that of natural frequency. Numerical simulations provide a good explanation for damping increase due to delamination, i.e., the energy dissipation is mostly induced by the interfacial slip across the delamination and the tendency of penetration between the upper and lower surfaces in the delamination region. The results of this study show that the location of internal delamination in multilayer composite plate can be estimated using a combination of measured modal damping change with computed modal strain energy distribution. It is convenient and feasible to predict delamination location using the method proposed in this paper, because experimental modal analysis is till now one of the most practical and reliable methods for structural vibration measurements. The results of this study show that the presented method for determination of damage location is quite promising. The proposed method has attractive application to damage detection of composites, especially for smart structures because of their inherent ability to provide excitation to the structure without requiring much additional equipment.

ACKNOWLEDGMENTS

The work described in this paper is supported by the Research Grants Council of Hong Kong Special Administrative Region, China (Project No. PolyU 5174/01E and PolyU 5313/03E). The authors are also grateful for the support of The Hong Kong Polytechnic University to carry out this project.

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