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Vibration analysis of a piezoelectric composite plate with cracks

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Abstract

The vibration behavior of a piezoelectric composite plate with cracks is analyzed in this paper. Based on the principle of minimum energy, a dynamical model is established, and the effects of cracks and piezoelectric materials on mode shapes are analyzed. Numerical simulations for a rectangular aluminum plate with and without cracks are conducted to validate the model. The contours of the displacement and strain mode shapes are compared. It is shown that the strain mode is more sensitive to cracks rather than the displacement mode. The approach is expected to detect damages of piezoelectric composite plates.
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Keywords: Piezoelectric composite plate; Cracks; Vibration; Mode shape

1. Introduction

The piezoelectric composite plates have extensively been used in engineering due to the high strength and stiffness, corrosion resistance and low cost of piezoelectric materials. In general, the piezoelectric materials may be distributed piecewise on the structural surface or occupy a whole layer. In the latter case, the host plate and the piezoelectric layer form a composite multi-layer laminate. However, this kind of structure is easily subjected to damage (e.g., cracks or delaminations) under loadings, such as the transverse time-varying loading, which reduces the structural safety, reliability and operational life [1–4]. Usually, the appearance of cracks changes the structural dynamic characteristics (natural frequencies and mode shapes) [5,6]. For example, Yuen [7] and Ratcliffe [8] discussed the sensitivity of displacement mode shapes and strain/curvature mode shapes to damage for a beam. Li et al. [9] presented a numerical model of a damaged plate with piezoelectric actuation based on variational methods, and verified its validity using indices related to frequency variation and energy change in both frequency and time domains. The sensitivity of static and dynamic parameters to damage occurring in plate-like structures was systematically investigated by Yam et al. [10]. These studies have covered many important aspects of physics associated with the modeling and damage detection of plate-like structures. However, as far as the piezoelectric composite laminates are concerned, few results have been reported so far, especially for structures with multi-cracks. In such case, it is of interest to investigate how the modeling and dynamic characteristics of piezoelectric composite plates are affected by the cracks.

This paper attempts to answer these questions and to potentially provide an effective approach to determine the existence of damage. The paper is organized as follows. Section 2 addresses the modeling approach. By using the principle of minimum energy, the modeling of a rectangular composite plate with a defect is performed. In Section 3, numerical simulations are studied. The effects of cracks on natural frequencies and displacement/strain mode shapes of a piezoelectric plate with different boundary conditions are considered.

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Comparison between existing results and the numerical solutions presented showed excellent agreement. Finally, some conclusions are drawn.

2. An analytical model of a piezoelectric composite plate with cracks

The configuration under investigation is a rectangular plate ($a \times b \times h_{\mathrm{p}}$) with perfect bonded piezoelectric patches $\Omega_{\mathrm{pe}_\ell}$:$\{x_{\mathrm{pe}_{\ell 1}} \leqslant x \leqslant x_{\mathrm{pe}_{\ell 2}},\ y_{\mathrm{pe}_{\ell 1}} \leqslant y \leqslant y_{\mathrm{pe}_{\ell 2}},\ \ell = 1, \ldots, r\}$ on both surfaces of the plate, which are symmetrical to the mid-plane of the structure (see Fig. 1). The ith crack in the structure is simulated by a small defective area $\Omega_{\mathrm{d}i}$:$[x_{\mathrm{d}i1}\ x_{\mathrm{d}i2}] \times [y_{\mathrm{d}i1}\ y_{\mathrm{d}i2}]$ $(i = 1, \ldots, s)$ with a reduction of thickness $h_{\mathrm{d}i}$ to ensure the linear strain distribution along with the thickness and zero transverse shear stress. According to the classical laminated theory, the constitutive relation of the structure is expressed as

$$\{\sigma\} = [S]\{\varepsilon\}, \tag{1a}$$

where $\{\sigma\}$ and $\{\varepsilon\}$ are the stress vector and the strain vector, respectively, and

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x + z\kappa_x \\ \varepsilon_y + z\kappa_y \\ \varepsilon_{xy} + z\kappa_{xy} \end{Bmatrix}, \quad \{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}, \tag{1b}$$

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \kappa_x = -\frac{\partial^2 w}{\partial x^2},$$
$$\kappa_y = -\frac{\partial^2 w}{\partial y^2}, \quad \kappa_{xy} = -\frac{2\partial^2 w}{\partial x \partial y}. \tag{1c}$$

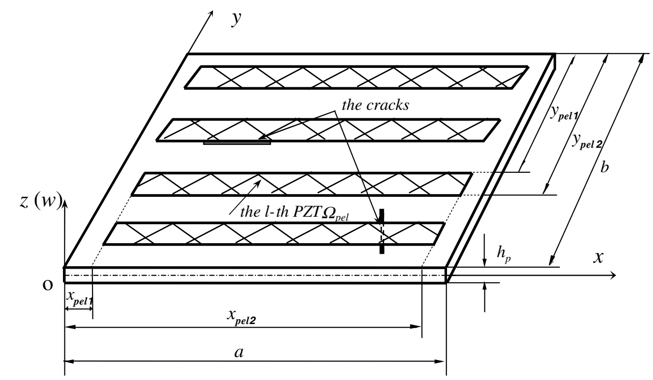


Fig. 1. Schematic diagram of a piezoelectric plate with cracks.

u, v and w denote the mid-plane displacements either on the plate or on the piezoelectric elements. $[S]$ is the elastic stiffness matrix and

$$[S]_{\mathrm{k}} = \frac{E_{\mathrm{k}}}{1 - v_{\mathrm{k}}^2} \begin{bmatrix} 1 & v_{\mathrm{k}} & 0 \\ v_{\mathrm{k}} & 1 & 0 \\ 0 & 0 & \frac{(1-v_{\mathrm{k}})}{2} \end{bmatrix},$$
$$\mathrm{k} = \mathrm{p}\ (\text{plate}),\ \mathrm{pe}\ (\text{piezoelectric material}), \tag{2}$$

where E_{k} and v_{k} are the Young's modulus and Poisson's ratio, respectively. In light of the stress resultants

$$\begin{bmatrix} N_x & M_x \\ N_y & M_y \\ N_{xy} & M_{xy} \end{bmatrix} = \int \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} (1,z)\mathrm{d}z, \tag{3}$$

the resultants of forces and moments in the presence of piezoelectric patches are determined as

$$\left.\begin{aligned} N_x &= \frac{E_{\mathrm{p}} h(x,y)}{1 - v_{\mathrm{p}}^2}(\varepsilon_x + v_{\mathrm{p}}\varepsilon_y) + \sum_{\ell=1}^{r} \frac{E_{\mathrm{pe}_\ell}}{1 - v_{\mathrm{pe}_\ell}} \cdot \frac{2t}{1 + v_{\mathrm{pe}_\ell}}(\varepsilon_x + v_{\mathrm{pe}_\ell}\varepsilon_y)\chi_{\mathrm{pe}_\ell}(x,y) \\ N_y &= \frac{E_{\mathrm{p}} h(x,y)}{1 - v_{\mathrm{p}}^2}(\varepsilon_y + v_{\mathrm{p}}\varepsilon_x) + \sum_{\ell=1}^{r} \frac{E_{\mathrm{pe}_\ell}}{1 - v_{\mathrm{pe}_\ell}} \cdot \frac{2t}{1 + v_{\mathrm{pe}_\ell}}(\varepsilon_y + v_{\mathrm{pe}_\ell}\varepsilon_x)\chi_{\mathrm{pe}_\ell}(x,y) \\ N_{xy} &= \left[\frac{E_{\mathrm{p}} h(x,y)}{2(1 + v_{\mathrm{p}})} + \sum_{\ell=1}^{r} \frac{E_{\mathrm{pe}_\ell} t}{(1 + v_{\mathrm{pe}_\ell})} \cdot \chi_{\mathrm{pe}_\ell}(x,y)\right]\varepsilon_{xy} \end{aligned}\right\} \tag{4}$$

and

$$\left.\begin{aligned} M_x &= \frac{E_{\mathrm{p}} h(x,y)^3}{12(1 - v_{\mathrm{p}}^2)}(\kappa_x + v_{\mathrm{p}}\kappa_y) + \sum_{\ell=1}^{r} \frac{E_{\mathrm{pe}_\ell}}{1 - v_{\mathrm{pe}_\ell}} \cdot \frac{2l_1(\kappa_x + v_{\mathrm{pe}_\ell}\kappa_y)}{3(1 + v_{\mathrm{pe}_\ell})}\chi_{\mathrm{pe}_\ell}(x,y) \\ M_y &= \frac{E_{\mathrm{p}} h(x,y)^3}{12(1 - v_{\mathrm{p}}^2)}(\kappa_y + v_{\mathrm{p}}\kappa_x) + \sum_{\ell=1}^{r} \frac{E_{\mathrm{pe}_\ell}}{1 - v_{\mathrm{pe}_\ell}} \cdot \frac{2l_1(\kappa_y + v_{\mathrm{pe}_\ell}\kappa_x)}{3(1 + v_{\mathrm{pe}_\ell})}\chi_{\mathrm{pe}_\ell}(x,y) \\ M_{xy} &= \left[\frac{E_{\mathrm{p}} h(x,y)^3}{24(1 + v_{\mathrm{p}})} + \sum_{\ell=1}^{r} \frac{E_{\mathrm{pe}_\ell} l_1}{3(1 + v_{\mathrm{pe}_\ell})} \cdot \chi_{\mathrm{pe}_\ell}(x,y)\right]\kappa_{xy} \end{aligned}\right\}, \tag{5}$$

where $h(x,y)$ is the thickness of the damaged plate and $h(x,y) = h_p - h_{di}\chi_{di}(x,y)$, while t the thickness of the piezoelectric patch; $l_1 = (h_p/2 + t)^3 - h_p^3/8$ (Ref. [11]). $\chi_{pe_\ell}(x,y)$ (or $\chi_{di}(x,y)$) is the generalized location function

$$\chi_{pe_\ell}(x,y) = \begin{cases} 1, & (x,y) \subset \Omega_{pe_\ell} \\ 0, & \text{elsewhere} \end{cases}. \tag{6}$$

For the sake of analysis, it can be assumed that all piezoelectric patches have the identical properties, viz. $E_{pe_1} = \ldots = E_{pe_r} = E_{pe}$, $\nu_{pe_1} = \ldots = \nu_{pe_r} = \nu_{pe}$.

In general, the equilibrium equations of a structure induced by the applied electric field (Ref. [12]) can be written as

$$\left.\begin{aligned} &\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \hat{\rho} h(x,y)\frac{\partial^2 u}{\partial t^2} \\ &\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = \hat{\rho} h(x,y)\frac{\partial^2 v}{\partial t^2} \\ &\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \hat{\rho} h(x,y)\frac{\partial^2 w}{\partial t^2} \end{aligned}\right\}, \tag{7}$$

where $\hat{\rho}$ is the mass per unit volume resulting from the material properties of the plate (ρ_p) and piezoelectric patch (ρ_{pe}), and $\hat{\rho} = \rho_p + \rho_{pe}\chi_{pe}(x,y)$. Substituting Eqs. (4) and (5) into (7) yields

$$\left.\begin{aligned} &D_1\frac{\partial^2 u}{\partial x^2} + (D_2 + D_3)\frac{\partial^2 v}{\partial x \partial y} + D_2\frac{\partial^2 u}{\partial y^2} = \hat{\rho} h(x,y)\frac{\partial^2 u}{\partial t^2} \\ &D_2\frac{\partial^2 v}{\partial x^2} + (D_1 + D_3)\frac{\partial^2 u}{\partial x \partial y} + D_1\frac{\partial^2 v}{\partial y^2} = \hat{\rho} h(x,y)\frac{\partial^2 v}{\partial t^2} \\ &D_4\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = f(x,y,t) - \hat{\rho} h(x,y)\frac{\partial^2 w}{\partial t^2} \end{aligned}\right\}, \tag{8a–c}$$

where $f(x,y,t)$ is the distributed transverse loading,

$$\begin{cases} D_1 = \left[\frac{E_p h(x,y)}{1-\nu_p^2} + \sum_{\ell=1}^{r} \frac{2E_{pe}t}{1-\nu_{pe}^2}\chi_{pe_\ell}(x,y)\right], \\ D_2 = \left[\frac{E_p h(x,y)}{2(1+\nu_p)} + \sum_{\ell=1}^{r} \frac{E_{pe}t}{(1+\nu_{pe})}\cdot\chi_{pe_\ell}(x,y)\right], \\ D_3 = \left[\frac{E_p \nu_p h(x,y)}{1-\nu_p^2} + \sum_{\ell=1}^{r} \frac{2E_{pe}\nu_{pe}t}{1-\nu_{pe}^2}\chi_{pe_\ell}(x,y)\right], \\ D_4 = \left[\frac{E_p h(x,y)^3}{12(1-\nu_p^2)} + \sum_{\ell=1}^{r} \frac{2E_{pe}l_1}{3(1-\nu_{pe}^2)}\chi_{pe_\ell}(x,y)\right]. \end{cases} \tag{9}$$

The above development established the formulation method which describes the displacement equations of motion for extension and flexure of a damaged plate under the action of an applied electric field and transverse mechanical loading. According to Eqs. (8) and (9), the effects of the piezoelectric patches and cracks on deflections are introduced by the variations of the density $\hat{\rho}$ and coefficients ($D_1, \ldots, D_4$). Note that for the case of the "bimorph" arrangement of the piezoelectric materials, the deflections u and v are relatively small compared with the deflection w [13]. To this end, the solution of w is of great interest. It is clear that an exact solution of Eq. (8c) is not available due to the variation of coefficients $\hat{\rho}h(x,y)$ and D_4. In such case, the parallel formulation based on the Rayleigh–Ritz method is developed.

In general, the existence of the piezoelectric material and cracks may change the configuration of the structure to some extent. However, the continuity condition of deflection must be satisfied in the interfaces between the plate and piezoelectric patches in defective areas. In light of mode superposition theory, the transverse deflection $w(x,y,t)$ can be formulated by

$$w(x,y,t) = \sum_i \sum_j c_{ij} W_{ij}(x,y)\eta_{ij}(t), \tag{10}$$

where $\eta_{ij}(t)$ and $W_{ij}(x,y)$ are the ijth modal co-ordinate and shape function, respectively. c_{ij} the coefficients to be determined. Evidently, the effect of cracks on mode shapes (or the deflection) is embodied by the coefficients c_{ij}. The results of vibration analysis for a plate with cracks have been reported by Li et al. [14]. In this section, the analysis is extended to a piezoelectric composite plate with cracks. Using the Rayleigh–Ritz method, c_{ij} can be obtained by seeking the minima of the energy function

$$J(w) = E_U - E_T, \tag{11a}$$

i.e.,

$$\frac{\partial J(W)}{\partial c_{ij}} = \frac{\partial (E_U - E_T)}{\partial c_{ij}} = 0, \tag{11b}$$

where E_U and E_T are respectively the strain energy function and kinetic energy function,

$$\begin{aligned} E_U = &\int_0^a \int_0^b \frac{E_p h(x,y)^3}{12(1-\nu_p^2)}\left[\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right)^2 \right. \\ &\left. -2(1-\nu_p)\left(\frac{\partial^2 W}{\partial x^2}\cdot\frac{\partial^2 W}{\partial y^2} - \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right)\right] \mathrm{d}y\,\mathrm{d}x \\ &+ \sum_{\ell=1}^{r} \int_{x_{pe_{\ell 1}}}^{x_{pe_{\ell 2}}} \int_{y_{pe_{\ell 1}}}^{y_{pe_{\ell 2}}} \frac{2E_{pe}l_1\chi_{pe_\ell}(x,y)}{3(1-\nu_{pe}^2)}\left[\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right)^2 \right. \\ &\left. -2(1-\nu_{pe})\left(\frac{\partial^2 W}{\partial x^2}\cdot\frac{\partial^2 W}{\partial y^2} - \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right)\right] \mathrm{d}y\,\mathrm{d}x \end{aligned} \tag{12a}$$

$$\begin{aligned} E_T = \omega^2 \Bigg\{ &\frac{h_p}{2}\int_0^a \int_0^b \rho_p\left(1 - \chi_{di}(x,y)\cdot\frac{h_{di}}{h_p}\right) W^2 \mathrm{d}y\,\mathrm{d}x \\ &+ \sum_{\ell=1}^{r} \int_{x_{pe_{\ell 1}}}^{x_{pe_{\ell 2}}} \int_{y_{pe_{\ell 1}}}^{y_{pe_{\ell 2}}} t\rho_{pe} W^2 \mathrm{d}y\,\mathrm{d}x \Bigg\} \end{aligned} \tag{12b}$$

In Eq. (12a), the first term on the right hand represents the strain energy contributed by the cracked plate, while the second term is contributed by the rigidity of the piezoelectric materials. In Eq. (12b), the first term reflects the effect of cracks, which is included by the loss of

Table 1
Geometric size and material properties of the specimen

	Host plate (aluminum)	Piezoelectric layer (PZT)
Dimension (m)	0.15×0.1	0.15×0.1
Thickness (m)	2E-3	5E-4
Mass density (kg/m^3)	2700	7650
Young's modulus (Pa)	7.1E+10	6.45E+10
Poisson's ratio	0.3	0.3
Piezoelectric constant (e_{31}) (N/m V)	–	1.032E+01
Loss factor	0.01	–

the kinetic energy. The last term describes the contribution of piezoelectric materials.

Table 2
The first four natural frequencies of the piezoelectric plate with cracks

Mode	Status	
	Intact	Damaged
Free–free (B.C.)		
(1,1)	330.8	328.8
(2,0)	357.5	353.1
(2,1)	763.3	759.9
(0,2)	836.0	827.4
Simply-supported (B.C.)		
(1,1)	160.5	158.8
(1,2)	355.7	354.6
(2,1)	651.3	647.2
(2,2)	908.3	904.4

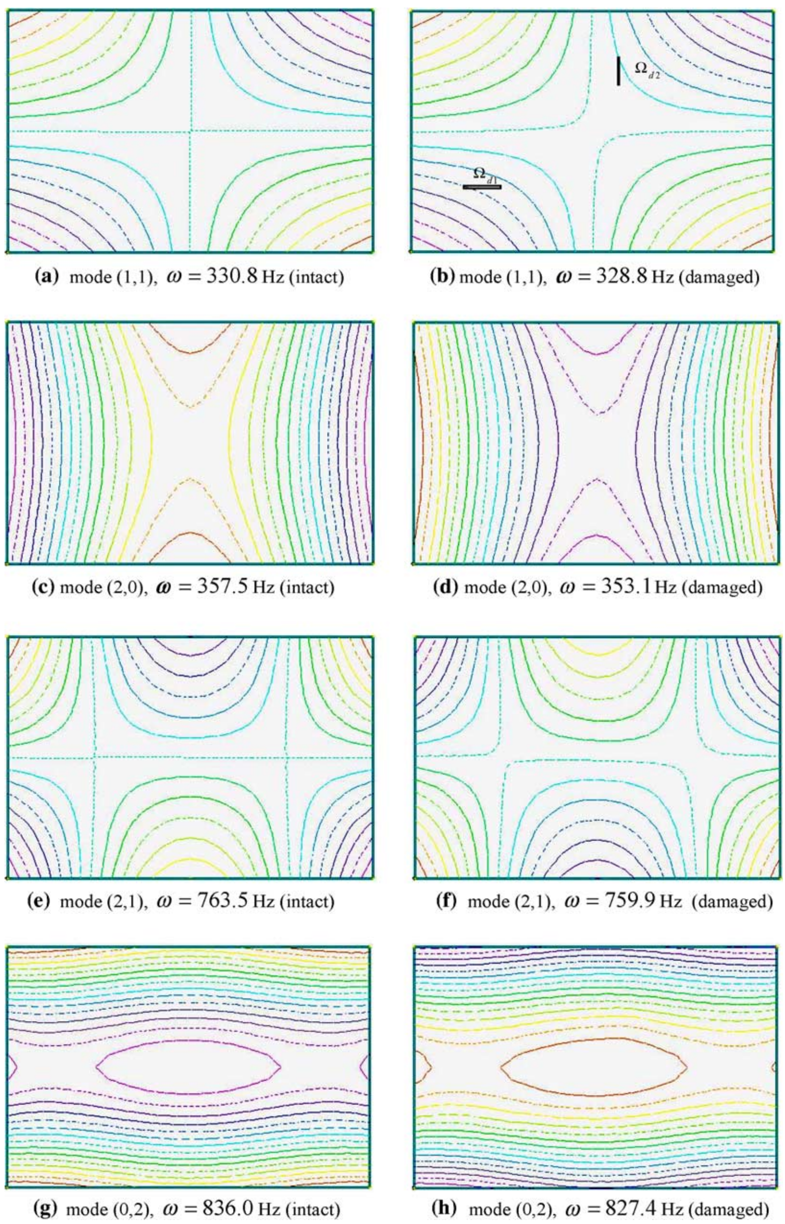


Fig. 2. Contours of displacement mode shapes for a free–free piezoelectric plate with and without cracks.

Substituting Eq. (12a) and (12b) into Eq. (11b) yields

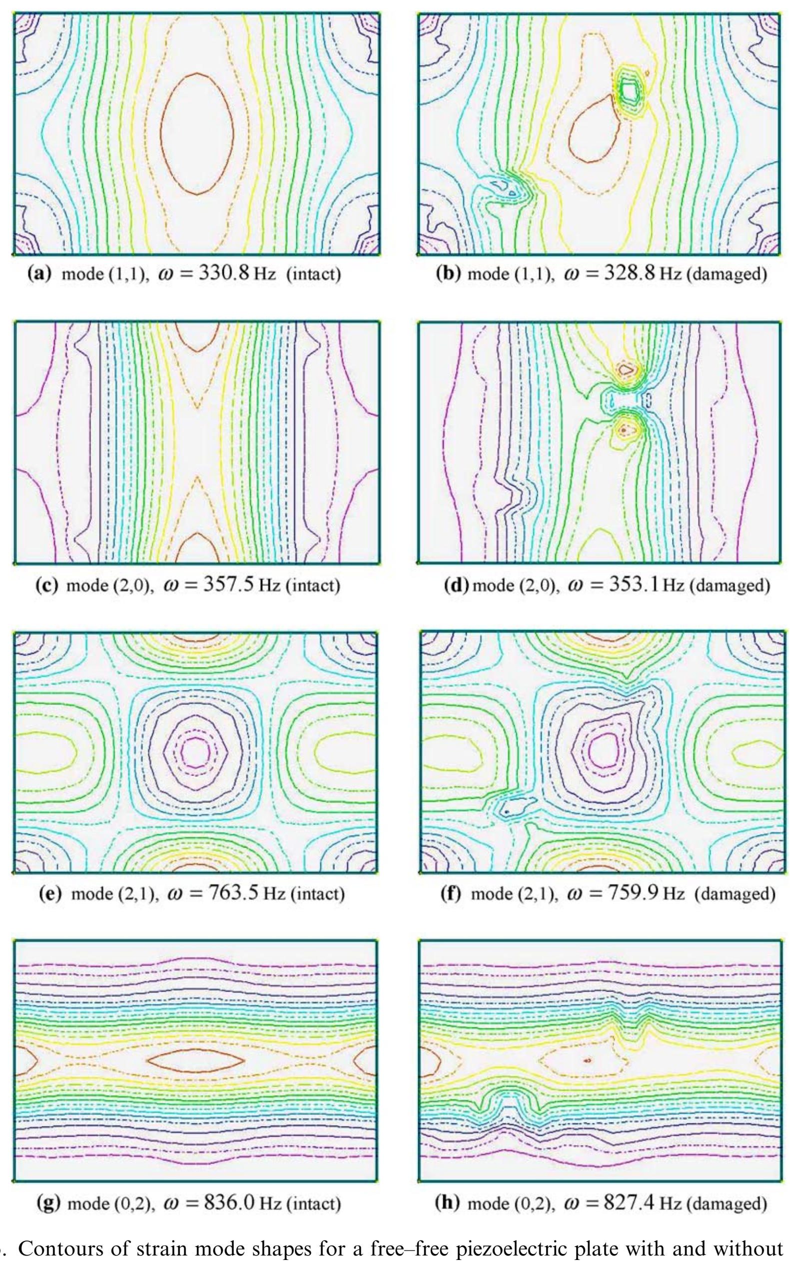
$$([\mathbf{K}] - \lambda^2[\mathbf{M}])\begin{Bmatrix} c_{11} \\ \vdots \\ c_{mn} \end{Bmatrix} = \mathbf{0}, \tag{13}$$

where λ is the frequency parameter, $[\mathbf{M}]$ and $[\mathbf{K}]$ are respectively the mass and stiffness matrices in intricate expressions. Eq. (13) describes the vibration behavior of the damaged piezoelectric plate, which can be used to calculate coefficients c_{ij} for constructing the deflection (or displacement mode shapes) of the structure. As a result, the strain mode shape, which describes the distribution of the in-plane strain components at the top or bottom surface of the plate according to the corresponding natural vibration mode, can be obtained in view of the strain–displacement relationship. For example, the strain mode shapes in the x and y directions can be derived

$$\varepsilon_{xx}(x,y) = -h(x,y)\frac{\partial^2 W(x,y)}{\partial x^2},$$
$$\varepsilon_{yy}(x,y) = -h(x,y)\frac{\partial^2 W(x,y)}{\partial y^2}, \tag{14}$$

and the total strain mode shape is accordingly defined as

$$\varepsilon(x,y) = \sqrt{\varepsilon_{xx}^2(x,y) + \varepsilon_{yy}^2(x,y)}. \tag{15}$$



(a) mode (1,1), $\omega = 330.8$ Hz (intact)
(b) mode (1,1), $\omega = 328.8$ Hz (damaged)
(c) mode (2,0), $\omega = 357.5$ Hz (intact)
(d) mode (2,0), $\omega = 353.1$ Hz (damaged)
(e) mode (2,1), $\omega = 763.5$ Hz (intact)
(f) mode (2,1), $\omega = 759.9$ Hz (damaged)
(g) mode (0,2), $\omega = 836.0$ Hz (intact)
(h) mode (0,2), $\omega = 827.4$ Hz (damaged)

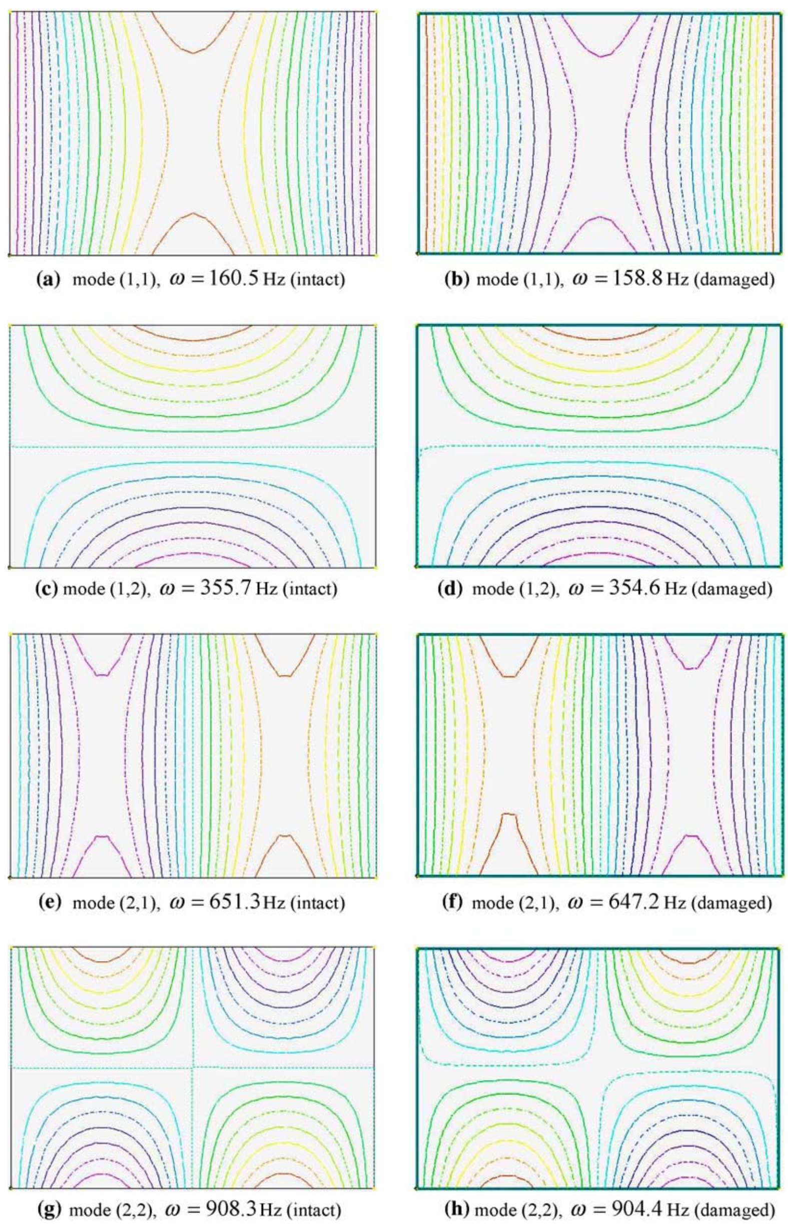
Fig. 3. Contours of strain mode shapes for a free–free piezoelectric plate with and without cracks.

3. Numerical results and discussions

The formulation described in Section 2 is implemented to analyze the natural frequencies and displacement/strain mode shapes of a damaged aluminum plate covered with a whole layer PZT. Dimensions and material properties are tabulated in Table 1. Two cracks are simulated by the defective areas $\Omega_{d1} = [0.03\ \ 0.045] \times [0.025\ \ 0.03]\ \mathrm{m}^2$ and $\Omega_{d2} = [0.09\ \ 0.095] \times [0.06\ \ 0.075]\ \mathrm{m}^2$ with a thickness reduction $h_d = 0.001$ m.

Firstly, the *first four* natural frequencies of the structure with and without cracks are calculated and listed in Table 2 for comparison. Two cases of boundary conditions, i.e., the free–free and the simply-supported boundary condition, are adopted for analysis. It can be found that a systematic decrease of natural frequencies appears due to the existence of cracks, which is in consistent with that reported in Ref. [9]. Although changes in natural frequencies are small, it is an intuitive parameter for health monitor of the piezoelectric composite plate, and is easy to be obtained using vibration test.

The effect of cracks on displacement mode shapes is then investigated. Fig. 2(a)–(h) illustrate the contour plot of the *first four* displacement mode shapes for the intact and damaged piezoelectric plates with *free–free* boundary condition. It can be found that no obvious change is observed except for a slight deformation occurring at nodal lines of modes $(1,1)$ (Fig. 2(b)) and $(2,1)$ (Fig. 2(f)). Special attention is paid to the defective



(a) mode (1,1), $\omega = 160.5$ Hz (intact)
(b) mode (1,1), $\omega = 158.8$ Hz (damaged)
(c) mode (1,2), $\omega = 355.7$ Hz (intact)
(d) mode (1,2), $\omega = 354.6$ Hz (damaged)
(e) mode (2,1), $\omega = 651.3$ Hz (intact)
(f) mode (2,1), $\omega = 647.2$ Hz (damaged)
(g) mode (2,2), $\omega = 908.3$ Hz (intact)
(h) mode (2,2), $\omega = 904.4$ Hz (damaged)

Fig. 4. Contours of displacement mode shapes for a simply-supported piezoelectric plate with and without cracks.

areas Ω_{d1} and Ω_{d2}, in which the appearance of cracks cannot lead to a significant change in displacement modes. Apparently, the displacement mode is not a sensitive parameter to defects.

However, things will be changed when strain mode shapes are taken into account. The contours of the corresponding strain mode shapes are plotted in Fig. 3. Comparing with the results obtained in Fig. 2 for displacement modes, an evident variation of strain mode shapes at Ω_{d1} and Ω_{d2} can be detected. Obviously, this local feature makes it possible to identify the damage locations in a piezoelectric composite plate using strain mode shapes.

This deduction is further verified in Figs. 4 and 5, which illustrate the displacement and strain mode shapes for the same configuration with *simply-supported* boundary condition. Similar to the results obtained for the *free–free* one (Figs. 2 and 3), strain mode shapes are verified as the more sensitive parameter to defects than displacement mode shapes.

4. Conclusions

An analytical model is developed to conduct vibration analysis of a damaged piezoelectric composite plate with different boundary conditions. Numerical results show that the present model can effectively simulate the dynamic characteristics of a damaged plate. The natural frequencies and displacement/strain mode shapes

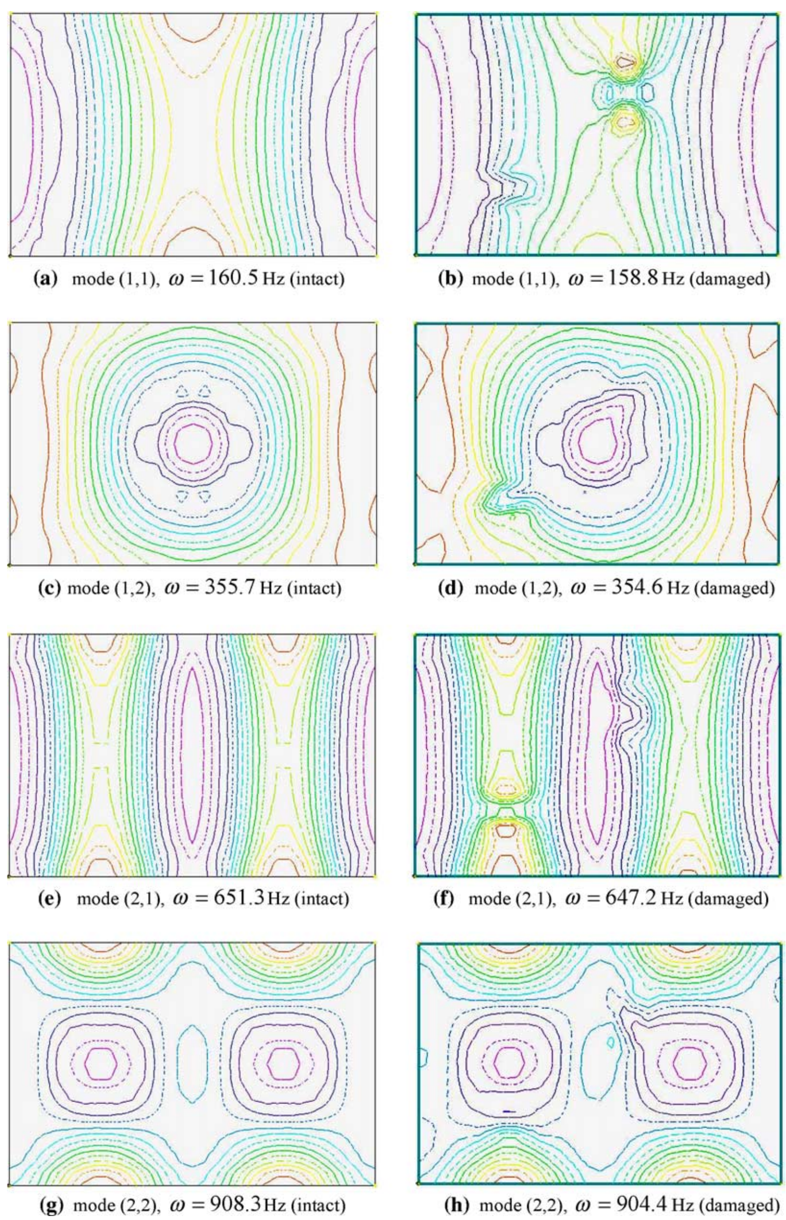


Fig. 5. Contours of strain mode shapes for a simply-supported piezoelectric plate with and without cracks.

are predicted and compared between the intact and damaged cases, leading to the following conclusions:

The cracks in a piezoelectric composite plate alter modal characteristics of the structure to a different level. The strain mode shape is the most sensitive parameter comparing with the parameters of natural frequencies and displacement mode shapes. This local behavior can therefore be adopted to identify damage locations in a piezoelectric composite plate. The method should be useful to future damage detection in piezoelectric composite plates.

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