Available online at www.sciencedirect.com
ScienceDirect
Journal of Sound and Vibration 301 (2007) 898–908
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Design of a dynamic vibration absorber for vibration isolation of beams under point or distributed loading

W.O. Wong*, S.L. Tang, Y.L. Cheung, L. Cheng

Department of Mechanical Engineering, The Hong Kong Polytechnic University, Hung Hom, Hong Kong SAR, China

Received 28 April 2006; received in revised form 23 August 2006; accepted 28 October 2006
Available online 8 December 2006

Abstract

A new dynamic vibration absorber combining a translational-type absorber and a rotational-type absorber is proposed for isolation of beam vibration under point or distributed harmonic excitation. Finite element analysis and Euler–Bernouli beam theory are used for evaluation of the performance of vibration isolation of the proposed absorber mounted on a beam. It is proved theoretically that the absorber can isolate the vibration in one part of a beam when it is subjected to a point or distributed harmonic excitation. A prototype of the combined translational and rotational dynamic absorber had been designed and made for evaluation. Both numerical and experimental tests have been done for verification of the theoretical prediction of vibration isolation in beam vibration. The proposed absorber is also suitable for vibration isolation of beam with nonuniform cross-section.

1. Introduction

The traditional dynamic vibration absorber is an auxiliary mass-spring system which, when correctly tuned and attached to a vibrating system subject to harmonic excitation, causes to cease the steady-state motion at the point to which it is attached [1–3]. It has the advantage of providing a cheap and easy-to-maintain solution for suppressing vibration in vibrating systems with harmonic excitation. However, when applying dynamic vibration absorber to a continuous structure such as a beam, vibration can be eliminated only at the attachment point of the vibrating beam while amplification of vibration may occur in other parts of the beam. Research on suppressing vibration in a region or the whole span of a beam structure by using the dynamic vibration absorber have been reported recently [4,5]. These methods require the use of many translational-type mass-spring absorbers for creating a region of nearly zero amplitude in the beam structure. In this paper, it is proved that a region of zero amplitude in the beam structure can be obtained if a translational-type absorber and a rotational-type absorber are combined as one and attached at a suitable location on a beam structure under distributed or point load of harmonic excitation. The performance of the proposed absorber for vibration isolation in beam structures was investigated via finite element analysis and verified by numerical and experimental tests.

*Corresponding author. Tel.: +852 2766 6667; fax: +852 2365 4703.
E-mail address: mmwowong@polyu.edu.hk (W.O. Wong).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2006.10.028

2. Theory

The finite element model of a beam with a translational absorber and a rotational absorber attached at node i is shown in Fig. 1. The finite element equation of the beam under excitation may be written as

$$M\frac{\partial^2 u(x,t)}{\partial t^2} + Ku(x,t) = f(x,t), \tag{1}$$

where u and f are the nodal displacement and load vectors, respectively. M and K are the mass and stiffness matrices, respectively.

For a distributed harmonic excitation with frequency ω, f may be written as $F(x)\sin\omega t$, where F is a vector of the nodal amplitudes of excitation written as

$$F = [F_1, M_1, F_2, M_2, \ldots, F_n, M_n]^{\mathrm{T}}, \tag{2}$$

where superscript T represent the transpose of the matrix. F_j and M_j where $j = 1,2,3,\ldots,n$ are the force and moment applied at node j, respectively.

For the steady state response of the nodal displacement of the beam in the equilibrium state,

$$u = U \sin \omega t, \tag{3}$$

where U is the vector of nodal amplitudes of vibration written as

$$U = [U_1, \Theta_1, U_2, \Theta_2, \ldots, U_n, \Theta_n]^{\mathrm{T}}, \tag{4}$$

where U_j and Θ_j where $j = 1,2,3,\ldots,n$ are the nodal amplitudes of translation and rotation at node j, respectively.

Eq. (1) may be written as

$$\left(K - \omega^2 M\right) U = F. \tag{5}$$

Based on Euler–Bernoulli beam theory with Hermite shape functions [6], the stiffness matrix of the jth element of the beam is

$$k_{e_j} = \frac{E_e I_e}{l_e^3}\begin{array}{c} \text{Column}: 2j-1 \quad 2j \quad (2j+1) \quad (2j+2) \\ \begin{bmatrix} 0 & & & & & & & & \\ & \ddots & & & & & & & \\ & & 0 & & & & & & \\ & & & 12 & 6l_e & -12 & 6l_e & & \\ & & & 6l_e & 4l_e^2 & -6l_e & 2l_e^2 & & \\ & & & -12 & -6l_e & 12 & -6l_e & & \\ & & & 6l_e & 2l_e^2 & -6l_e & 4l_e^2 & & \\ & & & & & & & 0 & \\ & & & & & & & & \ddots \\ & & & & & & & & & 0 \end{bmatrix} \end{array}. \tag{6}$$

According to Fig. 1, a translational absorber in form of a spring-mass and a rotational absorber in form of a pendulum are attached at node i of the beam. The stiffness matrix of the absorber K_a to be added to the global

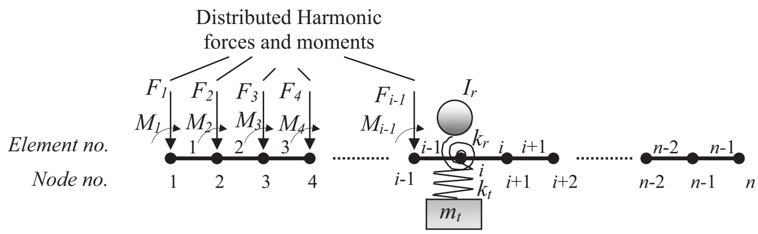

Fig. 1. A finite element model of a flexural beam with a translational and rotational dynamic absorber at node i.

stiffness matrix of the beam is

$$K_a = \begin{array}{c} \text{Column}: \quad 2i-1 \quad 2i \quad 2n-1 \quad 2n \\ \begin{bmatrix} 0 & & & & & & & & 0 & 0 \\ \vdots & \ddots & & & & & & & \vdots & \vdots \\ & & 0 & & & & & & 0 & 0 \\ & & & -k_t & & & & & k_t & 0 \\ & & & & -k_r & & & & 0 & k_r \\ & & & & & 0 & & & \vdots & 0 \\ & & & & & & \ddots & & & \vdots \\ & & & & & & & 0 & & \\ 0 & \cdots & 0 & k_t & 0 & & \cdots & & 0 & \\ 0 & \cdots & & 0 & k_r & 0 & \cdots & & & 0 \end{bmatrix} \end{array}, \tag{7}$$

where k_t and k_r are the stiffness of the translational and rotational absorbers, respectively.

The global stiffness matrix K of the beam with the absorbers may be written as

$$K = \sum_{j}^{n} k_{e_j} + K_a. \tag{8}$$

The mass matrix of the jth finite element of the beam is [6],

$$m_{e_j} = \frac{\rho_e A_e l_e}{420} \begin{array}{c} \text{Column}: \quad 2j-1 \quad 2j \quad (2j+1) \quad (2j+2) \\ \begin{bmatrix} 0 & & & & & & & & & \\ & \ddots & & & & & & & & \\ & & 0 & & & & & & & \\ & & & 156 & 22l_e & 54 & -13l_e & & & \\ & & & 22l_e & 4l_e^2 & 13l_e & -3l_e^2 & & & \\ & & & 54 & 13l_e & 156 & -22l_e & & & \\ & & & -13l_e & -3l_e^2 & -22l_e & 4l_e^2 & & & \\ & & & & & & & 0 & & \\ & & & & & & & & \ddots & \\ & & & & & & & & & 0 \end{bmatrix} \end{array}. \tag{9}$$

The mass matrix of the absorber M_a to be added to the global mass matrix of the beam is

$$
M_a = \begin{array}{c}
\begin{array}{ccccccc} & & & & \text{Column :} & 2n-1 & 2n \end{array} \\
\begin{bmatrix}
0 & \cdots & & & \cdots & 0 & 0 \\
\vdots & \ddots & & & & \vdots & \vdots \\
 & & & & & & \\
 & & & & & & \\
 & & & \ddots & & \vdots & \\
\vdots & & & & 0 & 0 & \vdots \\
0 & \cdots & & \cdots & 0 & m_t & 0 \\
0 & \cdots & & & \cdots & 0 & I_r
\end{bmatrix}
\end{array}, \tag{10}
$$

where m_t and I_r are the lumped mass of the translational absorber and the moment of inertia of the rotational absorber, respectively.

The global mass matrix M is the sum of the element mass matrices and the mass matrix of the absorber:

$$M = \sum_i m_{e_j} + M_a. \tag{11}$$

The matrix in Eq. (5) may be written as

$$
(K - \omega^2 M) = \Delta =
$$

$$
\begin{bmatrix}
\Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} & & & & & & & & & & & & & & & & \\
\Delta_{21} & \Delta_{22} & \Delta_{23} & \Delta_{24} & & & & & & & & & & & & & & & & \\
\Delta_{31} & \Delta_{32} & \Delta_{33} & \Delta_{34} & \Delta_{35} & \Delta_{36} & & & & & & & & & & & & & & \\
\Delta_{41} & \Delta_{42} & \Delta_{43} & \Delta_{44} & \Delta_{45} & \Delta_{46} & & & & & & & & & & & & & & \\
 & & \Delta_{53} & \Delta_{54} & \Delta_{55} & \Delta_{56} & & & & & & & & & & & & & & \\
 & & \Delta_{63} & \Delta_{64} & \Delta_{65} & \Delta_{66} & \ddots & & & & & & & & & & & & & \\
 & & & & & & & \Delta_{2i-32i-3} & \Delta_{2i-32i-2} & \Delta_{2i-32i-1} & \Delta_{2i-32i} & & & & & & & & & \\
 & & & & & & & \Delta_{2i-22i-3} & \Delta_{2i-22i-2} & \Delta_{2i-22i-1} & \Delta_{2i-22i} & & & & & & & & & \\
 & & & & & & & \Delta_{2i-12i-3} & \Delta_{2i-12i-2} & \Delta_{2i-12i-1} & \Delta_{2i-12i} & \Delta_{2i-12i+1} & \Delta_{2i-12i+2} & & & & & & -k_t & \\
 & & & & & & & \Delta_{2i2i-3} & \Delta_{2i2i-2} & \Delta_{2i2i-1} & \Delta_{2i2i} & \Delta_{2i2i+1} & \Delta_{2i2i+2} & & & & & & & -k_r \\
 & & & & & & & & & \Delta_{2i+12i-1} & \Delta_{2i+12i} & \Delta_{2i+12i+1} & \Delta_{2i+12i+2} & & & & & & & \\
 & & & & & & & & & \Delta_{2i+22i-1} & \Delta_{2i+22i} & \Delta_{2i+22i+1} & \Delta_{2i+22i+2} & \ddots & & & & & & \\
 & & & & & & & & & & & & & & \Delta_{2n-32n-3} & \Delta_{2n-32n-2} & \Delta_{2n-32n-1} & \Delta_{2n-32n} & & \\
 & & & & & & & & & & & & & & \Delta_{2n-22n-3} & \Delta_{2n-22n-2} & \Delta_{2n-22n-1} & \Delta_{2n-22n} & & \\
 & & & & & & & & & & & & & & \Delta_{2n-12n-3} & \Delta_{2n-12n-2} & \Delta_{2n-12n-1} & \Delta_{2n-12n} & & \\
 & & & & & & & & & & & & & & \Delta_{2n2n-3} & \Delta_{2n2n-2} & \Delta_{2n2n-1} & \Delta_{2n2n} & & \\
 & & & & & & & & & -k_t & & & & & & & & & k_t-\omega^2 m_t & \\
 & & & & & & & & & & -k_r & & & & & & & & & k_r-\omega^2 I_r
\end{bmatrix} \tag{12}
$$

where $\Delta_{pp} = K_{pp} - \omega^2 M_{pp}$, $\Delta_{pq} = K_{pq}$; p, $q = 1, 2, 3, \ldots, n$, and $p \neq q$.

When the absorbers are tuned such that

$$\frac{k_t}{m_t} = \frac{k_r}{m_r} = \omega^2 \tag{13}$$

$$\Delta =$$

$$\left[\begin{array}{cccccccccccccccccccc}
\Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} \\
\Delta_{21} & \Delta_{22} & \Delta_{23} & \Delta_{24} \\
\Delta_{31} & \Delta_{32} & \Delta_{33} & \Delta_{34} & \Delta_{35} & \Delta_{36} \\
\Delta_{41} & \Delta_{42} & \Delta_{43} & \Delta_{44} & \Delta_{45} & \Delta_{46} \\
& & \Delta_{53} & \Delta_{54} & \Delta_{55} & \Delta_{56} \\
& & \Delta_{63} & \Delta_{64} & \Delta_{65} & \Delta_{66} \\
&&&&&& \ddots \\
&&&&&&& \Delta_{2i-32i-3} & \Delta_{2i-32i-2} & \Delta_{2i-32i-1} & \Delta_{2i-32i} \\
&&&&&&& \Delta_{2i-22i-3} & \Delta_{2i-22i-2} & \Delta_{2i-22i-1} & \Delta_{2i-22i} \\
&&&&&&& \Delta_{2i-12i-3} & \Delta_{2i-12i-2} & \Delta_{2i-12i-1} & \Delta_{2i-12i} & \Delta_{2i-12i+1} & \Delta_{2i-12i+2} &&&&&& -k_t \\
&&&&&&& \Delta_{2i2i-3} & \Delta_{2i2i-2} & \Delta_{2i2i-1} & \Delta_{2i2i} & \Delta_{2i2i+1} & \Delta_{2i2i+2} &&&&&&& -k_r \\
&&&&&&&&& \Delta_{2i+12i-1} & \Delta_{2i+12i} & \Delta_{2i+12i+1} & \Delta_{2i+12i+2} \\
&&&&&&&&& \Delta_{2i+22i-1} & \Delta_{2i+22i} & \Delta_{2i+22i+1} & \Delta_{2i+22i+2} \\
&&&&&&&&&&&&& \ddots \\
&&&&&&&&&&&&&& \Delta_{2n-32n-3} & \Delta_{2n-32n-2} & \Delta_{2n-32n-1} & \Delta_{2n-32n} \\
&&&&&&&&&&&&&& \Delta_{2n-22n-3} & \Delta_{2n-22n-2} & \Delta_{2n-22n-1} & \Delta_{2n-22n} \\
&&&&&&&&&&&&&& \Delta_{2n-12n-3} & \Delta_{2n-12n-2} & \Delta_{2n-12n-1} & \Delta_{2n-12n} \\
&&&&&&&&&&&&&& \Delta_{2n2n-3} & \Delta_{2n2n-2} & \Delta_{2n2n-1} & \Delta_{2n2n} \\
&&&&&&&&& -k_t \\
&&&&&&&&&& -k_r
\end{array}\right] \tag{14}$$

Referring to Fig. 1 and consider a harmonic distributed excitation with frequency ω, written as $F'(x)\sin\omega t$, where F' is a vector of the nodal amplitudes of excitation written as

$$F' = [F_1, M_1, F_2, M_2 \ldots F_i, M_i, 0 \ldots 0]^{\mathrm{T}}. \tag{15}$$

The vector of nodal amplitudes of the beam may be written as

$$\begin{aligned} U &= (K - \omega^2 M)^{-1} F' \\ &= \varDelta^{-1} F'. \end{aligned} \tag{16}$$

By Cramer's rule, the amplitude of the translation or rotation of the $(2i+a)$th degree-of-freedom, where $a = 1, 2, 3, \ldots, 2n-2i$, may be written as

$$U_{2i+a} = \frac{|S_{2i+a}|}{|\varDelta|}, \tag{17}$$

where matrix S_{a+2i} has F' as its $(2i+a)$th column and the corresponding columns of $\varDelta$ as its other columns. Determinant of S_{2i+a} may be written as

$$|S_{2i+a}| =$$

$$\left|\begin{array}{cccccccccccccccccccc}
\Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} &&&&&&&&&&& F_1 \\
\Delta_{21} & \Delta_{22} & \Delta_{23} & \Delta_{24} &&&&&&&&&&& M_1 \\
\Delta_{31} & \Delta_{32} & \Delta_{33} & \Delta_{34} & \Delta_{35} & \Delta_{36} &&&&&&&&& F_2 \\
\Delta_{41} & \Delta_{42} & \Delta_{43} & \Delta_{44} & \Delta_{45} & \Delta_{46} &&&&&&&&& M_2 \\
& & \Delta_{53} & \Delta_{54} & \Delta_{55} & \Delta_{56} &&&&&&&&& \vdots \\
& & \Delta_{63} & \Delta_{64} & \Delta_{65} & \Delta_{66} &&&&&&&&& \vdots \\
&&&&&& \ddots &&&&&&&& \vdots \\
&&&&&&& \Delta_{2i-32i-3} & \Delta_{2i-32i-2} & \Delta_{2i-32i-1} & \Delta_{2i-32i} &&&& \vdots \\
&&&&&&& \Delta_{2i-22i-3} & \Delta_{2i-22i-2} & \Delta_{2i-22i-1} & \Delta_{2i-22i} &&&& \vdots \\
&&&&&&& \Delta_{2i-12i-3} & \Delta_{2i-12i-2} & \Delta_{2i-12i-1} & \Delta_{2i-12i} & \Delta_{2i-12i+1} & \Delta_{2i-12i+2} && F_i &&&& -k_t \\
&&&&&&& \Delta_{2i2i-3} & \Delta_{2i2i-2} & \Delta_{2i2i-1} & \Delta_{2i2i} & \Delta_{2i2i+1} & \Delta_{2i2i+2} && M_i &&&&& -k_r \\
&&&&&&&&& \Delta_{2i+12i-1} & \Delta_{2i+12i} & \Delta_{2i+12i+1} & \Delta_{2i+12i+2} && 0 \\
&&&&&&&&& \Delta_{2i+22i-1} & \Delta_{2i+22i} & \Delta_{2i+22i+1} & \Delta_{2i+22i+2} && 0 \\
&&&&&&&&&&&&& \ddots & \vdots \\
&&&&&&&&&&&&&& 0 \\
&&&&&&&&&&&&&& \vdots & \ddots & \Delta_{2n-22n-1} & \Delta_{2n-22n} \\
&&&&&&&&&&&&&&& \Delta_{2n-12n-2} & \Delta_{2n-12n-1} & \Delta_{2n-12n} \\
&&&&&&&&&&&&&& 0 & \Delta_{2n2n-2} & \Delta_{2n2n-1} & \Delta_{2n2n} \\
&&&&&&&&& -k_t \\
&&&&&&&&&& -k_r
\end{array}\right| \tag{18}$$

Since there is only one nonzero element in the last two columns and the last two rows, the determinant in Eq. (18) can be simplified as

$$
\left|S_{2i+a}\right| = k_t{}^2 k_r{}^2 \left|\begin{array}{ccccccccc:ccccccc}
\Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} & & & & & & & & & F_1 & & & \\
\Delta_{21} & \Delta_{22} & \Delta_{23} & \Delta_{24} & & & & & & & & & M_1 & & & \\
\Delta_{31} & \Delta_{32} & \Delta_{33} & \Delta_{34} & \Delta_{35} & \Delta_{36} & & & & & & & F_2 & & & \\
\Delta_{41} & \Delta_{42} & \Delta_{43} & \Delta_{44} & \Delta_{45} & \Delta_{46} & & & & & & & M_2 & & & \\
 & & \Delta_{53} & \Delta_{54} & \Delta_{55} & \Delta_{56} & & & & & & & \vdots & & & \\
 & & \Delta_{63} & \Delta_{64} & \Delta_{65} & \Delta_{66} & \ddots & & & & & & \vdots & & & \\
 & & & & & & & \Delta_{2i-32i-3} & \Delta_{2i-32i-2} & & & & F_{i-1} & & & \\
 & & & & & & & \Delta_{2i-22i-3} & \Delta_{2i-22i-2} & & & & M_{i-1} & & & \\
\hdashline
 & & & & & & & & & \Delta_{2i+12i+1} & \Delta_{2i+12i+2} & & 0 & & & \\
 & & & & & & & & & \Delta_{2i+22i+1} & \Delta_{2i+22i+2} & \ddots & 0 & & & \\
 & & & & & & & & & & & & \vdots & & & \\
 & & & & & & & & & & & & 0 & & & \\
 & & & & & & & & & & & & \vdots & & & \\
 & & & & & & & & & & & & & \ddots & \Delta_{2n-22n-1} & \Delta_{2n-22n} \\
 & & & & & & & & & & & & 0 & \Delta_{2n-12n-2} & \Delta_{2n-12n-1} & \Delta_{2n-12n} \\
 & & & & & & & & & & & & 0 & \Delta_{2n2n-2} & \Delta_{2n2n-1} & \Delta_{2n2n}
\end{array}\right|
$$

$$
= \left[\begin{array}{c:c} A & B \\ \hdashline C & D \end{array}\right] \tag{19}
$$

where block matrices A and D are square matrices with $(2i-2)$ columns and $(2i-2)$ rows, and $(2n-2i+2)$ columns and $(2n-2i+2)$ rows, respectively; block matrices B and C are rectangular matrices with $(2n-2i+2)$ columns and $(2i-2)$ rows, and $(2i-2)$ columns and $(2n-2i+2)$ rows, respectively.

Eq. (19) may be rewritten as [7]

$$
\begin{aligned}
\left|S_{2i+a}\right| &= \left|\begin{array}{c:c} A & B \\ \hdashline C & D \end{array}\right| \\
&= |A|\left|D - CA^{-1}B\right|
\end{aligned} \tag{20}
$$

Examination of the block matrix C in Eq. (19) reveals that all elements of C are zeros. Determinant of block matrix D equals to zero because all elements of the $(a+2)$th column of D are all zeros. Eq. (20) may therefore be rewritten as

$$
\begin{aligned}
\left|S_{2i+a}\right| &= \left|\begin{array}{c:c} A & B \\ \hdashline 0 & D \end{array}\right| \\
&= |D||A| \\
&= 0
\end{aligned} \tag{21}
$$

Eq. (17) can be written as

$$
U_{2i+a} = \frac{\left|S_{2i+a}\right|}{|\Delta|} = 0. \tag{22}
$$

Referring to Fig. 1 and Eq. (22), the vibration amplitude of displacement and rotation at all the nodes on the right-hand side of the absorbers are zeros. Assuming linear vibration, it proves that the part of the beam in opposite to the point or distributed load measured from the absorbers is isolated from the forced vibration.

3. Numerical results and discussion

The vibration isolation effect of the proposed absorbers as predicted in the previous section was studied numerically and compared to the experimental result as shown in the next section. A computer program was written in Matlab for the numerical study of the vibration isolation of beam vibration using the proposed absorbers. An aluminum beam of dimensions 82.3 mm × 25 mm × 2.5 mm was modeled with fifty standard Euler beam elements. Young's modulus and density of the beam are 70 G N/m^2 and 2720 kg/m^3, respectively. The finite element equation was constructed according to Eqs. (6)–(15) of the previous section. Two different cases of boundary conditions were considered for the beam. In the first case, the beam was assumed as free at the left end and clamped at the right end as illustrated in Fig. 2(a), and the corresponding columns and rows of the matrices in the finite element equation were removed. The first two natural frequencies of the beam without the absorbers were found as 6 and 25.37 Hz, respectively. A uniformly distributed force of unit amplitude of frequency 14.64 Hz was applied from the left end of the beam to 30% of the beam span measured from the same end of the beam. The forced response vector was calculated according to Eq. (16) and the amplitude of translation at the nodes along the beam was plotted in Fig. 2(b). Forced response of the beam with only the traditional translational absorber was also calculated using Eq. (16) after removing the last column and row of Δ in Eq. (14) and the last element of F' in Eq. (15). The nodal vibration amplitudes were plotted in the Fig. 2(b) for comparison. In the second case, the beam was assumed as clamped at the two ends as illustrated in Fig. 3(a). The finite element equation was constructed as before with the corresponding columns and rows of the matrices in the finite element equation removed for the clamped–clamped boundary condition. The first two natural frequencies of the beam without the absorbers were found as 19 and 53 Hz, respectively. A point harmonic force of frequency 43 Hz was applied at a point measured 20% of the beam span from the left end of the beam. The forced response vector was calculated according to Eq. (16) and the

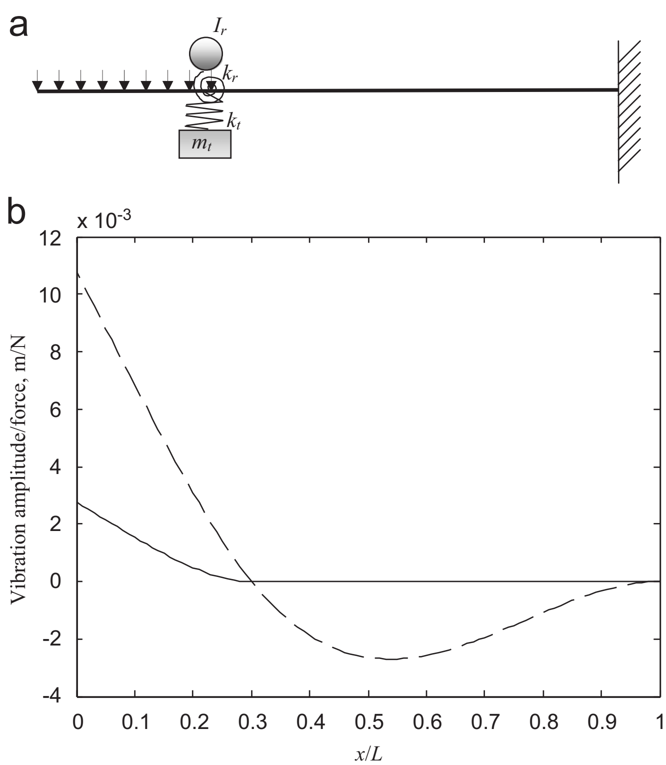

Fig. 2. (a) Schematic and (b) deflection shape of a free-clamped beam under harmonic excitation distributed uniformly in the region $0<x/L<0.3L$. - - - - - with only the translational dynamic absorber attached at $x/L = 0.3$, ——— with both the translational and rotational dynamic absorbers attached at $x/L = 0.3$.

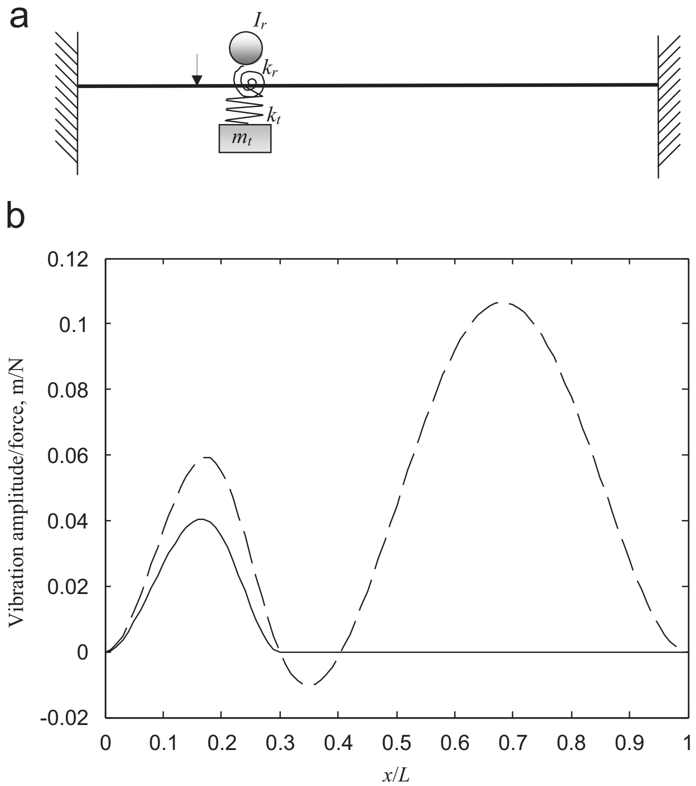

Fig. 3. (a) Schematic and (b) deflection shape of a clamped–clamped beam under harmonic excitation at $x/L = 0.2$. - - - - - with only the translational dynamic absorber attached at $x/L = 0.3$, ——— with both the translational and rotational dynamic absorbers attached at $x/L = 0.3$.

amplitude of translation at the nodes along the beam was plotted in Fig. 3(b). Forced response of the beam with only the traditional translational absorber was also calculated for comparison. In both Figs. 2 and 3, the results shows that the proposed absorber combining a translational and a rotational vibration absorber can isolate the vibration in the region of excitation applied onto the beam while the traditional dynamic vibration absorber can only enforce zero vibration at the point of attachment of the absorber.

4. Experimental test and design of the absorber

Experiments were done for validation of the theoretical predictions of vibration isolation using the proposed absorber in beam vibration. Since only one vibration exciter and a force transducer were available, the test rig was designed according to the model as shown in Fig. 3(a) and the experimental setup is illustrated in Fig. 4. An aluminum beam of dimensions 832 mm × 25 mm × 2.5 mm was clamped at the two ends in the experiment. Before fixing the absorbers onto the beam, it was excited with an impact hammer (B & K 8202) and the frequency response of the beam were measured by a light weight piezoelectric accelerometer (Endevco Model 22 Picomin) of 0.14 g and read from a spectrum analyser (B & K Pulse 3560c). The first two natural frequencies were measured to be 19.5 and 53 Hz.

A prototype of the absorber was designed and made for experimental verification of the theoretical prediction of vibration isolation in beam vibration. The prototype consisted of a translational absorber and a rotational absorber. The translational one was made by fixing a cylindrical block of copper of mass 13 g to one end of a compression spring of stiffness 900 N/m as shown in Fig. 5(a). The other end of the compression spring is fixed onto the beam by adhesive (Pattex Repair Express) at 30% of the beam span measured from one end of the beam. The mass of the absorber can be adjusted by fixing additional copper washers onto the

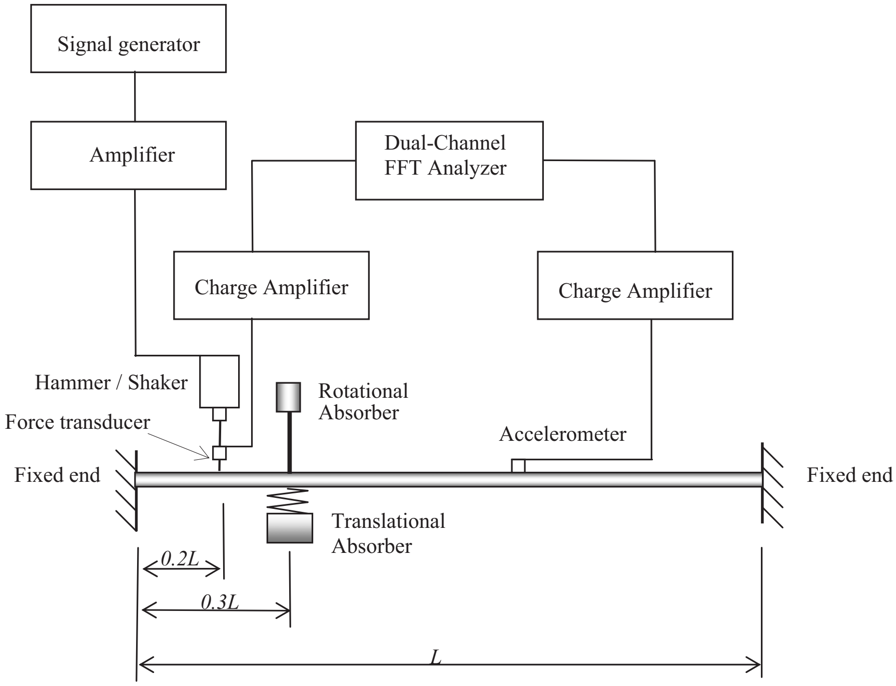

Fig. 4. Illustration of the experimental setup for testing of isolation of beam vibration using the combined vibration absorbers.

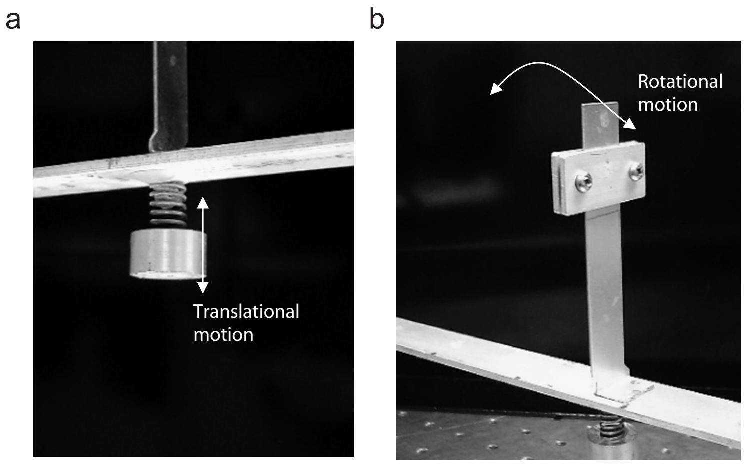

Fig. 5. Design of the (a) translational and (b) rotational absorber vibration absorbers.

copper block. The part of the beam carrying the translational absorber was rigidly clamped and the resonant frequency of the up-and-down vibration of the translational absorber was measured by the light weight accelerometer attached onto the copper block and read from the spectrum analyser. The resonant frequency of the up-and-down vibration of the translational absorber was tuned at 43 Hz which is between the first and the second frequencies of bending vibration of the beam.

The rotational absorber was made by attaching two rectangular aluminum plates of total mass 13 g onto a piece of steel strip of dimensions $100\,\text{mm} \times 18.3\,\text{mm} \times 0.7\,\text{mm}$ as shown in Fig. 5(b). One end of the steel strip is fixed on the beam by adhesive (Pattex Repair Express) to form a cantilever. The aluminum plates are clamped onto the steel strip by bolts and nuts so that their mounting position on the steel strip can be changed easily. The fundamental frequency of the rotational absorber can be tuned easily by adjusting the location of

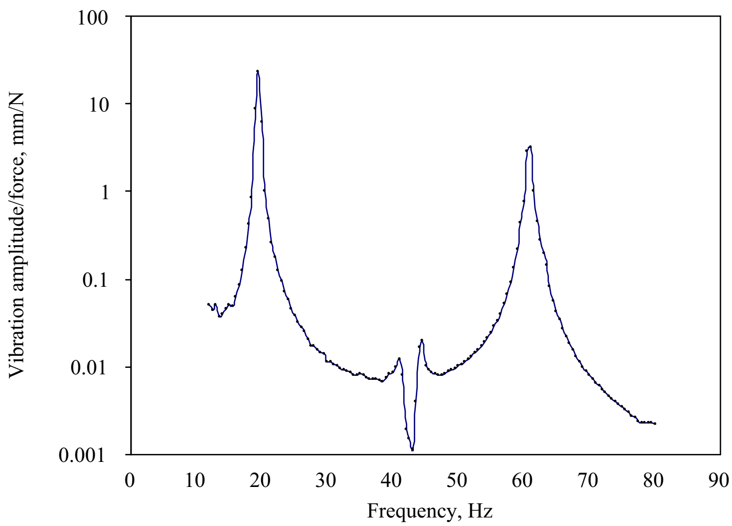

Fig. 6. Frequency spectrum of the beam with the proposed absorber.

the aluminum plates on the steel strip. Similar to the tuning procedure of the translational absorber, the fundamental frequency of the rotational motion of the rotational absorber was measured by the light weight accelerometer attached onto the surface of one of the aluminum plates and read from the spectrum analyser. The resonant frequency of the rotational oscillation of the rotational absorber was tuned at 43 Hz by carefully adjusting the mounting position of the aluminum plates on the steel strip as shown in Fig. 5(b).

After mounting the absorbers onto the beam, the beam was clamped at the two ends again. The beam was excited with an impact hammer (B & K 8202) applied at 166 mm ($0.2L$) and the response of the beam were measured by the light weight accelerometer at the attachment point of the absorbers ($0.3L$). The frequency spectrum was generated with the spectrum analyser and it is shown in Fig. 6. As observed in Fig. 6, two small peaks appeared around the absorption frequency and the response drops to a minimum at the absorption frequency, 43 Hz.

To test the theoretical prediction of vibration isolation using the absorbers, an electromagnetic shaker (B & K 4810) was used to excite the beam at 166 mm ($0.2L$) with a single harmonic excitation at 43 Hz as illustrated in Fig. 4. Vibration amplitudes were measured at twenty points on the beam surface by the light weight accelerometer and recorded by the spectrum analyser. The measurement procedure of the forced vibration amplitudes was repeated for the case of the beam with only the translational absorber and also the case of the beam without any absorber. Vibration amplitudes of the beam of all three cases were plotted in Fig. 7. As shown in Fig. 7, the vibration level of the whole beam was suppressed greatly when the combined type of absorber was implemented. The experimental result showed that the combined type of absorber could not completely isolate the vibration of the beam in the region $0<x/L<0.3$. This might due to the presence of damping in the vibration absorbers. Nevertheless, the effectiveness of using the combined type of absorber in vibration suppression and isolation was better than just using a translational absorber at all points of the beam. At some points such as $x/L = 0.7$, the vibration amplitude of the beam after using the combined type of absorber was 22 times lower than that of the case without any absorber and 4 times lower than that of the case with the translational vibration absorber alone.

5. Conclusion

It has been proved with finite element analysis using Euler beam theory that the forced beam vibration under point or distributed harmonic excitation can be isolated in a region of the beam by attaching a translational dynamic absorber and a rotational dynamic absorber at the same location on the beam surface. A practical design of the dynamic absorbers has been made and tested. Both numerical and experimental tests have been done for verification of the theoretical prediction of vibration isolation using the proposed dynamic

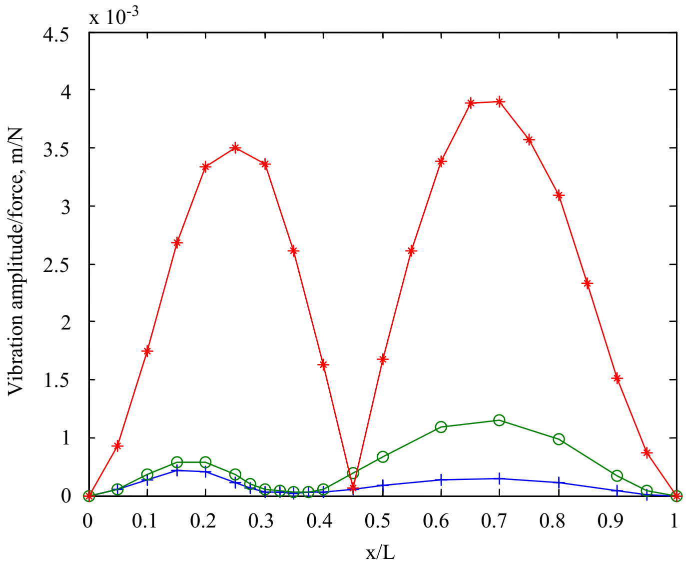

Fig. 7. Measured vibration amplitudes of the beam at the nodes with and without vibration absorbers. ✱— beam without absorber, ○— beam with the translational absorber at $x/L = 0.3$, +—beam with the combined translational and rotational absorbers at $x/L = 0.3$.

absorber. The numerical tests show that beam vibration under harmonic excitations can be isolated in a region of the beam by the proposed absorber but not the traditional translational absorber alone. In the experimental test, forced vibration of a beam were measured and compared for the cases of the beam carrying the proposed absorber, the beam carrying only the traditional translational absorber, and the beam carrying no absorber. The case of the beam with the proposed absorber has the best performance of vibration suppression and isolation in comparison of the case of using only the translational absorber. However, the region of the beam expected to be isolated from vibration was found to have vibration of relatively small amplitudes. This imperfection of vibration isolation would be due to the presence of damping in the prototype of the absorber because adhesive was used in joining the absorber onto the beam.

Acknowledgement

The authors wish to acknowledge support given to them by the Central Research Grant of The Hong Kong Polytechnic University.

References

[1] B.G. Korenev, L.M. Reznikov, *Dynamic Vibration Absorbers, Theory and Technical Applications*, Wiley, New York, 1993.
[2] J.P. Den Hartog, *Mechanical Vibrations*, Dover Publications Inc., 1985.
[3] J.B. Hunt, *Dynamic Vibration Absorbers*, Mechanical Engineering Publications Ltd., 1979.
[4] J. Dayou, Fixed-points theory for global vibration control using vibration neutralizer, *Journal of Sound and Vibration* 292 (2006) 765–776.
[5] C.P. Cha, Enforcing nodes at required locations in a harmonically excited structure using simple oscillators, *Journal of Sound and Vibration* 279 (3–5(21)) (2005) 799–816.
[6] L. Meirovitch, *Fundamentals of Vibrations*, McGraw-Hill, New York, 2001.
[7] M. Brookes, *The Matrix Reference Manual*. [online] ⟨http://www.ee.ic.ac.uk/hp/staff/dmb/matrix/intro.html⟩ 2005.