SHIP TRAFFIC IN CONTAINER PORT: MODELLING METHODOLOGY AND PERFORMANCE EVALUATION

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1. Introduction

This paper gives modeling methodology of ship traffic in container port and performance evaluation. The basic approach used mathematical and simulation models. These models are developed for impact analysis of the ship traffic and patterns of arrival ships at terminal performance. The results, analysis and conclusions given in this paper also addresses issues such as the performance criteria and the models parameters to propose an operational method that reduces average cost per ship served and increases the terminal efficiency. We present effect on capacity performance with numerical results and computational experiments which are reported to evaluate the efficiency of Pusan East Container Terminal (PECT). This paper describes the criteria for the combined evaluation of important parameters of the total port cost function such as: ship service and waiting time, the number of berths and related average container ship cost in port and the optimal combination of berths/terminal and quay cranes/berth. These parameters are the basis of simulation and analytical approaches used in this paper.
The rest of this paper is organized as follows:

- In the next section we provide an overview of the literature related to port simulation and analytical modeling.

- The following section presents a brief description of ship-berth-yard link modeling procedure. Also, this section is concerned with the evaluation of functional estimation models in container port.

- This is followed by the next section which gives model validation and simulation and analytical results for PECT.

- Finally, we conclude by summarizing the results and contributions of this paper.
## 2. Related literature

Most studies and papers focus their attention on the strategic and tactical problems in the applications of the simulation and analytical models. The determination of optimum number and capacity of berths have been treated both theoretically and practically in many studies.

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<td>Berth allocations and scheduling</td>
<td>Heuristic algorithm for BAPD. Heuristic algorithm (HA) for SBAP. Graphs; Heuristic.Genetic algorithm(GA). Lagrangian relaxation (LR) - based HA; Subgradient method (SM). LR; GA. HA for BAPD and BAPC. Tabu search (TS) heuristics. Mixed integer programming; Simulated annealing. Tree search procedure.</td>
<td>Lai and Shih (1992); Imai et al. (1997); Lim (1998); Nishimura et al. (2001); Imai et al. (2001); Imai et al. (2003); Imai et al. (2005); Zhou Xu (2002); Cordeau et al. (2003); Kim and Moon (2003); Guan and Cheung (2004);</td>
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<td>Crane scheduling</td>
<td>Heuristic method Branch &amp; Bound (B&amp;B). B&amp;B; Simulated annealing. Dynamic programming, TS and squeaky wheel heuristic. B&amp;B; GRASP.</td>
<td>Daganzo (1989); Peterkofsky and Daganzo (1990); Zhu and Lim (2004); Lim et al. (2004); Kim and Park (2004);</td>
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<td>Simulation of container terminals (CT) and ports</td>
<td>Modsim III Object oriented programming (ORP), C++. ARENA. ARENA, SLX. Visual SLAM. AweSim. Witness software. Taylor II. GPSS/H. Extend-version 3.2.2. Scenario generator. ORP, Java.</td>
<td>Gambradella et al.(1998,2001); Yun and Choi (1999); Tahar and Hussain (2000); Merkuryeva et al. (2000); Legato and Mazza (2001); Nam et al. (2002); Demirci (2003); Shabayek and Yeung (2002); Kia et al. (2002); Pachakis and Kiremidjian (2003); Dragović et al. (2005a,2005b,2005c); Sgouridis et al. (2003); Hartmann (2004); Bielli et al. (2005);</td>
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3. Problem description and modeling procedure

Container port system is complex owing to the different interarrival times of ships, different dimensions of ships, multiple quays and berths, different capabilities of QCs and QCs productivity and so on. Therefore, simulation and analytical models are used to describe this system.

The analytical modeling of container terminal consists of setting up mathematical models and equations which describe certain stages in the functioning of the system. Specifically, the probabilistic models are, often, used to describe the evolution of these systems in the process of their modeling. Simulation modeling is better than analytical one in representing random and complex environment of container terminal.

In the simulation and analytical models, customers and servers are designated to container ships and terminal operators, respectively. The basic structure of the models is shown in next Figure.
Port operation with ship movement in port and process flow diagram of the terminal transport operations
3.1 Simulation Model Structure

Most container terminal systems are sufficiently complex to warrant simulation analysis to determine systems performance. The GPSS/H simulation language, specifically designed for the simulation of manufacturing and queueing systems, has been used in this paper (Schriber, 1991).

In order to present the ship-berth link processes as accurate as possible the following phases need to be included into simulation model (Dragović et al. 2005a,b; 2006a,b):

**Model structure:** Ship-berth-yard link is complex due to different interarrival times of ships, different dimensions of ships, multiple quays and berths, different capabilities of QCs and so on. The modeling of these systems must be divided into several segments, each of which has its own specific input parameters. These segments are closely connected with the stages of ship service.
Data collection: All input values of parameters within each segment are based on data collected in the context of this research. The main input data consists of ship interarrival times, lifts per ship, number of allocated QCs per ship call, and QC productivity. Existing input data are subsequently aggregated and analyzed so that an accurate simulation algorithm is created in order to evaluate ship-berth-yard link parameters.

Inter-arrival times of ships: The inter-arrival time distribution is a basic input parameter that has to be assumed or inferred from observed data. The most commonly assumed distributions in literature are the exponential distribution (Demirci 2003; Pachakis and Kiremidjian 2003; Dragović et al. 2006a,b); the negative exponential distribution (Shabayek and Yeung 2002) or the Weibull distribution (Tahar and Hussain 2000; Dragović et al. 2005a,b).

Loading and unloading stage: Accurate representation of number of lifts per ship call is one of the basic tasks of ship-berth link modeling procedure. It means that, in accordance with the division of ships in different classes, the distribution corresponding to those classes has to be determined.
**Flowchart:** Upon arrival, a ship needs to be assigned a berth along the quay. The objective of berth allocation is to assign the ship to an optimum position, while minimizing costs, such as berth resources (Frankel 1987). After the input parameter is read, simulation starts by generating ship arrivals according to the stipulated distribution. Next, the ship size is determined from an empirical distribution. Then, the priority of the ship is assigned depending on its size. The ship size is important for making the ship service priority strategies. For the assumed number of lifts per ship to be processed, the number of QCs to be requested is chosen from empirical distribution. If there is no ship in the queue, the available berths are allocated to each arriving ship. In other cases ships are put in queue. The first come first served principle is employed for the ships without priority and ships from the same class with priority. After berthing, a ship is assigned the requested number of QCs. In case all QCs are busy, the ship is put in queue for QCs. Finally, after completion of the loading and unloading process, the ship leaves the port. This procedure is presented in the algorithm shown in next Figure.
Figure 3. Flowchart for a ship arrival/departure
LOGIC OF ALGORITHM FOR SIMULATION MODEL

Berths are not available! Wait in queue!
Compare priorities
Berth 4 available!!!
Berths are not available! Wait in queue!

Cranes are available!!!

Service completed
Wait for crane!
Service completed
Service completed
Service completed

Berth 1  Berth 2  Berth 3  Berth 4
**Number of QCs per ship:** The data available on the use of QCs in ship-berth link operations have to be considered too, as this is another significant issue in the service of ships. This is especially important as total ship service time depends not only on the number of lifts but also on the number of QCs allocated per ship. Different rules and relationships can be used in order to determine adequate number of QCs per ship. On the other hand, in simulation models, it is enough to determine the probability distribution of various numbers of QCs assigned per ship.

In order to calculate the ship-berth-yard performance, it is essential to have a through understanding of the most important elements in a port system including ship berthing/unberthing, crane allocation per ship, yard tractor allocation to a container and crane allocation in stacking area. As described in next Figure - process flow diagram of the terminal transport operations, the scope of simulation, strategy and initial value and performance measure will have to be defined. In addition, the operational aspect such as machine failures having a direct impact on ship, crane and vehicle will have to be considered. To move containers from apron to stacking area, four tractors are provided for each container crane.
Flowchart of the terminal transport operations

YT loaded/unloaded by TC → YT go to apron area → YT arrive at QC buffer

TC available:

- Yes → YT go to TC
- No → Wait for TC

QC buffer full:

- Yes → Wait for QC
- No → YT go to QC

TC buffer full:

- Yes → Wait in TC buffer
- No → YT arrive at TC buffer

Wait for QC:

- Yes → YT loaded/unloaded by QC
- No → YT go to container yard
3.2. Analytical Model Structure

Queueing theory (QT) models for analyzing movements of ships in port is proposed and shown in previous Figs.

In the analysis of various aspects of average time that ships spend in port, $t_{ws}$, including $n_s$, $n_b$, $\lambda$, $\mu$, $n_c$ and $\rho$, (e.g., [Plumlee (1966), Nicolaou (1967, 1969), Wanhill (1974), Noritake (1985), Noritake and Kimura (1983, 1990), Shabauek and Yeung (2001), Taniguchi et al. (1999)) defined $t_{ws}$ as the sum of the average waiting time and average service time.

The average service time, $t_s = 1/\mu$ where $\mu = \left( t_c + t_{du} \right)^{-1}$

includes ships loading/unloading time in hours per containership, $t_c$, expressed as

$$
t_c = \left( n_{con} \cdot r_{con} \right) / \left( n_c \right)^{k_c} \tag{1}
$$

$$
k_c = \left( \ln(n_{con} \cdot r_{con}) - \ln(t_c) \right) / \ln(n_c) \tag{2}
$$

It follows that

$$
n_c = \left( \frac{\lambda n_{con} r_{con}}{\theta - \lambda t_{du}} \right)^{1/k_c} \tag{3}
$$
Further, it can be shown that

\[ t_{ws} = t_w + t_s \] (4)

where

\[ t_w(\theta) = \frac{\theta^{n_b}}{(n_b - 1)! \mu (n_b - \theta)^2 \left( \sum_{n_s=0}^{n_b-1} \frac{\theta^{n_s}}{n_s!} \right) + \theta^n (n_b - \theta) \mu} \] (5)

for the \((M/M/n_b)\) model.

In this study, formulae due to Lee and Longton (1959) and Cosmetatos (1975, 1976) have been adapted concerning the average port waiting time of ships (Noritake 1985, Radmilović 1992). Accordingly with it, when the ships service time has an Erlang distribution with \(k\) phases, the following equations are obtained

\[ t_{ws} = t_w V_c + t_s \] (6)

\[ V_c = \frac{1}{2} \left( \frac{1}{k} + 1 \right) \] the coefficient of variation of ships service time distribution and

\[ k = \text{the number of phases of an Erlang distribution} \]

\[ t_{ws} = t_w \left[ \frac{1}{2} \left( \frac{1}{k} + 1 \right) + \left( 1 - \frac{1}{k} \right) \left( 1 - \frac{\theta}{n_b} \right) (n_b - 1) \left( \frac{4 + 5n_b}{32\theta} \right)^{\frac{1}{2}} - 2 \right] + \frac{1}{\mu} \] (7)

for the \((M/Ek/nb)\) model.
3.3. Ship traffic

A queueing theory model for analyzing traffic of $N \geq 1$ ships in port is proposed at interval $(0, T)$. Suppose that interarrival times of ships are random variables determined by the distribution function $A(t)$ with $A(+0) < 1$, and the expectation value $\alpha = \int_0^\infty t dA(t) < +\infty$. The function $A(t)$ is used below to determine the probability distribution for the number $p_i(t), i = 0, 1, ..., N$ of ships that arrive in port at an interval $(0, t)$, so that $\sum_{i=0}^N p_i(t) = 1$.

Such introduced random flow, determined for $0 \leq t < +\infty$ is called limited recurrent flow. The probability $p_i(t)$ can be determined (Cox and Smith, 1961) as

$$p_i(t) = A^{(i)}_*(-t) - A^{(i+1)}_*(-t), \quad i = 0, 1, ..., N - 1;$$

$$p_N(t) = 1 - \sum_{i=0}^{N-1} \left( A^{(i)}_*(-t) - A^{(i+1)}_*(-t) \right) = A^{(N)}_*(-t),$$

(8)
where $A_\ast^{(i)}(t)$ is a $i$'th convolution of the function $A(t)$ with itself; $A_\ast^{(0)}(t) \equiv 1$.

Suppose that ships arrive according to a limited Poisson process, so that

$$A(t) = \begin{cases} 1 - e^{-\lambda t} & \text{for} \quad t \geq 0 \\ 0 & \text{for} \quad t < 0 \end{cases}$$

(9)

Then (8) (Cox and Smith, 1961) implies

$$p_i(t) = \frac{(\lambda t)^i}{i!} e^{-\lambda t}, \quad 0 \leq i < N$$

(10)

$$p_N(t) = 1 - \sum_{i=0}^{N-1} \frac{(\lambda t)^i}{i!} e^{-\lambda t}$$

(11)

It is well known that a flow composed from finite number $m > 1$ of simple mutually independent flows with rates $\lambda_1, \ldots, \lambda_m$, is also a simple flow with rate $\lambda_1 + \ldots + \lambda_m$. The same property is also true for the limited Poisson process described above.
Suppose that on the interval \((0, T)\) is planed arrival of \(N_1 + N_2 + N_3\) ships, where the number \(N_i\) related to \(i^{th}\) flow, \(1 \leq i \leq 3\), and \(N_1 \leq N_2 \leq N_3\). Next suppose that mean interarrival time of ships in \(i^{th}\) flow, \(1 \leq i \leq 3\), is \(\lambda_i^{-1}\). Denote by \(p_n(t)\), \(0 \leq n \leq N_1 + N_2 + N_3\), the probability \(p_n(t)\) at the interval \((0,t)\) arrive \(n\) ships from described flow. Then since the above three flows are mutually independent, we have

\[
p_n(t) = \sum_{\sum_{i=1}^{3}n_i = n} p_{n_i}^{(1)}(t)p_{n_2}^{(2)}(t)p_{n_3}^{(3)}(t),
\]

(12)

where the probabilities \(p_i^{(i)}(t)\), \(1 \leq i \leq 3\), can be computed by the Eqs. (10) and (11).

by changing \(\lambda\) with \(\lambda_i\) and \(N\) with \(N_i\).

In order to determine the sum on the right hand side of the Eq. (12), we consider the following cases:

**Case 1**, \(n_i \neq N_i\) for all \(1 \leq i \leq 3\). Then by Eq. (10) the corresponding term in the sum given by Eq. (12) is
\[
\prod_{j=1}^{3} \left( \frac{\lambda_j t}{n_j!} \right)^{n_j} e^{-\lambda_j t} = \left( \prod_{j=1}^{3} \frac{\lambda_j}{n_j!} \right)^{t} e^{-\sum_{j=1}^{3} \lambda_j t} 
\]  

\text{(13)}

Case 2, \( n_i = N_i \) for exactly one \( i \), \( 1 \leq i \leq 3 \). Then for such an \( i \) by Eqs. (10) and (11) we have the corresponding term in the sum expressed by Eq. (12) is

\[
\left( 1 - \sum_{k=0}^{N_i-1} \frac{(\lambda_i t)^k}{k!} e^{-\lambda_i t} \right) \prod_{j \neq i}^{3} \frac{\lambda_j^{n_j}}{n_j!} e^{-\lambda_j t} 
\]

\[
= \left( 1 - \sum_{k=0}^{N_i-1} \frac{(\lambda_i t)^k}{k!} e^{-\lambda_i t} \right) \left( \prod_{j=1}^{3} \frac{\lambda_j^{n_j}}{n_j!} \right) t^{n_{j=1}^{3} n_j - n_i} e^{-\sum_{j=1}^{3} \lambda_j t - \lambda_i t} 
\]

\text{(14)}

Because of \( \sum_{j=1}^{3} n_j - n_i = n - n_i = N_i \), the above sum is

\[
\left( 1 - \sum_{k=0}^{N_i-1} \frac{(\lambda_i t)^k}{k!} e^{-\lambda_i t} \right) \left( \prod_{j=1}^{3} \frac{\lambda_j^{n_j}}{n_j!} \right) t^{n-N_i} e^{-\sum_{j=1}^{3} \lambda_j t - \lambda_i t}. 
\]

\text{(15)}
Case 3, \( n_i = N_i \) and \( n_l = N_l \) for exactly two values \( i \) and \( l \) with \( 1 \leq i < l \leq 3 \). Then for such \( i \) and \( l \) by (10) and (11) we obtain that the corresponding term in the sum given by (12) is

\[
\left(1 - \sum_{k=0}^{N_i-1} \frac{(\lambda_i t)^k}{k!} e^{-\lambda_i t}\right) \left(1 - \sum_{k=0}^{N_l-1} \frac{(\lambda_l t)^k}{k!} e^{-\lambda_l t}\right) \frac{(\lambda_i t)^{n_r}}{n_r!} e^{-\lambda_i t} \tag{16}
\]

where \( 1 \leq i < l \leq 3 \) and \( r \) is the element from \( \{1,2,3\} \) distinct of \( i \) and \( l \).

Case 4, \( n_i = N_i \) for all \( 1 \leq i \leq 3 \). Then \( n = N_1 + N_2 + N_3 \) and by Eq. (11) we obtain that the corresponding term in the sum expressed by Eq. (12) is

\[
\prod_{i=1}^{3} \left(1 - \sum_{k=0}^{N_i-1} \frac{(\lambda_i t)^k}{k!} e^{-\lambda_i t}\right) \tag{17}
\]

Finally, it is obvious that \( p_n(t) \) determined by (12), can be expressed as a sum of three subsums whose general terms are given by (13), (15), (16) and (17), respectively. Recall that in the whole sum given by (12) the second mentioned subsum must be written for any \( i \) with \( 1 \leq i \leq 3 \), and the third mentioned subsum must be written for any pair \( i,l \) with \( 1 \leq i < l \leq 3 \).
### 3.3. Modeling of ship loading/unloading operations

In general, this model integrates main actual operations of the container terminal by simplifying complex activities, and these operations are defined according to ship class. In this section, various objects were observed in the real terminal and model elements. Model elements of the container terminal can be separated into follow group:

- berth cost in $ per hour,  
  \[ c_1 = n_b c_{n_b} \]
- QCs cost in $ per hour,  
  \[ c_2 = n_b n_c c_{n_c} \]
- storage yards cost in $ per hour,  
  \[ c_3 = \theta \mu n_{con} t_{con} a_{cony} c_{cy} \]
- transportation cost by yard transport equipment between quayside and storage yard in $ per hour  
  \[ c_4 = \theta \mu n_{tc} n_{cy} c_t \]
- labor cost for QC gangs in $ per hour,  
  \[ c_5 = \theta \mu n_{t_l} c_l \]
- ships cost in port in $ per hour,  
  \[ c_6 = \theta \mu t_{ws} c_s \]
- containers cost and its contents in $ per hour  
  \[ c_7 = \theta \mu t_{ws} n_{r_con} c_w \]

The total cost function, would be concerned with the combined terminals and containerships cost as  
\[ TC = \sum_{i=1}^{7} c_i \]
It is necessary to know that only the total port cost function computes the number of berths/terminal and QCs/berth that would satisfy the basic premise that the service port cost plus the cost of ships in port should be at a minimum. This function was introduced by Schonfeld and Sharafeldien (1985). We point out that their solutions may not be as good as ours because we have simulation approach to determine key parameters $t_w$, $t_s$, $\lambda$, $\mu$, $\rho$ and especially $k_c$. Therefore, to find the optimal solution, their function can be obtained in the following form

$$
TC = f(\theta) = n_b\left(c_{n_b} + n_c c_{n_c}\right) + \\
+ \theta \mu \left(n_{\text{con}} t_{\text{con}} a_{\text{con}} c_{\text{cy}} + n_c t_l\left(c_l + n_c c_t\right) + t_{\text{ws}}(\theta)\left(c_s + n_{\text{rcon}} c_w\right)\right)
$$

where $TC$ - total port system costs in $$/hour.

By substituting the Eq. (3) into Eq. (18) yields

$$
TC = f(\theta) = n_b c_{n_b} + \lambda n_{\text{con}} t_{\text{con}} a_{\text{con}} c_{\text{cy}} + \left(\frac{\lambda n_{\text{con}} r_{\text{con}}}{\theta - \lambda t_{du}}\right)^{1/k_c} \times \\
\left(n_b c_{n_c} + \lambda t_l\left(c_l + n_c c_t\right)\right) + \lambda t_{\text{ws}}(\theta)\left(c_s + n_{\text{rcon}} c_w\right)
$$

where $t_{\text{ws}}(\theta)$ is defined by the Eq. (6) or the Eq. (7) or or by a result of simulation modeling.
From the total port cost function per average arrival rate, we can obtain

\[ AC = \frac{f(\theta)}{\lambda} = \frac{f(\theta)}{\theta \mu} \]  \hspace{1cm} (20)

Eq. (20) shows the average container ship cost in $/ship, \( AC \). In this study, the trade-off will be simulative and analytically resolved by minimizing the sum of the relevant cost components associated with the number of berths/terminal and QCs/berth, and average arrival rate. These three parameters are key to the analysis of facility utilization and achieving major improvements in container port efficiency, increasing terminal throughput, minimizing terminal traffic congestion and reducing re-handling time. A reduction in operating cost can be achieved by jointly optimizing these parameters. In solving the berths/terminal and QCs/berth, analysts and planners are concerned primarily with the average time that ships spend in port and the average cost per ship serviced.
4. Computational results

This section gives a ship-berth-yard link modeling methodology based on statistical analysis of container ship traffic data obtained from the PECT. The efficiency of operations and processes on the ship-berth link has been analyzed through the basic operating parameters such as traffic intensity, average number of ships in waiting line, average time that ships spend in waiting line, average service time, average total time in port, average QC productivity and average number of QCs per ship.

4.1. Parameters Involved

An important part of the model implementation is the correct choice of the values of the simulation parameters. The input data for the both simulation and analytical models are based on the actual ship arrivals at the PECT for the ten months period from January 1, 2005 to October 31, 2005 (Fig. 3) and January 1, 2006 to October 31, 2006 (Fig. 4), respectively (PECT website, PECT Management reports). This involved approximately 1,225 ship calls in 2005 and 1,285 in 2006. The ship arrival rate was 0.168 ships/hour in 2005 and 0.176 in 2006. Total throughput during the considering period was 1,704,173 TEU in 2005 and 1,703,662 TEU in 2006. Also, the berthing/unberthing time of ships was assumed to be 1 hour.
The ships were categorized into the following three classes according to the number of lifts: under 500 lifts; 501 – 1,000 lifts; and over 1,000 lifts per ship. Ship arrival probabilities were as follows: 23.8% for first class, 40.8% for second and 35.4% for third class of ships in 2005 and 29.9% for first class, 37.7% for second and 32.4% for third class of ships in 2006.

Suppose that on the interval $(0, T)$ is planned the arrival of $N_1 + N_2 + N_3$ ships, where the number $N_i$ related to $i$ th flow (class), $1 \leq i \leq 3$, and $N_1 \leq N_2 \leq N_3$. Next suppose that mean inter-arrival time of ships in $i$ th flow, $1 \leq i \leq 3$, is $\lambda_i^{-1}$. Since numbers of ships in each of three ship classes given in the Table 1, we can assume that $\lambda_1 : \lambda_2 : \lambda_3 = N_1 : N_2 : N_3$. In particular, for $T = 1$ hour we obtain $\lambda_1 = 0.0399$, $\lambda_2 = 0.0685$ and $\lambda_3 = 0.0594$ for 2005. Similarly, by using the same notation for 2006 from Table 1 we have $\lambda_1 = 0.0526$, $\lambda_2 = 0.0663$ and $\lambda_3 = 0.0570$. By using these values from Eqs. (12) and (13), we can compute the probabilities $p_n(t)$ for small values of $n$. The analogous results may be obtained for arbitrary times less than 10 months.
Fig. 3. PECT layout, 2005

Fig. 4. PECT layout, 2006
Table 1. Actual ship arrivals at the PECT in 2005 and 2006

<table>
<thead>
<tr>
<th>Class of ships</th>
<th>Number of ships</th>
<th>Average lifts per ship</th>
<th>Total lifts</th>
</tr>
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<tbody>
<tr>
<td>&lt; 500 lifts</td>
<td>292</td>
<td>384</td>
<td>313</td>
</tr>
<tr>
<td>501 – 1000 lifts</td>
<td>500</td>
<td>485</td>
<td>782</td>
</tr>
<tr>
<td>&gt; 1000 lifts</td>
<td>433</td>
<td>416</td>
<td>1,444</td>
</tr>
<tr>
<td>All classes</td>
<td>1,225</td>
<td>1,285</td>
<td>882</td>
</tr>
</tbody>
</table>
The interarrival time distribution (IATD) is plotted in the Figures 5a and 6a. Interestingly, even though ship arrivals of the ships are scheduled and not random, the distribution of interarrival times fitted very well the exponential distribution.

Service times were calculated by using the Erlang distribution with different phases. To obtain accurate data, we have first fitted the empirical distribution of service times of ships to the appropriate theoretical distribution. Service time distributions are given in Figure 5b-e for 2005 and in Figure 6b-e for 2006.

Goodness-of-fit was evaluated, for all tested data, by both chi-square and Kolmogorov-Smirnov tests at a 5 % significance level.

We have carried out extensive numerical work for high/low values of the PECT model characteristics. Our numerical experiments are based on different parameters of various PECT characteristics such as: number of containers loading/unloading from container ship, the QC move time, hourly berth cost, average yard container dwell time, transportation cost by yard transport equipment between quayside and storage yard, number of m² of storage yard per container, storage yard cost, paid labor time, labor cost, ship cost in port and average payload of containers, presented in next Table.
Fig. 5. (a) IATD of ships at PECT in 2005; (b) Service distribution (SD) of first class of ships, the 4-phase Erlang distribution, (E₄); (c) SD of second class of ships, (E₄); (d) SD of third class of ships, (E₅); (e) SD of all classes of ships, (E₃)
Fig. 6. (a) IATD of ships at PECT in 2006; (b) Service distribution (SD) of first class of ships, ($E_5$); (c) SD of second class of ships, ($E_6$); (d) SD of third class of ships, ($E_2$); (e) SD of all classes of ships, ($E_4$)
Table 2. Input data – Terminal characteristics

<table>
<thead>
<tr>
<th>Class of ships</th>
<th>$n_{con}$ (no. of containers)</th>
<th>$r_{con}$ (hours/ container)</th>
<th>$t_{l}$ (hours/ gang/ship)</th>
<th>$c_s$ ($/ship$ hour)</th>
<th>$n_c^*$</th>
<th>$k_c$</th>
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<tr>
<td>I class</td>
<td>313</td>
<td>0.055</td>
<td>8.87</td>
<td>745</td>
<td>1.76</td>
<td>0.89</td>
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<tr>
<td>II class</td>
<td>782</td>
<td>0.051</td>
<td>13.91</td>
<td>1098</td>
<td>2.55</td>
<td>0.931</td>
</tr>
<tr>
<td>III class</td>
<td>1444</td>
<td>0.038</td>
<td>20.29</td>
<td>1354</td>
<td>3.14</td>
<td>0.979</td>
</tr>
<tr>
<td>All classes</td>
<td>862</td>
<td>0.044</td>
<td>14.35</td>
<td>1164</td>
<td>2.50</td>
<td>0.918</td>
</tr>
</tbody>
</table>

$n_c^*$ - average number of QCs assigned per ship (Real data and Simulation results); $C_{n_{b1}} = 62$ million $\$/; $i = 0.0663$; $n_y - 40$, $C_{n_{lm}} = 6.2$ million $\$/; $C_{n_b} = 1215$ $\$/; $C_{n_c} = 38.8$ $\$/QC hour; $t_{con} = 188$ hours; $a_{con_{cy}} = 63.9$ m$^2$/container; $C_{cy} = 0.000292$ $\$/m$^2$ hour; $n_{cye} = 9$; $C_i = 5$ $\$/cycle; $C_i = 357$ $\$/gang hour; $n_{r_{con}} (601$ for I class, 1085 for II class, 1312 for III class, 999 for all classes in 2005; and 642 for I class, 1114 for II class, 1371 for III class, 1042 for all classes in 2006; $C_w = 1.4$ $\$/container hour. To move containers from apron to stacking area, four tractors are provided for each QC. It takes average 10 minutes from apron to stacking area including unloading/loading time by transfer crane. The average distance between apron and stacking area is assumed to be 850 meters.
4.2. Model Validation

For purposes of validation of simulation model and verification of simulation computer program, the results of simulation model were compared with the actual measurement. Three statistics were used as a comparison between simulation output and real data: traffic intensity, average service time and average number of serviced ships. The simulation model was run for 44 statistically independent replications. The average results were recorded and used in comparisons. After analysis of the port data, it was determined that traffic intensity is about 2.55 in 2005 and 2.25 in 2006, while the simulation output shows the value of 2.61 in 2005 and 2.28 in 2006, respectively. Average service time shows very little difference between the simulation results and actual data, that is, 14.07 h and 14.35 h in 2005 and 12.60 h and 12.88 in 2006, respectively. The simulation results of the number of serviced ships completely correspond with the real data (i.e. the simulation result of the total number of ships are 1224.1 in 2005 and 1285.88 in 2006, and the real data are 1225 and 1285; the first class of ships: 291.11 in 2005 and 383.3 in 2006, and 291 and 384; the second class: 502.16 in 2005 and 486,01 in 2006, and 501 and 485; and third class: 434,17 in 2005 and 415.02 in 2006, and 433 and 416). All the above shows that simulation results are in agreement with real data.
The attained agreement of the results obtained by using simulation model with real parameters has also been used for validation and verification of applied analytical model. In accordance with it, the correspondence between simulation and analytical results gives, in full, the validity to the applied analytical model to be used for the optimization of servicing ships processes at PECT, i.e., at the considered ship-berth link, see Tables 3 and 4.

Table 3. Average service time of ships

<table>
<thead>
<tr>
<th>Results</th>
<th>Average service time of ships in hours</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(All classes)</td>
</tr>
<tr>
<td>Real data</td>
<td>14.07</td>
</tr>
<tr>
<td>Simulation results</td>
<td>14.35</td>
</tr>
<tr>
<td>Analytical results</td>
<td>14.51</td>
</tr>
<tr>
<td>Table 4. Average waiting time of ships</td>
<td>Average waiting time of ships in hours</td>
</tr>
<tr>
<td>--------------------------------------</td>
<td>---------------------------------------</td>
</tr>
<tr>
<td></td>
<td>(All classes)</td>
</tr>
<tr>
<td>Simulation results</td>
<td>2.429</td>
</tr>
<tr>
<td>Analytical results, AM II</td>
<td>2.632</td>
</tr>
</tbody>
</table>

Table 5. Average time that ships spend in port

<table>
<thead>
<tr>
<th>Table 5. Average time that ships spend in port</th>
<th>Average time that ships spend in port in hours</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(All classes)</td>
</tr>
<tr>
<td>Analytical results, AM II</td>
<td>18.142</td>
</tr>
</tbody>
</table>
4.3.1. Simulation and Analytical Results for PECT

The impact of the different models is determined by comparing the key performance measures of simulation and analytical approaches to those of the real data of PECT. Table 3 presents average service time of ships (all classes, I class, II class and III class), while Table 4 shows average average waiting time of ships (all classes, I class, II class and III class). In addition, Table 5 gives average time that ships spend in port. According to this, judging from the computational results for some numerical examples of the ($M/Ek/nb$) – using average waiting time, from Eq. (6) (for brevity analytical model I (AM I)) and ($M/Ek/nb$) – using average waiting time, from Eq. (7) (for brevity analytical model II (AM II)) models, it can be confirmed that Eq. (6) is inclined to estimate the values of average time that container ships spend in port, i.e. average waiting time of ships.

The average time that ships spend in port for simulation model (SM) is 15.036 h for all classes of ships in 2006. This is about 15% shorter than that of SM, 17.799 h in 2005 and about 1.5% shorter than that of AM II, 15.245 h in 2006. For first class of ships, the average time that ships spend in port is 10.380 h for AM II in 2006, about 0.6% shorter than SM, 10.441. This time is 15.232 h for second class of ships (AM II) in 2006, about 12% shorter than AM II in 2005. Finally, the average time that ships spend in port for third class of ships is 19.818 h (SM) in 2006, about 16% shorter than SM in 2005 or 2% shorter than AM II, 20.270 h in 2006.
The results presented here support the argument that average cost per ship or container served, can be easily obtained by the use of the average cost curves in function of traffic intensity and QCs/berth. The described and tested numerical experiments contain more segments in relation to the input variables. All numerical results presented in Figure 7 are obtained by using the input data from Table 2. Simulation testing (Simulation model (SM)) was then carried out by using the GPSS/H. The solution procedure for AM I and AM II models was programmed using the MATLAB program.

Figure 7 compares the average ship costs of different ship classes taken by SM, AM I and AM II models at a PECT in 2005 and 2006. They graphically show the sensitivity of the average ship costs to the various values of $\theta$. In curve SM for all classes of ships in 2006, the minimum cost per ship served decreases by about 3.3% in 2006 with respect to 2005. However, the average costs per first class of ships served decrease in 2006 by about 7% than the minimum cost in 2005, see curve AM II. This decrease for second class of ships is about 2% in 2006 with respect to the minimum cost in 2005 for curve AM II. Finally, in curve SM for third class of ships, the minimum cost per ship served decreases by about 2.6% ($138,019) than the minimum cost in 2005 ($141,697).
5. Conclusions
A simulation model employing the GPSS/H has been developed to ship-berth-yard link performance evaluation of PECT. It is shown to provide good results in predicting the actual ship-berth-yard link operations system of the PECT. The attained agreement of the results obtained by using simulation model with real parameters has been also used for validation and verification of applied analytical model. In accordance with that, the correspondence between simulation and analytical results gives, in full, the validity to the applied analytical model to be used for optimization of processes of servicing ships at PECT. Finally, these models also address the issues such as the performance criteria and the model parameters to propose an operational method that reduces average cost per ship served and increases the terminal efficiency. However, presented simulation and analytical methodology and results are convenient for different analyses, planning and development of port system, for example, increasing the number of berths or traffic intensity depending on the optimum berth capacity and average ship cost.
Container terminals in Busan Port, especially PECT, are trying to expand capacity and increase performance at a maximum of investments. Often the container terminal operations are changing to meet increased customer demands as well as to adapt to new technologies. Reasons for the decrease of the average cost per ship served with the introduction of new container berth, QCs, container yard area and automated stacking cranes (ASC) include that waiting time of ships and the average time that ships spend in port decrease with the advanced handling systems improving the operations procedures.

We develop simulation and analytical models, which provides solutions to large-sized problems usually encountered in practice in reasonable computational times, and analyze its effectiveness. In addition, computational analysis shows that these models are quite effective in realistic settings, when the OCs are assigned closer related to the apron area, and when the number of containers to be unloaded from/loaded onto each ship is in the hundreds. Thus, these models can be used to obtain a good solution to the real problem.
--- THANK YOU ---