Appendix A: Screen captures of module in Blackboard

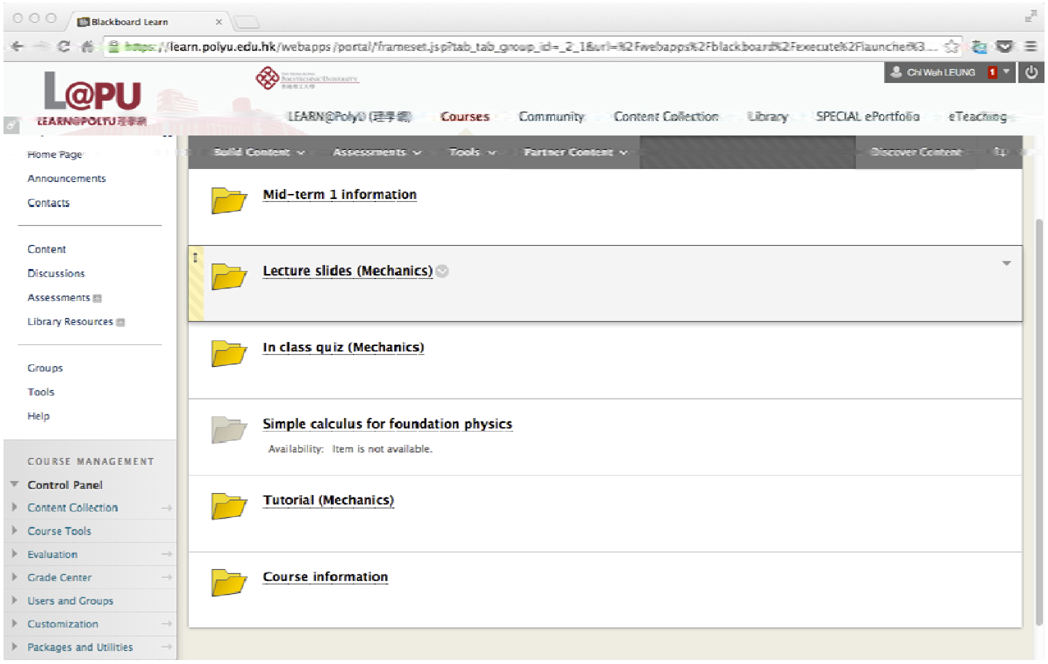


Fig. A1 Snapshot of calculus module in the course page of AP10005 (Physics i) in Blackboard.

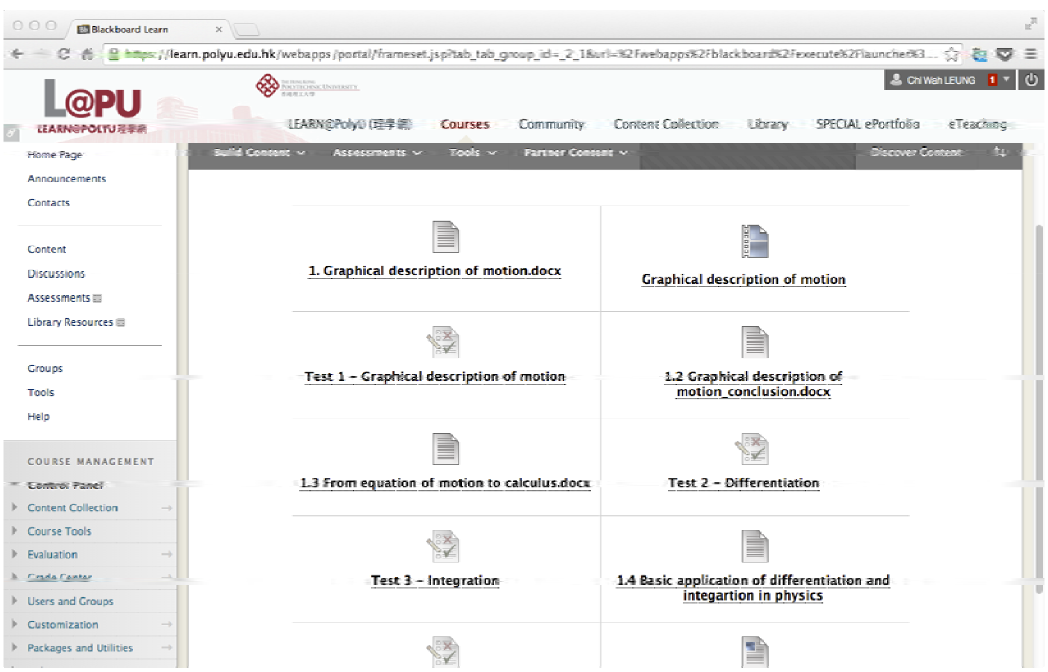


Fig. A2 Snapshot of the contents page of the calculus module, with notes, videos and self-assessment tests.

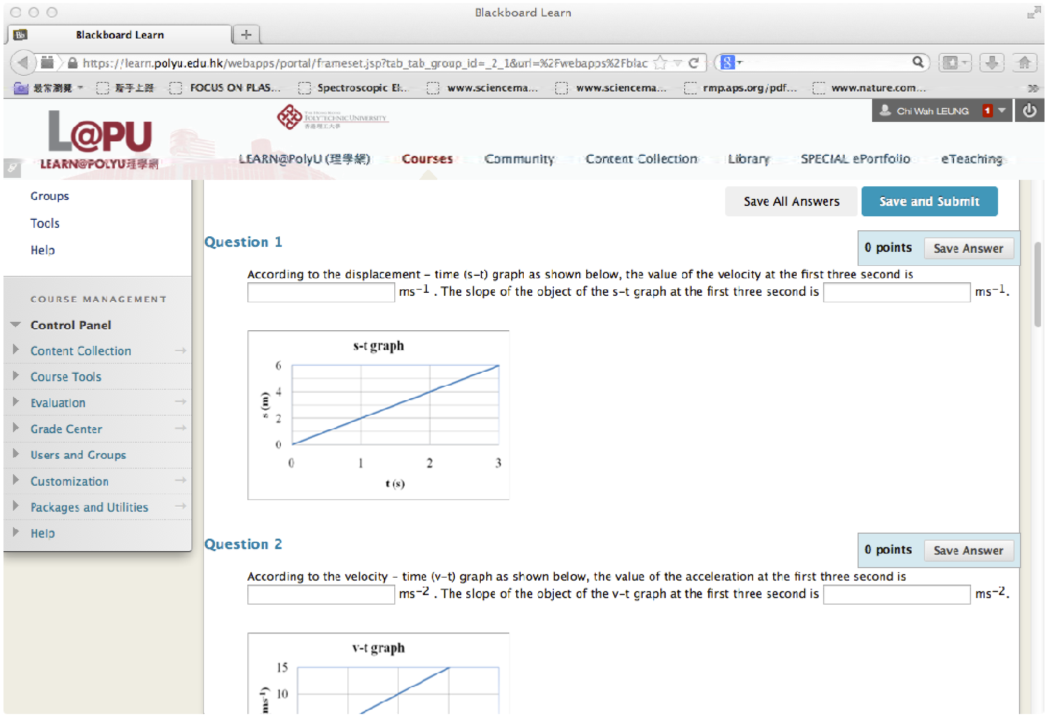


Fig. A3 Snapshot of one of the progress tests.

1.2 From graphical description of motion to fundamental calculus

As mention in the previous section, the slope and area of s-t, v-t and a-t graphs are important physical quantities for interpretation of motion.

	Slope	Area
s-t graph	v	
v-t graph	a	s
a-t graph		v

Table.1.2. Relationship of slope and area in s-t, v-t and a-t graphs.

It also mentions that if the curves of those graphs are not linear, a new mathematical tool is needed to figure out the values of the slope and area.

In this section, we will go through 6 different rules in calculus. 3 of them are used to find the value of slope (differentiation) and 3 of them are used to find the value of area under curve (integration).

Differentiation:

a) $\frac{d}{dx}[k]=0$

b) $\frac{d}{dx}[kf(x)]=k\frac{d}{dx}[f(x)]$

c) $\frac{d}{dx}[x^n]=nx^{n-1}$

Integration:

d) $\int k dx = kx + C$

e) $\int k f(x) dx = k\int f(x) dx$

f) $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

where *k* and *C* are constants, *n* is an integer. (Detailed explanation and the derivation of the above rules can be found in the powerpoint which will be adaptive released after you finishing all the tests in this short course.)

Before going into details of the rules, let's introduce the mathematic notation of both methods. If we want to find the slope of a function *f(x)*, a symbol " $\frac{d}{dx}f(x)$ " would be used which means “to find the slope of *f(x)*” (to use a more precise word, $\frac{d}{dx}f(x)$ is a derivative of *f(x)*.) Similarly, if we want to find the area of a function *f(x)*, a symbol ”$\int f(x)dx$” would be used which means “to find the area of *f(x)*”. (to use a more precise word, $\int f(x)dx$ is an integral of *f(x)*.)

For instance, for a particle moving along a straight line, its velocity *v* and acceleration *a* at time *t* can be expressed as:

$$v = \frac{ds}{dt}$$

It means “the velocity *v* is equal to the slope of the displacement *s*.”

$$a = \frac{dv}{dt}$$

It means “the velocity *a* is equal to the slope of the velocity.”

Similarly:

$$s = \int vdt$$

It means “the displacement *s* is equal to the area of the velocity *v*.”

$$v = \int adt.$$

It means “the velocity *v* is equal to the area of the acceleration *a*.”

1.2.1 Examples of applying the fundamental rules of calculus

Ex 1. If *f(x) = 2* , find the derivative of *f(x).*

By rule (a), $\frac{d}{dx}[k]=0$

$$\frac{d}{dx}f(x)=\frac{d}{dx}[2]=0$$

Ex 2. If *f(x)* = x^5 , find the derivative of *f(x).*

By rule (c), $\frac{d}{dx}[x^n]=nx^{n-1}$

$$\frac{d}{dx}f(x)=\frac{d}{dx}x^5=5x^{5-1}=5x^4$$

Ex.3. If *f(x)* = x^2, find the derivative of *f(x).*

By rule (c), $\frac{d}{dx}[x^n]=nx^{n-1}$

$$\frac{d}{dx}f(x)=2x^{2-1}=2x$$

Ex.4. If *f(x)* = $5x^5$, find the derivative of f(x).

By rule (c) and (d), $\frac{d}{dx}[x^n]=nx^{n-1}$, $\frac{d}{dx}[kf(x)]=k\frac{d}{dx}[f(x)]$

$$\frac{d}{dx}f(x)=\frac{d}{dx}5x^5=5\frac{d}{dx}x^5=5x^{5-1}=5x^4$$

Ex 5. If *f(x) = 2* , find the integral of *f(x).*

By rule (d), $\int kdx=kx+C$

$$\int 2dx=2x+C$$

Ex 6. If *f(x)* = x^2 , find the integral of *f(x).*

By rule (f), $\int x^n dx=\frac{x^{n+1}}{n+1}+C$

$$\int x^2 dx=\frac{x^{2+1}}{2+1}+C=\frac{x^3}{3}+C$$

Ex 7. If $f(x) = x^5$, find the integral of $f(x)$.

By rule (f), $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$$\int x^5 dx = \frac{x^{5+1}}{5+1} + C = \frac{x^6}{6} + C$$

Ex 8. If $f(x) = 3x^5$, find the integral of $f(x)$.
By rule (e) and (f)

$$\int 3x^5 dx = 3\int x^5 dx = 3\frac{x^{5+1}}{5+1} + C = 3\frac{x^6}{6} + C = \frac{x^6}{2} + C$$

Ex 9. If $s(t) = ut + \frac{1}{2}at^2$ where u and a are constant, find the derivative of $s(t)$.

$$\frac{d}{dt}s(t) = \frac{d}{dt}ut + \frac{d}{dt}\frac{1}{2}at^2$$

$$= u\frac{d}{dt}t + \frac{1}{2}a\frac{d}{dt}t^2 \qquad \text{By rule (b)}$$

$$= u + \frac{1}{2}a(2t) \qquad \text{By rule (c)}$$

$$= u + at$$

Ex 10. If $v(t) = u + at$ where u and a are constant, find the derivative of $v(t)$.

$$\frac{d}{dt}v(t) = \frac{d}{dt}u + \frac{d}{dt}at \qquad \text{By rule (a) and rule (c)}$$

$$= 0 + a$$

$$= a$$

Ex 11. If $v(t) = u + at$ where u and a are constant, find the integral of $v(t)$.

$$\int v(t)dt = ut + \frac{1}{2}at^2 + C$$

Ex 12. If $a(t) = a$ where a is constant, find the integral of $a(t)$.

$$\int a(t)dt = at + C$$

Ex. 9 to Ex. 12 illustrate the relationship between *s(t), v(t)* and *a(t)* by the use of differentiation and integration when the acceleration *a(t)* is a constant. The equations obtained are called equation of motion. Appendix I show another method to derive the equation of motion. It is noted that there is a constant term C in the example Ex. 11 and Ex. 12. Next section will discuss the method of finding this constant C.

Appendix I:

Equations of motion

Derivation of equation of motion from v-t graph: (Under constant acceleration)

Uniform motion

Uniform motion is a motion which travels with constant velocity.

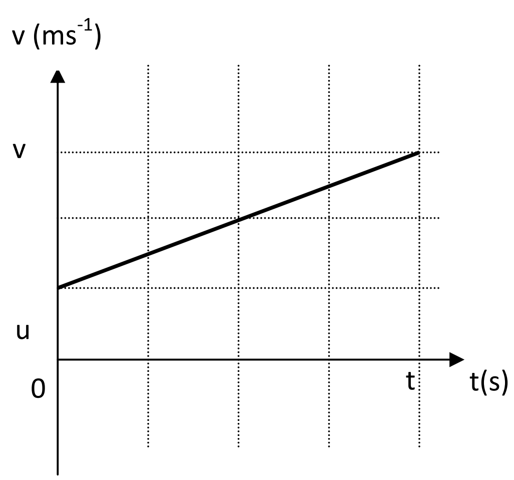
(i.e. Constant speed and moving in a straight line with the same direction)

$$s = vt$$

Uniform acceleration

Uniform acceleration is a motion which traval with constant acceleration.

(i.e. moving in a straight line with the same direction)



Study the v-t graph.

The slope of the straight line is $\frac{v-u}{t}$.

➔The acceleration of the straight line equal to:

$$a = \frac{v-u}{t} .$$

$$\therefore v = u + at \qquad (1)$$

Displacement s = area under the graph

$$\therefore s = \frac{(u+v)t}{2} \qquad (2)$$

Combine (1) and (2) together:

$$s = \frac{(u+u+at)t}{2}$$

$$\therefore s = ut + \frac{1}{2}at^2 \qquad (3)$$

By taking square of equation (1)

$$\therefore v^2 = u^2 + 2as \qquad (4)$$

Summary :

$$\begin{cases} v = u + at \\ s = ut + \frac{1}{2}at^2 \\ v^2 = u^{2+}2as \\ s = \frac{u+v}{2}t \end{cases}$$

Each of the above equations has four unknowns.

➔If we know three of them

➔The rest unknown can be found.

	u	v	a	s	t
$v = u + at$	✓	✓	✓	X	✓
$s = ut + \frac{1}{2}at^2$	✓	X	✓	✓	✓
$v^2 = u^{2+}2as$	✓	✓	✓	✓	X
$s = \frac{u+v}{2}t$ (optional)	✓	✓	X	✓	✓