Sampling Distribution of Difference of Proportions

$$\mu_{\hat{p}_1 - \hat{p}_2} = E \begin{bmatrix} \hat{p}_1 \end{bmatrix} - E \begin{bmatrix} \hat{p}_2 \end{bmatrix}$$

$$= \mu_{\hat{p}_1} - \mu_{\hat{p}_2}$$

$$= p_1 - p_2$$

$$\begin{split} \sigma_{\hat{p}_1 - \hat{p}_2}^2 &= Var \Big(\hat{p}_1 - \hat{p}_2 \Big) \\ &= Var \Big(\hat{p}_1 \Big) + Var \Big(\hat{p}_2 \Big) \qquad \qquad \because \hat{p}_1 \text{ and } \hat{p}_2 \text{ are independent} \\ &= \sigma_{\hat{p}_1}^2 + \sigma_{\hat{p}_2}^2 \\ &= \frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2} \end{split}$$