Group sparse optimization via $\ell_{p,q}$ regularization

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Abstract In this paper, we investigate a group sparse optimization problem via $\ell_{p,q}$ regularization in three aspects: theory, algorithm and application. In the theoretical aspect, by introducing a notion of group restricted eigenvalue condition, we establish an oracle property and a global recovery bound of order $O(\lambda^{\frac{2}{2-q}})$ for any point in a level set of the $\ell_{p,q}$ regularization problem, and by virtue of modern variational analysis techniques, we also provide a local analysis of recovery bound of order $O(\lambda^2)$ for a path of local minima. In the algorithmic aspect, we apply the well-known proximal gradient method to solve the $\ell_{p,q}$ regularization problems, either by analytically solving some specific $\ell_{p,q}$ regularization subproblems, or by using the Newton method to solve general $\ell_{p,q}$ regularization subproblems. In particular, we establish a local linear convergence rate of the proximal gradient method for solving the $\ell_{1,q}$ regularization problem under some mild conditions and by first proving a second-order growth condition. As a consequence, the local linear convergence rate of proximal gradient method for solving the usual ℓ_q regularization problem ($0 < q < 1$) is obtained. Finally in the aspect of application, we present some numerical results on both the simulated data and the real data in gene transcriptional regulation.

Key words Group sparse optimization, lower-order regularization, nonconvex optimization, restricted eigenvalue condition, proximal gradient method, iterative thresholding algorithm, gene regulation network.

Reference:

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