Title: Mixed Contingent Coderivative and Relative Lipschitz-like Property in Banach Spaces

Infinite dimensional optimization problems have found many applications in engineering, dynamic user equilibrium problem, finance and stochastic programming. Stability analysis is to establish verifiable optimality conditions to guarantee that the accuracy of the solutions obtained increases with the degree of approximation of the initial data. The Lipschitz-like property is at the core of stability theory, which is closely related to metric regularity, the latter is often used in convergence analysis of numerical optimization algorithms. Various generalized differential theories and solution characterizations for the Lipschitz-like property of set-valued mappings have been developed in finite dimensional spaces. The study in infinite dimensional spaces is very limited. By virtue of a mixed coderivative, a complete characterization of the Lipschitz-like property for a set-valued mapping has been obtained in Asplund spaces under a partially sequentially normally compact condition. However this characterization requires that the reference point lies in the interior of the domain.

In this project we will introduce a mixed contingent coderivative for set-valued mappings in Banach spaces by virtue of an epsilon-normal cone and a duality mapping and apply it to establish a complete characterization of the relative Lipschitz-like property under a relative partially sequentially normally compact condition by using Ekeland variational principle. We will investigate equivalences between the relative metric regularity of a set-valued mapping and the relative Lipschitz-like property of its inverse. We will derive some chain rules for the compositions of two set-valued mappings and sum rules for the sum of the two set-valued mappings under inner semicontinuity, inner semicompactness and some constraint qualification conditions and apply the sum rule to study the relative Lipschitz-like property of the solution mapping of parametric systems. We will derive a generalized critical face condition for the relative Lipschitz-like property of the solution mapping of an affine variational inequality problem in a Banach space and show that it can be verified via its finite-dimensional counterpart. Our study does not require that the reference point lies in the interior of the domain as we are motivated by the fact that relative stability properties have been illustrated in extensive practical applications in infinite dimensional spaces.