Title: The Lipschitz-like Property Relative to a Set with Applications

Abstract: Stability analysis of set-valued mappings is to determine intuitively verifiable conditions to guarantee that the accuracy of the solutions obtained increases with the degree of approximation of the initial data. The literature on the subject is vast when the study is of global nature. Lipschitz-like property is one of the properties at the center of stability theory. By virtue of coderivative, Mordukhovich criterion provides a necessary and sufficient condition of the Lipschitz-like property and has widely been applied in the study of solution mappings of various linear and nonlinear systems. One important assumption of the Mordukhovich criterion is that the candidate point is in the interior of the domain of set-valued mappings. This is a very restrictive assumption. On one hand, the candidate point under consideration may be at the boundary of the domain of setvalued mappings. On the other hand, in order to guarantee some stationarity conditions one may only need a regular behavior of the constraint systems with respect to one single critical direction, not on the whole space. Thus there is a great demand to study characterizations of Lipschitz-like property relative to a set.

This project will investigate characterizations of the Lipschitz-like property relative to a set for set-valued mappings and their applications. We will employ regular normal cone and limiting normal cone of a restricted graph of the set-valued mapping to obtain some neighborhood necessary condition and sufficient condition respectively for set-valued mappings to have the Lipschitz-like property relative to a closed set. We will introduce a projectional coderivative of set-valued mappings and apply it to establish a verifiable generalized Mordukhovich criterion for the Lipschitz-like property relative to a closed and convex set. We will study the representation of the graphical modulus of a setvalued mapping relative to a closed and convex set by using the outer norm of the corresponding projectional coderivative value. We will apply the obtained results to investigate the Lipschitz-like property relative to a set for a level-set mapping and the stationary point set of an l_1-norm convex optimization problem.