Title: Variational Analysis of Piecewise Linear Vector Optimization

Abstract: Linear and continuous piecewise linear vector optimization have been extensively studied and applied to various decision-making problems in economics, management science and engineering. The construction theory of solution sets of vector optimization problem in the case of finite dimension spaces with the polyhedron ordering cone has played a key role in the development of its theory and applications. It is well recognized that continuous piecewise linearity is still very restrictive in both theory and application. Practical problems of piecewise linear functions include transaction costs in finance and sparse nature of the decision variables in statistics and compress sensing. Here by a piecewise linear function we mean a not necessarily continuous one. It is worth noting that the study of piecewise linear, in particular vector, optimization problems is very limited and immature. Therefore there is a need to establish a unified and in-depth theory for piecewise linear vector optimization problems.

In this project, we will study the fully piecewise linear vector optimization problem, that is, both objective and constraint functions can be piecewise linear. We will first explore the solution structures of the piecewise linear vector optimization problem in general Banach spaces and apply them to develop weak sharp minima for linear vector optimization problems and to explore the corresponding finite terminate rule for some piecewise linear vector optimization algorithms. On the other hand, in establishing mathematical models of practical problems, one often adopts piecewise linear functions as objectives and constraint functions, and these piecewise linear functions are determined by only finitely many available test data. However all test data obtained are not perfectly ideal. In general, the gap between the ideal data with the ``test'' data is unavoidable. In view of this, we will consider some stability issues when the objective and/or constraint functions undergo small perturbations.