Title: Linearized proximal algorithms with Bregman distance for convex composite optimization with applications

Abstract: The proximal point algorithm has been very popular in solving structured optimization problems for more than half a century. One of the main reasons is that they generate subproblems that always have a unique solution, and are able to avoid the ill conditioning in numerical implementation. A well-known drawback of the proximal point algorithms is that the computational cost of solving each subproblem is as high as solving the original problem. We will employ a linearization technique and a Bregman distance to overcome this drawback, such that the corresponding subproblem is convex, irrespective of the original problem being convex or not, and remains to have a unique solution which lies in the interior of the feasible set so that the constraints are thus eliminated automatically. In recent years, there has been an extensive study on the global convergence of proximal point algorithms for convex constrained optimization problems. However, there is a lack of research on the convergence rates of proximaltype algorithms with Bregman distance for both convex and nonconvex constrained optimization problems. It is well known that the establishment of convergence rates is important in guaranteeing the numerical performance of relevant algorithms.

In this project, we will develop a linearized proximal theory with Bergman distance for solving convex composite optimization, including local superlinear convergence rates in terms of both functional values and iterates, under the assumptions of local weak sharp minima of order p and a quasi-regularity condition. We will propose a globalization strategy for the linearized proximal algorithm with Bregman distance based on the Armijo-line search and establish global superlinear convergence results. We will apply the designed linearized proximal algorithms with Bergman distance to a feasibility problem and a sensor network localization problem. The project is of a long-term significant value, with an implication of further investigation on effective solution algorithms for constrained optimization problems and in the training of Operations Researchers with strong mathematical skills.