

Title: Nonlinear Constrained Sparse Optimization with Lower Order Regularization

Abstract: The sparse solutions of under-determined nonlinear measurements arise in the sparse inverse covariance selection while maximizing the log-likelihood, quadratic compressive sensing in diffraction imaging and sub-wavelength imaging, and nonlinear base pursuit. Nearly, all of the optimization theory and algorithms developed for compressed sensing signal recovery assume that samples are taken using linear measurements and without constraints. Thus it is of great demand to design optimal algorithms to solve constrained sparse optimization problems with nonlinear measurements.

In this project, we will address compressed sensing recovery problems in a setting where the observations are nonlinear measurements and there are constraints and develop optimization theory and algorithms by virtue of constrained lower order regularization problems for solving them. We will introduce the extended restricted isometry property and extended restricted eigenvalue condition, and apply them to establish the recovery bound for the lower order regularization problem of the constrained sparse optimization with nonlinear measurements. We will propose a projected proximal gradient method to solve the constrained lower order regularization problem, and establish the convergence theory by virtue of the projection operator and the Kurdyka-Lojasewicz theory. As optimization problems are of a nonconvex structure, it is difficult to establish that our algorithms converge to a global solution. However, we will aim to design algorithms to find a good enough sparse solution and compare their performance with some existing optimization algorithms in terms of computing time, accuracy and successful recovery rate.

We will also apply our theory and algorithms to solve (group) sparse constrained optimization problems arising in portfolio selection and gene regulatory network. The practical meaning of the sparsity requirement in the portfolio is that an investor aims to invest in a small number of assets in order to avoid the high cost of transaction fees. The group sparsity structure arises in prediction, dynamic MRI, and gene finding. We will show that exploiting the group sparsity structure can reduce the degrees of freedom in the solution, thereby lead to better recovery performance.