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AMA1D01C History of Ancient Chinese and World Mathematics

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Extra Notes to Lecture Notes Set #3

The Chinese New Year started in 05 Feb 2019 was 己亥年. Using the method discussed in Lecture Notes Set #3 from《孫子算經》物不知數, find the number of years from 甲子 (first year of the sexagenary cycle) to 己亥:

10 天干: 甲乙丙丁戊<u>己</u>庚辛壬癸

12 地支: 子丑寅卯辰巳午未申酉戌<u>亥</u>

Since 己 is the 6th of 10 天干, and 亥 is the 12th of 12 地支, thus, we have the following congruence relations:

$x \equiv 6 \pmod{10}$
$x \equiv 12 \pmod{12}$
Note that 12-6=6 is divisible by GCD(10,12), thus solvable

However, 10 and 12 are not coprime, so we take 5 and 12 instead. The LCM of 5 and 12 is still 60. So, we have

$x \equiv 6 \pmod{5}$
$x \equiv 12 \pmod{12}$

Now, find positive integers α and β so that

$12\times\alpha \equiv 1 \pmod{5}$
$5\times\beta \equiv 1 \pmod{12}$

Since 12×3 = 36 = 7×5+1, and 5×5 = 25 = 2×12+1, therefore α=3, and β=5.

Hence, $x = 12\times3\times6 + 5\times5\times12 = 516 = 60\times8+36 \equiv 36 \pmod{60}$.

Therefore, 己亥年 is the 36th year counting from 甲子年.

We have 10 天干, and 12 地支, but we do not have all 10×12=120 combinations of years. A 甲子 cycle only has 60 years, and some combination is not possible.

	甲	乙	丙	丁	戊	己	庚	辛	壬	癸
子	1		13		25		37		49	
丑		2		14		26		38		50
寅	51		3		15		27		39	
卯		52		4		16		28		40
辰	41		53		5		17		29	
巳		42		54		6		18		30
午	31		43		55		7		19	
未		32		44		56		8		20
申	21		33		45		57		9	
酉		22		34		46		58		10
戌	11		23		35		47		59	
亥		12		24		36		48		60

For example, the year 甲亥 is not possible.
甲 is the 1^{st} of 10 天干, and 亥 is the 12^{th} of 12 地支, thus, we have the following congruence relations:

$x \equiv 1 \pmod{10}$ this implies x is odd
$x \equiv 12 \pmod{12}$ this implies x is even
Note that 12-1=11 is ***<u>not</u>*** *divisible by GCD(10,12), no solution.*

If we take modulos 5 and 12 as before, we can still have the pair

$x \equiv 1 \pmod{5} \equiv 6 \pmod{5}$
$x \equiv 12 \pmod{12}$

The answer to this pair is 36, but of course, 甲亥 does not exist.

Conversion from western year to sexagenary cycle is relatively simplier. Take the year number and minus 3. Then take mod 10 and mod 12 to get the Heavenly Stems 天干 and Earthly Branches 地支.

For example, take the year 2019.
2019 – 3 = 2016. So,
2016 ≡ 6 (mod 10), therefore 天干 is 己, and
2016 ≡ 12 (mod 12), therefore 地支 is 亥.