

The Hong Kong Polytechnic University

Department of Applied Mathematics

AMA1007 Calculus and Linear Algebra

Tutorial 13

Matrices

1. Which of the following are elementary matrices

(a) $\begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$;

(b) $\begin{bmatrix} -5 & 1 \\ 1 & 0 \end{bmatrix}$;

(c) $\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{3} \end{bmatrix}$;

(d) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$;

(e) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$;

(f) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$;

(g) $\begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

2. Find the inverse of the following matrix if the matrix is invertible.

(a) $\begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$;

(b) $\begin{bmatrix} -3 & 6 \\ 4 & 5 \end{bmatrix}$;

(c) $\begin{bmatrix} 6 & -4 \\ -3 & 2 \end{bmatrix}$;

(d) $\begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}$;

(e) $\begin{bmatrix} -1 & 3 & 4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$;

(f) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$;

(g) $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$;

(h) $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

3. Find the inverse of each of the following matrices, where k_1, k_2, k_3, k_4 and k are all non-zero.

$$(a) \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix};$$

$$(b) \begin{bmatrix} 0 & 0 & 0 & k_1 \\ 0 & 0 & k_2 & 0 \\ 0 & k_3 & 0 & 0 \\ k_4 & 0 & 0 & 0 \end{bmatrix};$$

$$(c) \begin{bmatrix} k & 0 & 0 & 0 \\ 1 & k & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & k \end{bmatrix};$$

4. Let A and B be row-equivalent $n \times n$ matrices. If A is invertible, prove that so is B .
5. Let A and B be $n \times n$ matrices. If the product AB is invertible, prove that both A and B are invertible.
6. Let A and B be $n \times n$ matrices with A invertible, prove that $(A+B)A^{-1}(A-B) = (A-B)A^{-1}(A+B)$.
7. Let A and B be $n \times n$ matrices such that $AB - I_n$ is invertible. Show that $(BA - I_n)[B(AB - I_n)^{-1}A - I_n] = I_n$ and deduce that $(BA - I_n)$ is also invertible.
8. For each of the following matrices

$$(a) A = \begin{bmatrix} 2 & 3 \\ -1 & 6 \end{bmatrix};$$

$$(b) A = \begin{bmatrix} -1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

Find the eigenvalues and their associated eigenvectors of A .

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