

In [1]:

```
# Q2e
A=matrix([[ -1, 3, 4],[ 2, 4, 1],[ -4, 2, -9]])
show(A)
```

Out[1]:

$$\begin{pmatrix} -1 & 3 & 4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{pmatrix}$$

In [2]:

```
AI=A.augment(identity_matrix(3))
show(AI)
```

Out[2]:

$$\begin{pmatrix} -1 & 3 & 4 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{pmatrix}$$

In [3]:

```
# elementary matrix E1
E1=identity_matrix(3)
E1 = E1.change_ring(QQ)
# R1:= -R1
E1.rescale_row(0,-1)
show(E1)
```

Out[3]:

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In [4]:

```
show(E1*AI)
```

Out[4]:

$$\begin{pmatrix} 1 & -3 & -4 & -1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{pmatrix}$$

In [5]:

```
# elementary matrix E2
E2=identity_matrix(3)
E2 = E2.change_ring(QQ)
# R2:= R2-2*R1
E2.add_multiple_of_row(1, 0, -2)
# R3:= R3+4*R1
E2.add_multiple_of_row(2, 0, 4)
show(E2)
```

Out[5]:

$$\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & 0 & 1 \end{pmatrix}$$

In [6]:

```
show(E2*E1*AI)
```

Out[6]:

$$\begin{pmatrix} 1 & -3 & -4 & -1 & 0 & 0 \\ 0 & 10 & 9 & 2 & 1 & 0 \\ 0 & -10 & -25 & -4 & 0 & 1 \end{pmatrix}$$

In [7]:

```
# elementary matrix E3
E3=identity_matrix(3)
E3 = E3.change_ring(QQ)
# R2:= R2/10
E3.rescale_row(1,1/10)
show(E3)
```

Out[7]:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{10} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In [8]:

```
show(E3*E2*E1*AI)
```

Out[8]:

$$\begin{pmatrix} 1 & -3 & -4 & -1 & 0 & 0 \\ 0 & 1 & \frac{9}{10} & \frac{1}{5} & \frac{1}{10} & 0 \\ 0 & -10 & -25 & -4 & 0 & 1 \end{pmatrix}$$

In [9]:

```
# elementary matrix E4
E4=identity_matrix(3)
E4 = E4.change_ring(QQ)
# R3:= R3+10*R2
E4.add_multiple_of_row(2, 1, 10)
show(E4)
```

Out[9]:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10 & 1 \end{pmatrix}$$

In [10]:

```
show(E4*E3*E2*E1*AI)
```

Out[10]:

$$\begin{pmatrix} 1 & -3 & -4 & -1 & 0 & 0 \\ 0 & 1 & \frac{9}{10} & \frac{1}{5} & \frac{1}{10} & 0 \\ 0 & 0 & -16 & -2 & 1 & 1 \end{pmatrix}$$

In [11]:

```
# elementary matrix E5
E5=identity_matrix(3)
E5 = E5.change_ring(QQ)
# R3:= -R3/16
E5.rescale_row(2,-1/16)
show(E5)
```

Out[11]:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{16} \end{pmatrix}$$

In [12]:

```
show(E5*E4*E3*E2*E1*AI)
```

Out[12]:

$$\begin{pmatrix} 1 & -3 & -4 & -1 & 0 & 0 \\ 0 & 1 & \frac{9}{10} & \frac{1}{5} & \frac{1}{10} & 0 \\ 0 & 0 & 1 & \frac{1}{8} & -\frac{1}{16} & -\frac{1}{16} \end{pmatrix}$$

In [13]:

```
# elementary matrix E6
E6=identity_matrix(3)
E6 = E6.change_ring(QQ)
# R1:= R1+R3*4
E6.add_multiple_of_row(0, 2, 4)
# R2:= R2-R3*9/10
E6.add_multiple_of_row(1, 2, -9/10)
show(E6)
```

Out[13]:

$$\begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & -\frac{9}{10} \\ 0 & 0 & 1 \end{pmatrix}$$

In [14]:

```
show(E6*E5*E4*E3*E2*E1*AI)
```

Out[14]:

$$\begin{pmatrix} 1 & -3 & 0 & -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 1 & 0 & \frac{7}{80} & \frac{5}{32} & \frac{9}{160} \\ 0 & 0 & 1 & \frac{1}{8} & -\frac{1}{16} & -\frac{1}{16} \end{pmatrix}$$

In [15]:

```
# elementary matrix E7
E7=identity_matrix(3)
E7 = E7.change_ring(QQ)
# R1:= R1+R2*3
E7.add_multiple_of_row(0, 1, 3)
show(E7)
```

Out[15]:

$$\begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In [16]:

```
show(E7*E6*E5*E4*E3*E2*E1*AI)
```

Out[16]:

$$\begin{pmatrix} 1 & 0 & 0 & -\frac{19}{80} & \frac{7}{32} & -\frac{13}{160} \\ 0 & 1 & 0 & \frac{7}{80} & \frac{5}{32} & \frac{9}{160} \\ 0 & 0 & 1 & \frac{1}{8} & -\frac{1}{16} & -\frac{1}{16} \end{pmatrix}$$

In [17]:

```
# double check
show(A^(-1))
```

Out[17]:

$$\begin{pmatrix} -\frac{19}{80} & \frac{7}{32} & -\frac{13}{160} \\ \frac{7}{80} & \frac{5}{32} & \frac{9}{160} \\ \frac{1}{8} & -\frac{1}{16} & -\frac{1}{16} \end{pmatrix}$$

In [18]:

```
# Q8b
# we put 2 as (sqrt(2))^2 to trick CoCalc not to give numerical answer
B=matrix([[ -1, (sqrt(2))^2, 0], [1, 1, 0], [0, 0, 1]])
show(B)
```

Out[18]:

$$\begin{pmatrix} -1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In [19]:

```
show(B.eigenvectors_right())
```

Out[19]:

$$\left[\left(-\sqrt{3}, \left[\left(1, -\frac{1}{2}\sqrt{3} + \frac{1}{2}, 0 \right) \right], 1 \right), \left(\sqrt{3}, \left[\left(1, \frac{1}{2}\sqrt{3} + \frac{1}{2}, 0 \right) \right], 1 \right), (1, [(0, 0, 1)]) \right]$$

In [0]: