

# The Hong Kong Polytechnic University

## Department of Applied Mathematics

### AMA1007 Calculus and Linear Algebra

#### Tutorial 12

#### Determinants

1. Evaluate (a)  $\begin{vmatrix} 2 & 4 \\ 3 & -2 \end{vmatrix}$ ; (b)  $\begin{vmatrix} 3 & 1 & -2 \\ 8 & -5 & 7 \\ 4 & 0 & 1 \end{vmatrix}$ ; (c)  $\begin{vmatrix} 5 & 4 & 2 & -1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 9 \\ 1 & -2 & -1 & 4 \end{vmatrix}$ .

2. Express the following determinant as a product of simple factors.

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Check your answer with CoCalc Jupyter.

3. Solve by Cramer's rule the following linear systems.

(a)  $\begin{cases} 4x + 3y = -1 \\ 2x - y = 7 \end{cases}$ ;

(b)  $\begin{cases} 2x + 3y + z = 2 \\ -x + 2y + z = 1 \\ 2x + y + z = 3 \end{cases}$ .

4. Find the values of  $\lambda$  for which the system of equations

$$\begin{cases} x - z = \lambda x \\ x + 2y + z = \lambda y \\ 2x + 2y + 3z = \lambda z \end{cases}$$

has solutions for  $(x, y, z) \neq (0, 0, 0)$ .

Check your answer with CoCalc Jupyter.

5. Find the values of  $\lambda$  for which the system of equations

$$\begin{cases} (2-\lambda)x+2y+3=0 \\ 2x+(4-\lambda)y+7=0 \\ 2x+5y+(6-\lambda)=0 \end{cases}$$

is consistent (i.e. there are values of  $x$  and  $y$  satisfying all the three equations simultaneously). Find the values of  $x$  and  $y$  corresponding to each of these values of  $\lambda$ .

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