

The Hong Kong Polytechnic University

Department of Applied Mathematics

AMA1007 Calculus and Linear Algebra

Tutorial 10

Improper Integrals

1. Which of the following integrals is/are improper? Why?

(a) $\int_1^2 \frac{1}{2x-1} dx;$

(b) $\int_0^1 \frac{1}{2x-1} dx;$

(c) $\int_{-\infty}^{\infty} \frac{\sin x}{1+x^2} dx;$

(d) $\int_1^2 \ln(x-1) dx.$

2. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a) $\int_1^{\infty} \frac{1}{(3x+1)^2} dx;$

(b) $\int_{-\infty}^{-1} \frac{1}{\sqrt{2-w}} dw;$

(c) $\int_4^{\infty} e^{\frac{-y}{2}} dy;$

(d) $\int_{-\infty}^{\infty} \frac{x}{1+x^2} dx.$

(e) $\int_{-\infty}^{\infty} xe^{-x^2} dx;$

(f) $\int_{2\pi}^{\infty} \sin \theta d\theta;$

$$(g) \int_1^{\infty} \frac{x+1}{x^2+2x} dx;$$

$$(h) \int_0^{\infty} s e^{-5s} ds;$$

$$(i) \int_0^{\infty} \frac{1}{\sqrt{x}(1+x)} dx.$$

3. Find the values of p for which the integral $\int_0^1 x^p \ln x dx$ converges and evaluate the integral for those values of p .

4. (a) Show that $\int_{-\infty}^{\infty} x dx$ is divergent. (b) Show that $\lim_{t \rightarrow \infty} \int_{-t}^t x dx = 0$.

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