The Hong Kong Polytechnic University

Department of Applied Mathematics

AMA1007 Calculus and Linear Algebra

Tutorial 7

Mean Value Theorem and Indefinite Integrals

1. Apply Mean-value theorem to prove that if $0 < a < b$,

$$\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2},$$

and hence show that

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}.$$

2. Show that if $f(x)$ is continuous on $[a,b]$ and $f'(x) = 0$ for all $x \in (a,b)$, then $f(x)$ is constant on (a,b). (Hint: Use the Mean Value Theorem.)

3. Show that if $f(x)$ is continuous on $[a,b]$ and $f'(x) = c$ (c is a constant) for all $x \in (a,b)$, then $f(x) = cx + d$ for some real number d. (Hint: Use the result of 2.)

4. Let $f(x) = (x-a)^m (x-b)^n$ where $a < b$, and m, n are positive integers. Find a point c between a and b such that $f'(c) = 0$.

5. Suppose f is a function defined on an open interval I and $f''(x) \geq 0$ for all $x \in I$. For any $a, b \in I$ with $a < b$, show that $(1-t) f(a) + t f(b) \geq f[(1-t)a + tb]$ for $0 \leq t \leq 1$.

6. Evaluate

(a) $\int\left(1-\frac{1}{x^2}\right)\sqrt{x\sqrt{x}}\,dx$;

(b) $\int(1-x)(1-2x)(1-3x)\,dx$;

(c) $\int\left(2^x+3^x\right)^2\,dx$; Check your answer with CoCalc Jupyter.

(d) $\int(2x-3)^{10}\,dx$;

(e) $\int\frac{1}{\sqrt{x(1-x)}}\,dx$;

(f) $\int\frac{dx}{x\ln x\ln(\ln x)}$.

7. Find the following integrals by integration by parts:

(a) $\int x^2e^{-x}dx$;

(b) $\int e^x\sin x dx$;

(c) $\int(\ln x)^2dx$.

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