The Hong Kong Polytechnic University

Department of Applied Mathematics

AMA1007 Calculus and Linear Algebra

Tutorial 5

Higher derivatives and Application of Differentiation

1. (a) Find y'' if $y = x^3(x+1)^2$;

   (b) Find y''' if $y = x^3 \cos 2x$;

   (c) Show that $(x^2-1)y'' + xy' - y = 0$ if $y = x + \sqrt{x^2-1}$.

2. Let $y = (x^2-1)^n$, where n is a non-negative integer. Show that

   (a) $(x^2-1)y' - 2nxy = 0$;

   (b) $(x^2-1)y^{(n+2)} + 2xy^{(n+1)} - n(n+1)y^{(n)} = 0$.

3. Find the open intervals in which the function is increasing or decreasing.

   (a) $y = x^2 - 4$;

   (b) $y = \dfrac{1}{1+x^2}$;

   (c) $y = 2x^3 + x - 1$.

4. Sketch the graphs of the function by considering the x-, y-intercepts, local extrema, inflection points and asymptotes.

   (a) $y = \dfrac{x^2+x+1}{x+1}$; Check your answer with CoCalc Jupyter.

   (b) $y = x^3 + 6x^2$, $x \in [-4, 4]$.

5. A rectangular storage container with a lid on the top has a volume of 10m^3. The length of the base is twice the width. Material for the base costs \$10 per square metre. Material for the sides costs \$6 per square metre. The lid is made from the same material as the sides. Find the dimensions of the container for minimizing the material cost and the minimum material cost.

6. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r.

7. The position of a particle moving along a line is $s = \sqrt{1+4t}$, with s in metres and t in seconds. Find the velocity and acceleration of the particle at $t = 6$ seconds.

8. For the following, use parametric differentiation to find (i) the equation for the tangent to the curve and (ii) the value of d^2y/dx^2, at the point defined by the given value of t:
   (a) $x = 2\cos t, \quad y = 2\sin t, \quad t = \pi/4$;
   (b) $x = 2t^2 + 3, \quad y = t^4, \quad t = -1$.

9. Consider the following statements:
   I. If $f''(a) = 0$, then $f(x)$ has local minimum at $x = a$.
   II. if $f(x) = \begin{cases} x^2, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$, then $f''(0)$ doesn't exist.
   III. If $f(x) = x^2 + \frac{2}{x}$, then there is a relative minimum at $x = 1$.
   IV. If $f(x) = x^2 + \frac{2}{x}$, then there is one inflection point.
   V. If $f(x) = x^2 + \frac{2}{x}$, then there are two vertical asymptotes.

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   Which of the following statements is true? Briefly explain.
   (a) Only one of the above statements is correct.
   (b) Only two of the above statements are correct.
   (c) Only three of the above statements are correct.
   (d) Only four of the above statements are correct.
   (e) All of the above statements are correct.

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