

The Hong Kong Polytechnic University

Department of Applied Mathematics

AMA1007 Calculus and Linear Algebra

Tutorial 4

Differentiation techniques

1. Differentiate from (a) to (c) with respect to x .

(a) $y = \frac{5x+3}{x-3}e^{-2x}$; (b) $y = (\sin x)^{\tan^3 x}$; (c) $y = 5x^3 - 3x^5 + 5e^{\cos x}$.

2. Find the first order derivative $\frac{dy}{dx}$ of the following functions:

(a) $xy^4 + x^2y = x + 3y$; Check your answer with CoCalc Jupyter.

(b) $y \cos x = 1 + \sin(xy)$;

(c) $y = \frac{e^x}{x^2}$;

(d) $y\sqrt{1+x^2} = \ln(x + \sqrt{1+x^2})$;

(e) $y = 3^{x \ln x}$.

3. Find h' in terms of f, g, f' and g' for the following functions.

(a) $h(x) = \sqrt{\frac{f(x)}{g(x)}}$;

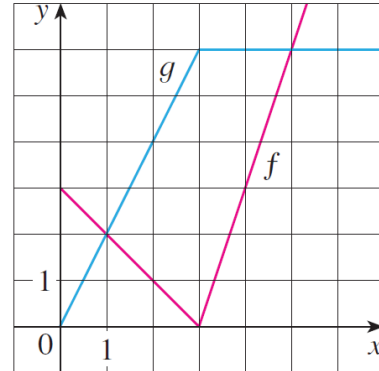
(b) $h(x) = f(g(\sin 4x))$.

4. If f and g are the functions whose graphs are shown below. Let $P(x) = f(x)g(x)$,

$$Q(x) = \frac{f(x)}{g(x)}, \text{ and } C(x) = f(g(x)).$$

Find

- (a) $P'(2)$;
- (b) $Q'(2)$;
- (c) $C'(2)$.



5. Find $\frac{dy}{dx}$ of the function $y^2 = x^2 + \sin(xy)$.

6. Consider the following statements:

- I. If $f(x) = \sin^{-1} x$, then $f'(x) = \frac{1}{\sqrt{1-x^2}}$.
- II. if $x = t^3$ and $y = 2t^2 - 1$, then $\frac{dy}{dx} = \frac{4}{3\sqrt[3]{x}}$.
- III. If $y = \frac{u^2-1}{u^2+1}$ and $u = \sqrt[3]{x^2+2}$, then $\frac{dy}{dx} = \frac{8x}{3\sqrt[3]{x^2+2} \left(\left(\sqrt[3]{x^2+2} \right)^2 + 1 \right)^2}$.
- IV. If f and g are differentiable functions such that $f' = -g$ and $g' = -f$, then h' cannot be determined where $h = f^2 - g^2$.
- V. $\frac{d^2y}{dx^2} = \frac{1}{\frac{d^2x}{dy^2}}$.

Which of the following statements is true? Briefly explain.

- (a) Only one of the above statements is correct.
- (b) Only two of the above statements are correct.
- (c) Only three of the above statements are correct.
- (d) Only four of the above statements are correct.
- (e) All of the above statements are correct.

-End-