The Hong Kong Polytechnic University

Department of Applied Mathematics

AMA1007 Calculus and Linear Algebra

Tutorial 3

Differentiability

1. Prove, from the first principle, that if $f(x)=\frac{1}{x^2}$, then $f'(a)=\frac{-2}{a^3}$, for $a\neq 0$.

2. Prove the following statements:

   (a) Suppose that f is differentiable and periodic, with period a, f' is also periodic.
   (b) If f is even, then f' is odd.
   (c) If f is odd, then f' is even.

3. Suppose that $f(a)=g(a)=h(a)$, $f(x)\le g(x)\le h(x)$ for all x, and $f'(a)=h'(a)$. Prove that g is also differentiable at a such that $f'(a)=g'(a)=h'(a)$.

4. Suppose that f is differentiable at x. Prove that $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x-h)}{2h}$.

5. Find $f'(x)$ for the following functions:

   (a) $f(x)=\sin\left((x+1)^2(x+2)\right)$;
   (b) $f(x)=(\cos x)^{31^2}$;
   (c) $f(x)=\sin\left(\sin\left(\sin\left(\sin x\right)\right)\right)$. Check your answer with CoCalc Jupyter.

6. Find the derivatives of the following functions:

(a) $y = -2x^5 + \sqrt{3}x^3 + 2\pi x^2 - 12$;

(b) $y = \dfrac{x^5 - x + 2}{x^3 + 7}$;

(c) $y = \sqrt{2x + 7}$;

(d) $y = x \sin x$;

(e) $y = \sin^3(5x + 4)$.

7. Consider the following statements:

I. If $f(x)$ is continuous, $f(x)$ is also differentiable.

II. $f(x) = \sqrt[3]{x}$ is differentiable at $x = 0$.

III. $f(x) = |x - 2|$ is not differentiable at $x = 2$.

IV. $\lim\limits_{h \to 0} \dfrac{(f(a+h))^2 - (f(a))^2}{h}$ does not exist.

V. $f(x) = \dfrac{3x + 3}{x^2 - 3x - 4}$ is differentiable everywhere.

Which of the following statements is true? Briefly explain.

(a) Only one of the above statements is correct.
(b) Only two of the above statements are correct.
(c) Only three of the above statements are correct.
(d) Only four of the above statements are correct.
(e) All of the above statements are correct.

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