The Hong Kong Polytechnic University

Department of Applied Mathematics

AMA1007 Calculus and Linear Algebra

Tutorial 2

Limits and Continuity

1. Evaluate the limit, if it exists.

   (a) $\lim\limits_{x\to 8}\left(1+x^{\frac{1}{3}}\right)\left(2-6x^2+x^3\right)$;

   (b) $\lim\limits_{x\to 1}\left(\dfrac{1+3x}{1+4x^2+3x^4}\right)^3$;

   (c) $\lim\limits_{x\to 16}\dfrac{4-\sqrt{x}}{16x-x^2}$;

   (d) $\lim\limits_{x\to 0}\left(\dfrac{1}{x\sqrt{1+x}}-\dfrac{1}{x}\right)$;

   (e) $\lim\limits_{x\to 0}\left(x^4\cos\dfrac{2}{x}\right)$;

   (f) $\lim\limits_{x\to 0^-}\left(\dfrac{1}{x}-\dfrac{1}{|x|}\right)$;

   (g) $\lim\limits_{x\to -1}\dfrac{\sin\left(x^2-x-2\right)}{x+1}$;

   (h) $\lim\limits_{x\to 0}\dfrac{\sin\left(\sin x\right)}{x}$.

2. Suppose that $f(x)$ is an even function. Does knowing that $\lim\limits_{x\to 2^-} f(x)=7$ tell you anything about $\lim\limits_{x\to -2^-} f(x)$ or $\lim\limits_{x\to -2^+} f(x)$? Give reasons for your answer.

3. Find the following limits

   (a) $\lim\limits_{x\to 0^+}\left(\dfrac{1}{x^{\frac{1}{3}}}-\dfrac{1}{(x-1)^{\frac{4}{3}}}\right)$

   and $\lim\limits_{x\to 0^-}\left(\dfrac{1}{x^{\frac{1}{3}}}-\dfrac{1}{(x-1)^{\frac{4}{3}}}\right)$;

   (b) $\lim\limits_{h\to 0^+}\dfrac{\sqrt{h^2+4h+5}-\sqrt{5}}{h}$;

   (c) $\lim\limits_{\theta\to 0}\left(2-\cot\theta\right)$.

4. Use the $\varepsilon - \delta$ definition of limit to show that:

   (a) $\lim_{x\to 1}(5x-3)=2$;

   (b) $\lim_{x\to 1}\frac{1}{x}=1$;

   (c) $\lim_{x\to -3}\frac{x^2-9}{x+3}=-6$.

5. Evaluate the following limits.

   (a) $\lim_{x\to\infty} x^3-3x-\frac{1}{x}$

   (b) $\lim_{x\to-\infty} x^3-3x-\frac{1}{x}$

   (c) $\lim_{x\to\infty}\frac{5x^2+8x-3}{3x^2+1}$

   (d) $\lim_{x\to\infty}\frac{11x+3}{2x^3-1}$

   (e) $\lim_{x\to-\infty}\frac{x^2-7x}{x+1}$

   (f) $\lim_{x\to\infty}\frac{x^{\frac{2}{3}}+x^{-1}}{x^{\frac{2}{3}}+\cos^2 x}$

   (g) $\lim_{x\to\infty}\frac{\sqrt{2x^2+1}}{3x-5}$

   (h) $\lim_{x\to\infty}\frac{x+\sin x+2\sqrt{x}}{x+\sin x}$

6. For what value of k is $f(x)$ continuous at every x where $f(x)=\begin{cases}x^2-1, & x<3\\ 2kx, & x\geq 3\end{cases}$?

7. Given $f(x)=\begin{cases}1+x^2, & 0\leq x<1\\ 1, & 1\leq x<2\\ 3-x, & x\geq 2\end{cases}$, consider the following statements:

   I. $\lim_{x\to 0} f(x)$ exists.

   II. $\lim_{x\to 1} f(x)$ exists.

   III. $\lim_{x\to 2} f(x)$ exists.

   IV. $f(x)$ is continuous at $x=1$.

   V. $f(x)$ is continuous at $x=2$.

   Which of the following statements is true? Briefly explain.

   (a) Only one of the above statements is correct.
   (b) Only two of the above statements are correct.
   (c) Only three of the above statements are correct.
   (d) Only four of the above statements are correct.
   (e) All of the above statements are correct.

-End-