The Hong Kong Polytechnic University

Department of Applied Mathematics

AMA1007 Calculus and Linear Algebra

Tutorial 1

Elementary function and Partial fractions

1. Consider the functions f and g defined by $f(x)=2-x^2$ and $g(x)=\sqrt{x+2}$.

   (a) Find domains and ranges of f and g respectively.

   (b) Find the composite function $f \circ g$ and $g \circ f$, and hence state their domains and ranges.

2. Determine whether the following functions is/are odd, even, or neither.

   (a) $y=\sec x \tan x$;

   (b) $y=\dfrac{x^4+1}{x^3-2x}$;

   (c) $y=1-\sin x$.

3. Find the asymptotes of the following rational functions and sketch their graphs.

   (a) $y=\dfrac{x^2-x+1}{x}$;

   (b) $y=\dfrac{x^2}{x^2-4}$.

4. Resolve the following functions into partial fractions.

   (a) $f(x)=\dfrac{x^4-2x^2+4x+1}{x^3-x^2-x+1}$;

   (b) $g(x)=\dfrac{1-x+2x^2-x^3}{x\left(x^2+1\right)^2}$.

and use CoCalc to reproduce / check your answers.

5. Solve the following inequality for x:

(a) $(x-1)\left(x+\frac{1}{2}\right)>0$;

(b) $\frac{2x-1}{3x+2}\geq 0$;

(c) $-2<\frac{x-1}{x+2}<2$.

6. Prove the following inequalities for all numbers a,b:

(a) $|a+b|\geq|a|-|b|$ [Hints: write $a=a+b+(-b)$];

(b) $|a-b|\geq|a|-|b|$;

(c) $|a-b|\leq|a|+|b|$.

7. Consider the following statements:

I. If both $f(x)$ and $g(x)$ are odd function, then $f(x)\,g(x)$ is also an odd function.

II. If both $f(x)$ and $g(x)$ are odd function, then $f(x)\pm g(x)$ is also an odd function.

III. If both $f(x)$ and $g(x)$ are periodic function, then $f(x)\pm g(x)$ is also a periodic function.

IV. Rational function is always a proper function.

V. Only one-to-one function f has the inverse of f.

VI. If $P(a)=0$, then the polynomial $P(x)$ is divisible by $x-a$.

Which of the following statements is true? Briefly explain.

(a) Only one of the above statements is correct.
(b) Only two of the above statements are correct.
(c) Only three of the above statements are correct.
(d) Only four of the above statements are correct.
(e) All of the above statements are correct.

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