In [1]:

```
# define A as a 2x5 matrix as follows
#
A=matrix(QQ,[[1,2,3,4,5],[6,7,8,9,10]])
show(A)
```

Out[1]:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \end{pmatrix}$$

In [2]:

```
# augment a 2x2 identity matrix to the right
#
AI=A.augment(identity_matrix(2) , subdivide=True )
show(AI)
```

Out[2]:

$$\left(\begin{array}{ccccc|cc} 1 & 2 & 3 & 4 & 5 & 1 & 0 \\ 6 & 7 & 8 & 9 & 10 & 0 & 1 \end{array}\right)$$

In [3]:

```
# R2:=R2-6*R1
# and capture the right block elementary matrix E1
# i.e., E1 captures the elementary row operation
#
AI.add_multiple_of_row(1,0,-6)
show(AI)
E1=AI.matrix_from_rows_and_columns([0,1], [5,6])
```

Out[3]:

$$\left(\begin{array}{ccccc|cc} 1 & 2 & 3 & 4 & 5 & 1 & 0 \\ 0 & -5 & -10 & -15 & -20 & -6 & 1 \end{array}\right)$$

In [4]:

```
# reset the right block identity
#
AI=AI.matrix_from_rows_and_columns([0,1], [0,1,2,3,4])
AI=AI.augment(identity_matrix(2) , subdivide=True )
show(AI)
```

Out[4]:

$$\left(\begin{array}{ccccc|cc} 1 & 2 & 3 & 4 & 5 & 1 & 0 \\ 0 & -5 & -10 & -15 & -20 & 0 & 1 \end{array}\right)$$

In [5]:

```
# R2:=(-1/5)*R2
# and capture the right block elementary matrix E2
# i.e., E2 captures the elementary row operation
#
AI.rescale_row(1,-1/5)
show(AI)
E2=AI.matrix_from_rows_and_columns([0,1], [5,6])
```

Out[5]:

$$\left(\begin{array}{ccccc|cc} 1 & 2 & 3 & 4 & 5 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 0 & -\frac{1}{5} \end{array}\right)$$

In [6]:

```
# reset the right block identity again
#
AI=AI.matrix_from_rows_and_columns([0,1], [0,1,2,3,4])
AI=AI.augment(identity_matrix(2) , subdivide=True )
show(AI)
```

Out[6]:

$$\left(\begin{array}{ccccc|cc} 1 & 2 & 3 & 4 & 5 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 0 & 1 \end{array}\right)$$

In [7]:

```
# R1:=R1-2*R2
# and capture the right block elementary matrix E3
# i.e., E3 captures the elementary row operation
#
AI.add_multiple_of_row(0,1,-2)
show(AI)
E3=AI.matrix_from_rows_and_columns([0,1], [5,6])
```

Out[7]:

$$\left(\begin{array}{ccccc|cc} 1 & 0 & -1 & -2 & -3 & 1 & -2 \\ 0 & 1 & 2 & 3 & 4 & 0 & 1 \end{array}\right)$$

In [8]:

```
# E=E3*E2*E1 is the matrix capturing all three
# elementary row operations
#
E=E3*E2*E1
show(E)
```

Out[8]:

$$\left(\begin{array}{cc} -\frac{7}{5} & \frac{2}{5} \\ \frac{6}{5} & -\frac{1}{5} \end{array}\right)$$

In [9]:

```
# to demonstrate, we multiply E to A, and it reproduce
# the desired reduced row-echelon form of A
show(E*A)
```

Out[9]:

$$\left(\begin{array}{ccccc} 1 & 0 & -1 & -2 & -3 \\ 0 & 1 & 2 & 3 & 4 \end{array}\right)$$

In [0]:

```

```