To determine if three given points on the xy-plane are collinear

Given three points (x_0, y_0), (x_1, y_1), (x_2, y_2) on the Cartesian plane (the xy-plane) in 2-D. They are collinear (that means, they are contained in one straight line) if the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \end{vmatrix} = 0.$$

Example Given the coordinates :

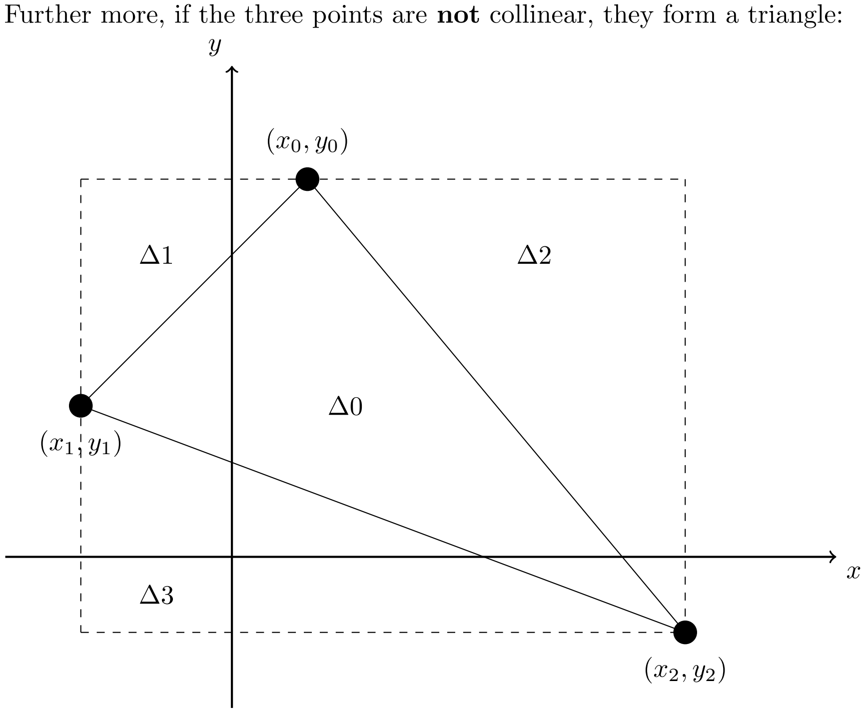
$$\begin{aligned} (x_0, y_0) &= (\sqrt{2}\ ,\ \sqrt{2}\sqrt{3} + \sqrt{7}) \\ (x_1, y_1) &= (\sqrt{3}\ ,\ \sqrt{7} + 3) \\ (x_2, y_2) &= (\sqrt{5}\ ,\ \sqrt{3}\sqrt{5} + \sqrt{7}) \end{aligned}$$

We compute the value of the determinant:

$$\begin{vmatrix} 1 & 1 & 1 \\ x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ \sqrt{2} & \sqrt{3} & \sqrt{5} \\ \sqrt{2}\sqrt{3} + \sqrt{7} & \sqrt{7} + 3 & \sqrt{3}\sqrt{5} + \sqrt{7} \end{vmatrix} = 0.$$

Therefore, the three given points are collinear (they are contained in one straight line).

Further more, if the three points are **not** collinear, they form a triangle:



Note that the determinant $\begin{vmatrix} 1 & 1 & 1 \\ x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \end{vmatrix}$ is giving **twice** the area of the triangle with vertices (x_0, y_0), (x_1, y_1), (x_2, y_2).

Area of big rectangle $= (x_2 - x_1) \times (y_0 - y_2)$.
Area of $\Delta 1 = \frac{1}{2}(x_0 - x_1) \times (y_0 - y_1)$.
Area of $\Delta 2 = \frac{1}{2}(x_2 - x_0) \times (y_0 - y_2)$.
Area of $\Delta 3 = \frac{1}{2}(x_2 - x_1) \times (y_1 - y_2)$.
Area of $\Delta 0 =$ Area of big rectangle - Area of $\Delta 1$ - Area of $\Delta 2$ - Area of $\Delta 3$.

$$\begin{aligned} &= (x_2 y_0 - x_2 y_2 - x_1 y_0 + x_1 y_2) - \frac{1}{2}(x_0 y_0 - x_0 y_1 - x_1 y_0 + x_1 y_1) \\ &\quad - \frac{1}{2}(x_2 y_0 - x_2 y_2 - x_0 y_0 + x_0 y_2) - \frac{1}{2}(x_2 y_1 - x_2 y_2 - x_1 y_1 + x_1 y_2) \\ &= -\frac{1}{2}\, x_1 y_0 + \frac{1}{2}\, x_2 y_0 + \frac{1}{2}\, x_0 y_1 - \frac{1}{2}\, x_2 y_1 - \frac{1}{2}\, x_0 y_2 + \frac{1}{2}\, x_1 y_2. \end{aligned}$$

$$\begin{aligned} \begin{vmatrix} 1 & 1 & 1 \\ x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \end{vmatrix} &= \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} - \begin{vmatrix} x_0 & x_2 \\ y_0 & y_2 \end{vmatrix} + \begin{vmatrix} x_0 & x_1 \\ y_0 & y_1 \end{vmatrix} \\ &= (x_1 y_2 - x_2 y_1) - (x_0 y_2 - x_2 y_0) + (x_0 y_1 - x_1 y_0) \\ &= -x_1 y_0 + x_2 y_0 + x_0 y_1 - x_2 y_1 - x_0 y_2 + x_1 y_2. \end{aligned}$$

Hence, the determinant is giving twice the area of the triangle with vertices (x_0, y_0), (x_1, y_1), (x_2, y_2).