In [1]:

```
f(x)=(x+1)/(x^2-2*x-7)
show(f)
```

Out[1]:

$$x \mapsto \frac{x + 1}{x^2 - 2\,x - 7}$$

In [2]:

```
f1(x)=taylor(f(x),x,0,1)
f2(x)=taylor(f(x),x,0,2)
f3(x)=taylor(f(x),x,0,3)
f4(x)=taylor(f(x),x,0,4)
f5(x)=taylor(f(x),x,0,5)
f6(x)=taylor(f(x),x,0,6)
f20(x)=taylor(f(x),x,0,20)
```

In [3]:

```
show(f6)
```

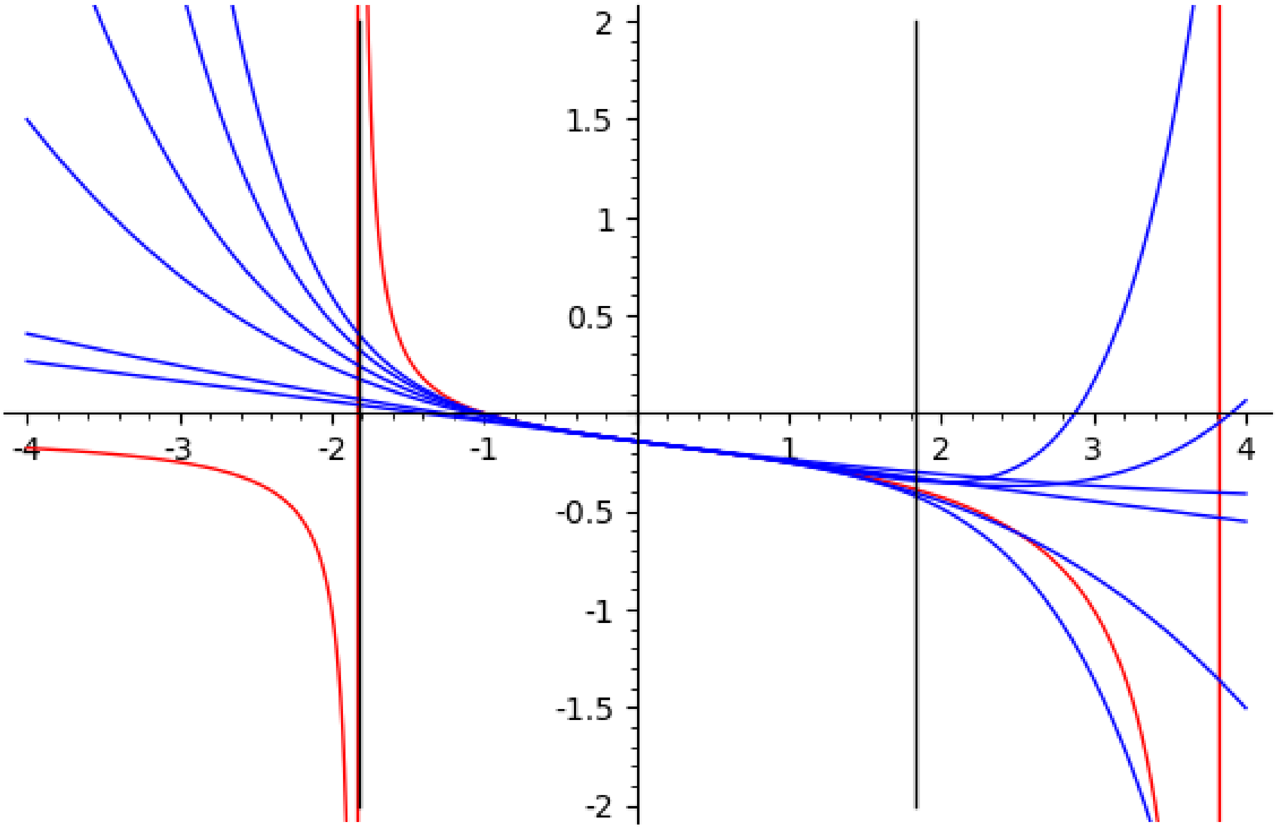
Out[3]:

$$x \mapsto \frac{1707}{823543}\,x^6 - \frac{493}{117649}\,x^5 + \frac{103}{16807}\,x^4 - \frac{41}{2401}\,x^3 + \frac{3}{343}\,x^2 - \frac{5}{49}\,x - \frac{1}{7}$$

In [4]:

```
p1=plot(f(x),x,-4,4, rgbcolor="red")
ps1=plot(f1(x),x,-4,4)
ps2=plot(f2(x),x,-4,4)
ps3=plot(f3(x),x,-4,4)
ps4=plot(f4(x),x,-4,4)
ps5=plot(f5(x),x,-4,4)
ps6=plot(f6(x),x,-4,4)
Q1 = line([(14/(2+sqrt(32)),-2),(14/(2+sqrt(32)),2)], rgbcolor="black")
Q2 = line([(-14/(2+sqrt(32)),-2),(-14/(2+sqrt(32)),2)], rgbcolor="black")
(p1+ps1+ps2+ps3+ps4+ps5+ps6+Q1+Q2).show(ymax=2,ymin=-2)
```

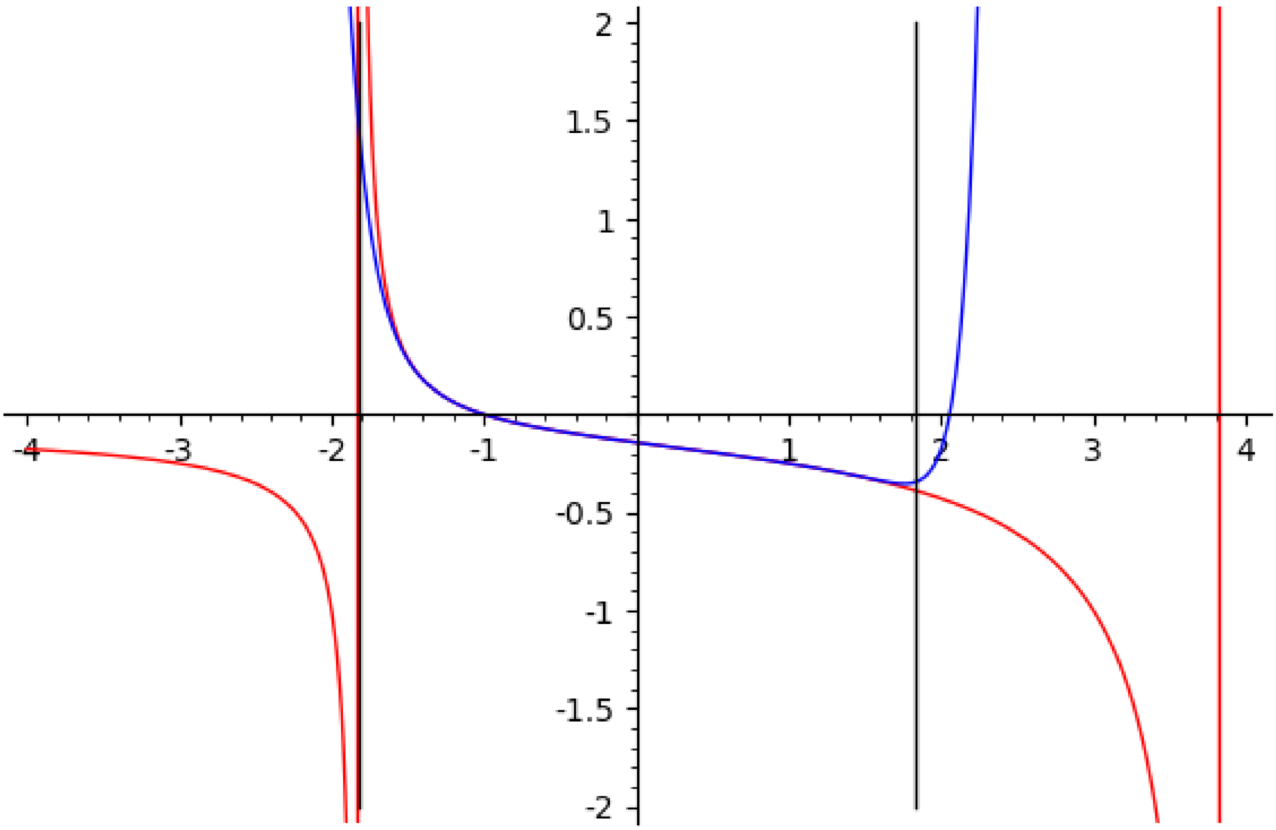
Out[4]:



In [5]:

```
p1=plot(f(x),x,-4,4, rgbcolor="red")
ps20=plot(f20(x),x,-4,4)
Q1 = line([(14/(2+sqrt(32)),-2),(14/(2+sqrt(32)),2)], rgbcolor="black")
Q2 = line([(-14/(2+sqrt(32)),-2),(-14/(2+sqrt(32)),2)], rgbcolor="black")
(p1+ps20+Q1+Q2).show(ymax=2,ymin=-2)
```

Out[5]:



In [6]:

```
solve(x^2-2*x-7==0,x)
```

```
[x == -2*sqrt(2) + 1, x == 2*sqrt(2) + 1]
```

Out[6]:

In [7]:

```
alpha=solve(x^2-2*x-7==0,x)[0].rhs()
show(alpha)
```

Out[7]:

$$-2\sqrt{2}+1$$

In [8]:

```
beta=solve(x^2-2*x-7==0,x)[1].rhs()
show(beta)
```

Out[8]:

$$2\sqrt{2}+1$$

In [9]:

```
show(expand((x-alpha)*(x-beta)))
```

Out[9]:

$$x^2-2x-7$$

In [10]:

```
var('a,b')
g(x)=a/(x-beta)+b/(x-alpha)
show(g)
```

Out[10]:

$$x \mapsto \frac{b}{x+2\sqrt{2}-1}+\frac{a}{x-2\sqrt{2}-1}$$

In [11]:

```
maxima_calculus('algebraic: true;')
```

Out[11]: true

In [12]:

```
var('u')
a=solve(u*(-alpha)+(1-u)*(-beta)==1,u)[0].rhs()
show(a)
```

Out[12]:

$$\frac{1}{4}\sqrt{2}+\frac{1}{2}$$

In [13]:

```
b=solve((1-u)*(-alpha)+u*(-beta)==1,u)[0].rhs()
show(b)
```

Out[13]:

$$-\frac{1}{4}\sqrt{2}+\frac{1}{2}$$

In [14]:

```
show((a/(x-beta)+b/(x-alpha)).canonicalize_radical())
```

Out[14]:

$$\frac{x+1}{x^2 - 2\,x - 7}$$

In [15]:

```
ga4(x)=taylor(a/(x-beta),x,0,4)
show(ga4)
```

Out[15]:

$$x \mapsto -\frac{x^4\left(\sqrt{2}+2\right)}{4\left(298\,\sqrt{2}+401\right)} - \frac{x^3\left(\sqrt{2}+2\right)}{4\left(72\,\sqrt{2}+113\right)} - \frac{x^2\left(\sqrt{2}+2\right)}{4\left(22\,\sqrt{2}+25\right)} - \frac{x\left(\sqrt{2}+2\right)}{4\left(4\,\sqrt{2}+9\right)} -$$

In [16]:

```
gb4(x)=taylor(b/(x-alpha),x,0,4)
show(gb4)
```

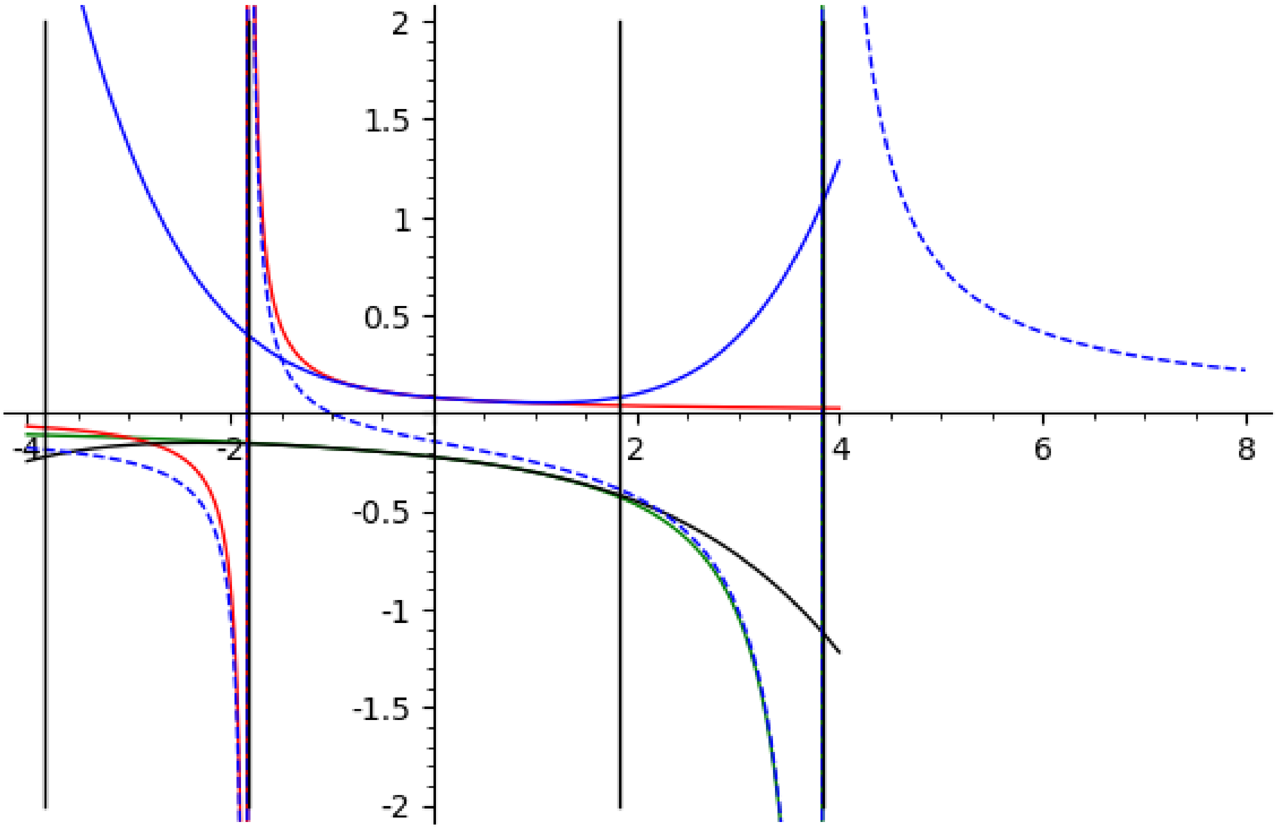
Out[16]:

$$x \mapsto -\frac{x^4\left(\sqrt{2}-2\right)}{4\left(298\,\sqrt{2}-401\right)} - \frac{x^3\left(\sqrt{2}-2\right)}{4\left(72\,\sqrt{2}-113\right)} - \frac{x^2\left(\sqrt{2}-2\right)}{4\left(22\,\sqrt{2}-25\right)} - \frac{x\left(\sqrt{2}-2\right)}{4\left(4\,\sqrt{2}-9\right)} -$$

In [17]:

```
pf1=plot(a/(x-beta),x,-4,4, rgbcolor="green")
pf2=plot(b/(x-alpha),x,-4,4, rgbcolor="red")
addtwo=plot(a/(x-beta)+b/(x-alpha),x,-4,8, linestyle="dashed")
pf3=plot(ga4(x),x,-4,4, rgbcolor="black")
pf4=plot(gb4(x),x,-4,4)
Q1 = line([(1-2*sqrt(2),-2),(1-2*sqrt(2),2)], rgbcolor="black")
Q1a = line([(-1+2*sqrt(2),-2),(-1+2*sqrt(2),2)], rgbcolor="black")
Q2 = line([(1+2*sqrt(2),-2),(1+2*sqrt(2),2)], rgbcolor="black")
Q2a = line([(-1-2*sqrt(2),-2),(-1-2*sqrt(2),2)], rgbcolor="black")
(pf1+pf2+pf3+pf4+addtwo+Q1+Q2+Q1a+Q2a).show(ymin=-2,ymax=2)
```

Out[17]:



In [0]: