In [1]:

```
hh(x)=(x-3)/(x^2-3*x-3)
show(hh)
```

Out[1]:

$$x \mapsto \frac{x-3}{x^2-3\,x-3}$$

In [2]:

```
hh6(x)=taylor(hh(x),x,0,6)
show(hh6)
```

Out[2]:

$$x \mapsto \frac{115}{27}\,x^6 - \frac{91}{27}\,x^5 + \frac{8}{3}\,x^4 - \frac{19}{9}\,x^3 + \frac{5}{3}\,x^2 - \frac{4}{3}\,x + 1$$

In [3]:

```
var("L")
show(solve(3*L^2+3*L-1==0,L))
```

Out[3]:

$$\left[L = -\frac{1}{6}\,\sqrt{21} - \frac{1}{2}, L = \frac{1}{6}\,\sqrt{21} - \frac{1}{2}\right]$$

In [4]:

```
maxima_calculus('algebraic: true;')
```

Out[4]: true

In [5]:

```
rneg=1/solve(3*L^2+3*L-1==0,L)[0].rhs().canonicalize_radical()
show(rneg)
```

Out[5]:

$$-\frac{6}{\sqrt{7}\sqrt{3}+3}$$

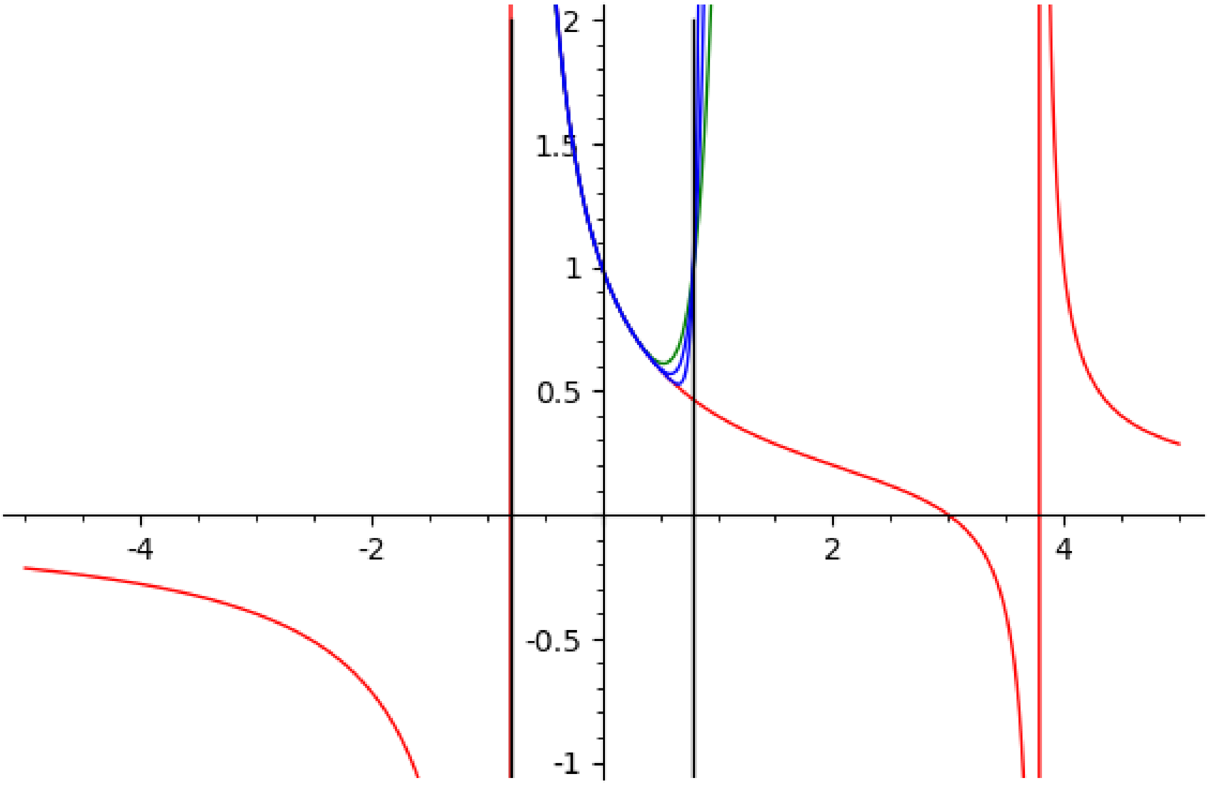
In [6]:

```
hh10(x)=taylor(hh(x),x,0,10)
hh20(x)=taylor(hh(x),x,0,20)
```

In [7]:

```
ph1=plot(hh(x),x,-5,5, rgbcolor="red")
phs6=plot(hh6(x),x,-5,5 , rgbcolor="green" )
phs10=plot(hh10(x),x,-5,5)
phs20=plot(hh20(x),x,-5,5)
Q1 = line([(rneg,-2),(rneg,2)], rgbcolor="black")
Q2 = line([(-rneg,-2),(-rneg,2)], rgbcolor="black")
(ph1+phs6+phs10+phs20+Q1+Q2).show(ymax=2,ymin=-1)
```

Out[7]:



In [8]:

```
numerator(x)=16*x
denominator(x)=4*x^2+1
g(x)=numerator(x)/denominator(x)
show(g)
```

Out[8]:

$$x \mapsto \frac{16\,x}{4\,x^2+1}$$

In [9]:

```
gddash(x)=diff(g(x),x,2).factor()
show(gddash)
```

Out[9]:

$$x \mapsto \frac{128\left(4\,x^2-3\right)x}{\left(4\,x^2+1\right)^3}$$

In [10]:

```
show(solve(gddash(x)==0,x))
```

Out[10]:

$$\left[x = -\frac{1}{2}\sqrt{3}, x = \frac{1}{2}\sqrt{3}, x = 0\right]$$

In [11]:

```
x0=solve(gddash(x)==0,x)[0].rhs()
y0=g(x0)
show(x0,',',y0)
x1=solve(gddash(x)==0,x)[1].rhs()
y1=g(x1)
show(x1,',',y1)
x2=solve(gddash(x)==0,x)[2].rhs()
y2=g(x2)
show(x2,',',y2)
```

Out[11]:

$$-\frac{1}{2}\sqrt{3},\ -2\sqrt{3}$$

Out[11]:

$$\frac{1}{2}\sqrt{3}, 2\sqrt{3}$$

Out[11]:

$$0, 0$$

In [12]:

```
A=matrix([[1,1,1],[x0,x1,x2],[y0,y1,y2]]);
show(A)
```

Out[12]:

$$\begin{pmatrix} 1 & 1 & 1 \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2}\sqrt{3} & 0 \\ -2\sqrt{3} & 2\sqrt{3} & 0 \end{pmatrix}$$

In [13]:

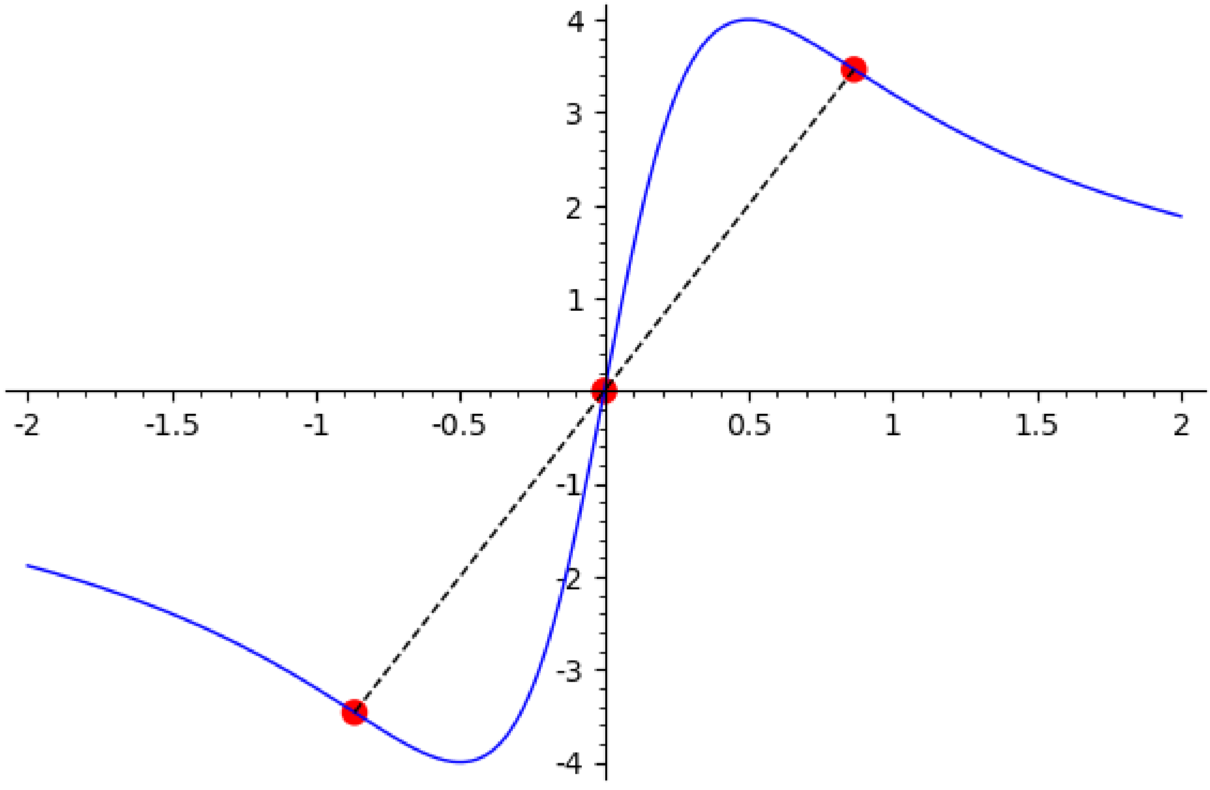
```
det(A)
```

Out[13]: 0

In [14]:

```
pr1=plot(g(x),x,-2,2)
ptr1 = point((x0,y0), rgbcolor='red', pointsize=80)
ptr2 = point((x1,y1), rgbcolor='red', pointsize=80)
ptr3 = point((x2,y2), rgbcolor='red', pointsize=80)
Q3 = line([(x0,y0),(x1,y1)], rgbcolor="black",linestyle="dashed")
pr1+ptr1+ptr2+ptr3+Q3
```

Out[14]:



In [15]:

```
AA=matrix([[1/2,1],[0,1/3]])
show(AA)
```

Out[15]:

$$\begin{pmatrix} \frac{1}{2} & 1 \\ 0 & \frac{1}{3} \end{pmatrix}$$

In [16]:

```
show(AA^8)
```

Out[16]:

$$\begin{pmatrix} \frac{1}{256} & \frac{6305}{279936} \\ 0 & \frac{1}{6561} \end{pmatrix}$$

In [17]:

```
show(identity_matrix(2)-AA)
```

Out[17]:

$$\begin{pmatrix} \frac{1}{2} & -1 \\ 0 & \frac{2}{3} \end{pmatrix}$$

In [18]:

```
show((identity_matrix(2)-AA)^(-1))
```

Out[18]:

$$\begin{pmatrix} 2 & 3 \\ 0 & \frac{3}{2} \end{pmatrix}$$

In [19]:

```
show((identity_matrix(2)-AA^8)*(identity_matrix(2)-AA)^(-1))
```

Out[19]:

$$\begin{pmatrix} \frac{255}{128} & \frac{137845}{46656} \\ 0 & \frac{3280}{2187} \end{pmatrix}$$

In [20]:

```
show(AA^0+AA^1+AA^2+AA^3+AA^4+AA^5+AA^6+AA^7)
```

Out[20]:

$$\begin{pmatrix} \frac{255}{128} & \frac{137845}{46656} \\ 0 & \frac{3280}{2187} \end{pmatrix}$$

In [21]:

```
show(AA.characteristic_polynomial().factor())
```

Out[21]:

$$(x - \frac{1}{2}) \cdot (x - \frac{1}{3})$$

In [0]:

```

```