

In [1]:

```
A=matrix([[1,-2,2],[2,-3,6],[1,1,7]])
show(A)
```

Out[1]:

$$\begin{pmatrix} 1 & -2 & 2 \\ 2 & -3 & 6 \\ 1 & 1 & 7 \end{pmatrix}$$

In [2]:

```
# elementary matrix E1
E1=identity_matrix(3)
E1 = E1.change_ring(QQ)
# R2:= R2-2*R1
E1.add_multiple_of_row(1, 0, -2)
# R3:= R3-R1
E1.add_multiple_of_row(2, 0, -1)
show(E1)
```

Out[2]:

$$\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

In [3]:

```
show(E1*A)
```

Out[3]:

$$\begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & 2 \\ 0 & 3 & 5 \end{pmatrix}$$

In [4]:

```
# elementary matrix E2
E2=identity_matrix(3)
E2 = E2.change_ring(QQ)
# R3:= R3-3*R2
E2.add_multiple_of_row(2, 1, -3)
show(E2)
```

Out[4]:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix}$$

In [5]:

```
show(E2*E1*A)
```

Out[5]:

$$\begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix}$$

In [6]:

```
# elementary matrix E3
E3=identity_matrix(3);
E3 = E3.change_ring(QQ)
# R3:= -R3
E3.rescale_row(2,-1)
show(E3)
```

Out[6]:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

In [7]:

```
show(E3*E2*E1*A)
```

Out[7]:

$$\begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

In [8]:

```
# elementary matrix E4
E4=identity_matrix(3)
E4 = E4.change_ring(QQ)
# R1:= R1+2*R2
E4.add_multiple_of_row(0, 1, 2)
show(E4)
```

Out[8]:

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In [9]:

```
show(E4*E3*E2*E1*A)
```

Out[9]:

$$\begin{pmatrix} 1 & 0 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

In [10]:

```
# elementary matrix E5
E5=identity_matrix(3)
E5 = E5.change_ring(QQ)
# R1:= R1-6*R3
E5.add_multiple_of_row(0, 2, -6)
# R2:= R2-2*R3
E5.add_multiple_of_row(1, 2, -2)
show(E5)
```

Out[10]:

$$\begin{pmatrix} 1 & 0 & -6 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

In [11]:

```
show(E5*E4*E3*E2*E1*A)
```

Out[11]:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In [12]:

```
show(E5*E4*E3*E2*E1)
```

Out[12]:

$$\begin{pmatrix} 27 & -16 & 6 \\ 8 & -5 & 2 \\ -5 & 3 & -1 \end{pmatrix}$$

In [13]:

```
#double check  
show(A(-1))
```

Out[13]:

$$\begin{pmatrix} 27 & -16 & 6 \\ 8 & -5 & 2 \\ -5 & 3 & -1 \end{pmatrix}$$

In [14]:

```
AI=A.augment(identity_matrix(3))  
show(AI)
```

Out[14]:

$$\begin{pmatrix} 1 & -2 & 2 & 1 & 0 & 0 \\ 2 & -3 & 6 & 0 & 1 & 0 \\ 1 & 1 & 7 & 0 & 0 & 1 \end{pmatrix}$$

In [15]:

```
show(E1*AI)
```

Out[15]:

$$\begin{pmatrix} 1 & -2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 3 & 5 & -1 & 0 & 1 \end{pmatrix}$$

In [16]:

```
show(E2*E1*AI)
```

Out[16]:

$$\begin{pmatrix} 1 & -2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & -1 & 5 & -3 & 1 \end{pmatrix}$$

In [17]:

```
show(E3*E2*E1*AI)
```

Out[17]:

$$\begin{pmatrix} 1 & -2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 1 & -5 & 3 & -1 \end{pmatrix}$$

In [18]:

```
show(E4*E3*E2*E1*AI)
```

Out[18]:

$$\begin{pmatrix} 1 & 0 & 6 & -3 & 2 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 1 & -5 & 3 & -1 \end{pmatrix}$$

In [19]:

```
show(E5*E4*E3*E2*E1*AI)
```

Out[19]:

$$\begin{pmatrix} 1 & 0 & 0 & 27 & -16 & 6 \\ 0 & 1 & 0 & 8 & -5 & 2 \\ 0 & 0 & 1 & -5 & 3 & -1 \end{pmatrix}$$