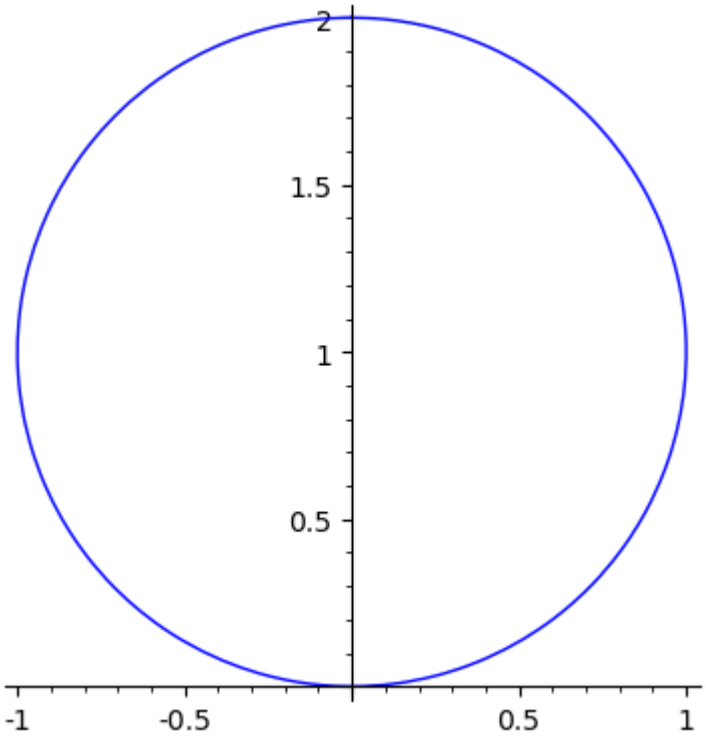
In [1]:
```
#Question 1
polar_plot(2*sin(x), (x, -pi/2, pi/2))
```

Out[1]:




In [2]:
```
f(x)=8*pi*(sin(x))^4*cos(2*x)
show(f(x).reduce_trig())
```

Out[2]:

$$-2\,\pi + \frac{1}{2}\,\pi\cos(6\,x) - 2\,\pi\cos(4\,x) + \frac{7}{2}\,\pi\cos(2\,x)$$

In [3]:
```
show(pi*integrate(f(x),x,pi,0))
```

Out[3]:

$$2\,\pi^3$$

In [4]:
```
show(integrate(x*arccos(x),x))
```

Out[4]:

$$\frac{1}{2}\,x^2\arccos(x) - \frac{1}{4}\,\sqrt{-x^2+1}x + \frac{1}{4}\arcsin(x)$$

In [5]:
```
#Question 2
show(integrate(x*arccos(x),x,0,1))
```

Out[5]:

$$\frac{1}{8}\,\pi$$

In [6]:
```
#Queastion 3
numerator(x)=10*x-610
denominator(x)=x^2-108*x+1691
f(x)=numerator(x)/denominator(x)
show(f)
```

Out[6]:

$$x \mapsto \frac{10\,(x - 61)}{x^2 - 108\,x + 1691}$$

In [7]:
```
show(f(x).partial_fraction())
```

Out[7]:

$$\frac{6}{x - 19} + \frac{4}{x - 89}$$

In [8]:
```
term1(x)=f.partial_fraction().operands()[0]
show(term1)
```

Out[8]:

$$x \mapsto \frac{6}{x - 19}$$

In [9]:
```
show(taylor(term1(x),x,0,2))
```

Out[9]:

$$-\frac{6}{6859}\,x^2 - \frac{6}{361}\,x - \frac{6}{19}$$

In [10]:
```
term2(x)=f.partial_fraction().operands()[1]
show(term2)
```

Out[10]:

$$x \mapsto \frac{4}{x - 89}$$

In [11]:
```
show(taylor(term2(x),x,0,2))
```

Out[11]:

$$-\frac{4}{704969}\,x^2 - \frac{4}{7921}\,x - \frac{4}{89}$$

In [12]:
```
show(taylor(f(x),x,0,2))
```

Out[12]:

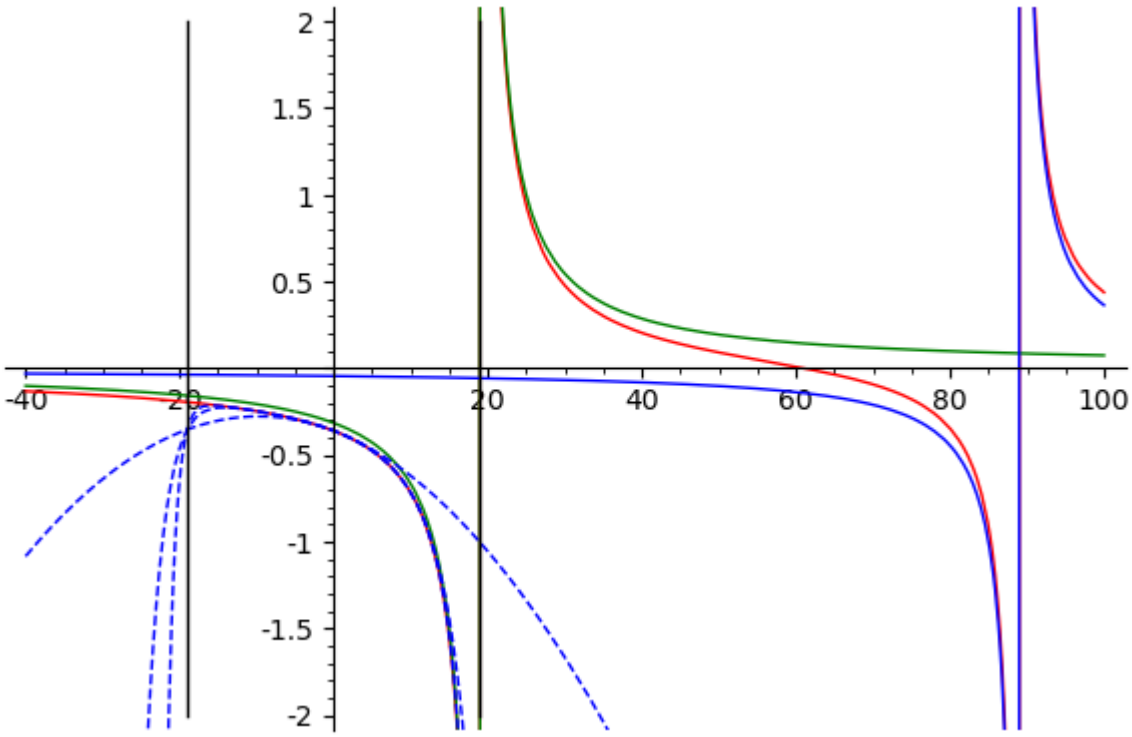
$$-\frac{4257250}{4835382371}\,x^2 - \frac{48970}{2859481}\,x - \frac{610}{1691}$$

In [13]:
```
#radius of convergence
min(19,89)
```

Out[13]:
```
19
```

In [14]:
```
#showing plots and radius of convergence
p1=plot(f(x),x,-40,100, rgbcolor="red")
p2=plot(term1(x),x,-40,100, rgbcolor="green")
p3=plot(term2(x),x,-40,100, rgbcolor="blue")
term1t2=taylor(term1(x),x,0,2)
term1t10=taylor(term1(x),x,0,10)
term1t20=taylor(term1(x),x,0,20)
term2t2=taylor(term2(x),x,0,2)
term2t10=taylor(term2(x),x,0,10)
term2t20=taylor(term2(x),x,0,20)
pp1=plot(term1t2(x)+term2t2(x),x,-40,100, linestyle="dashed")
pp2=plot(term1t10(x)+term2t10(x),x,-40,100, linestyle="dashed")
pp3=plot(term1t20(x)+term2t20(x),x,-40,100, linestyle="dashed")
Q1 = line([(19,-2),(19,2)], rgbcolor="black")
Q2 = line([(-19,-2),(-19,2)], rgbcolor="black")
(p1+p2+p3+pp1+pp2+pp3+Q1+Q2).show(ymin=-2,ymax=2)
```

Out[14]:




In [15]:
```
#Question 4
f(x)=x^2-10*x+25
g(x)=-x^2-6*x-10
fdash(x)=diff(f(x),x)
gdash(x)=diff(g(x),x)
var('u v')
parallel=solve(fdash(u)==gdash(v),v)
v=parallel[0].rhs()
show(v)
```

Out[15]:

$$-u + 2$$

In [16]:
```
D(u)=(v-u)^2+(g(v)-f(u))^2
show(expand(D(u)))
```

Out[16]:

$$4\,u^4 - 80\,u^3 + 608\,u^2 - 2048\,u + 2605$$

In [17]:
```
show(factor(diff(D(u),u)))
```

Out[17]:

$$16\left(u^2-11\,u+32\right)(u-4)$$

In [18]:
```
u=solve(diff(D(u),u)==0,u)[2].rhs()
show(u)
```

Out[18]:

$$4$$

In [19]:
```
var('uu')
diff(D(uu),uu,2).subs(uu==u)
```

Out[19]:
```
64
```

In [20]:
```
show(v(u))
```

Out[20]:

$$-2$$

In [21]:
```
show(f(u))
```

Out[21]:

$$1$$

In [22]:
```
show(g(v(u)))
```

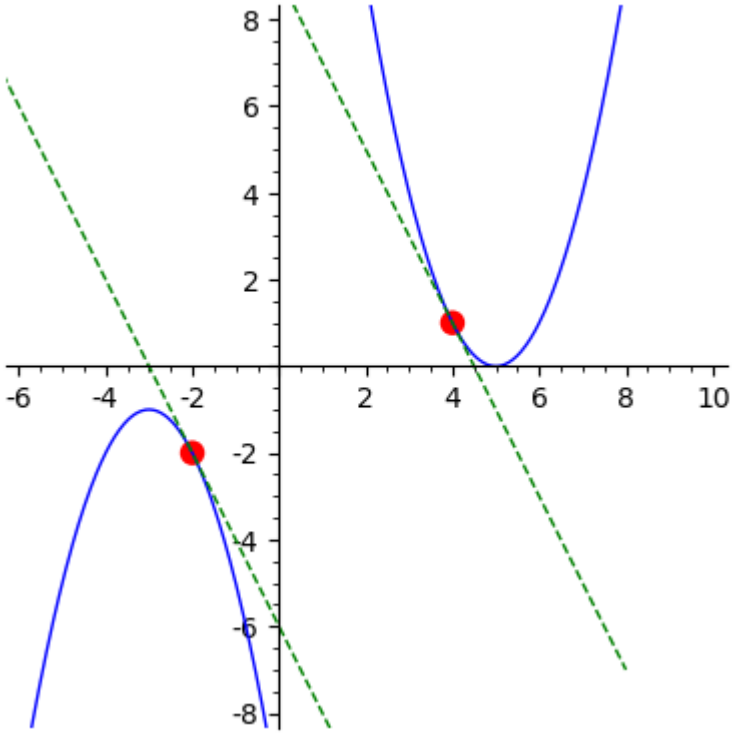
Out[22]:

$$-2$$

In [23]:
```
var('y')
ftangent(x)=solve((y-f(u))/(x-u)==fdash(u),y)[0].rhs()
gtangent(x)=solve((y-g(v(u)))/(x-v(u))==gdash(v(u)),y)[0].rhs()
```

In [24]:
```
p1=plot(f(x),x,-6,10)
p2=plot(g(x),x,-6,10)
p3=plot(ftangent(x),x,-3,8, rgbcolor='green', linestyle = "dashed")
p4=plot(gtangent(x),x,-8,2, rgbcolor='green', linestyle = "dashed")
pt1 = point((u,f(u)), rgbcolor='red', pointsize=80)
pt2 = point((v(u),g(v(u))), rgbcolor='red', pointsize=80)
(p1+p2+p3+p4+pt1+pt2).show(xmin=-6, xmax=10, ymin=-8, ymax=8,aspect_ratio=1)
```

Out[24]:




In [0]: