**The Hong Kong Polytechnic University**
**Department of Applied Mathematics**

# AMA1007 / AMA1120 (Calculus and Linear Algebra)

# Assignment 04

Students should submit their solutions **via Blackboard :**

(1) Sign the covering declaration statement and write your answers with proper steps (but do not include rough work) within the designated area (i.e., inside the designated boxes). Plan your space properly and do not use any other paper.

(2) Use **Microsoft Office Lens** on your mobile device to scan page-by-page into one single clear and readable PDF file, (pages must be in sequence following page numbering, and must be one full page per page scan, and all pages must be in the upright portrait orientation). This Microsoft Office Lens app provides an option to save a copy of your PDF file onto your **PolyU Connect OneDrive**, and you can then manipulate the file from your computer. You must not use any other scanner software or any other app other than Microsoft Office Lens. Check the ordering of pages to make sure it is in sequence.

(3) Make sure your file is of file size no bigger than 3MB, and the

(4) file name must be student's name with surname first.

(5) Then, make submissions from your computer (do not make submissions via your mobile device), submissions must be made **by 5:00pm** on the due date to **Blackboard**.

Solutions with detailed workings, presented in a clear, decent, formal, precise and concise mathematical way, in simple but grammatically correct English are required. Sketch diagrams whenever necessary.

## Covering declaration

By submitting this work through the online system, I affirm on my honour that I am aware of the Regulations on Academic Integrity in Student Handbook and

(i) have not given nor received any unauthorized aid to/from any person or persons, and

(ii) have not used any unauthorized materials in completing my answers to this submission.

Signature: ____________________________________

Name : ______________________________________

Student Number________________________________

**Question 1**

Determine whether the following series converge. Justify.

( a ) $\sum_{n=1}^{\infty} \frac{1-n}{n\,2^n}$

( b ) $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)}$

( c ) $\sum_{n=1}^{\infty} \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{4^n\, 2^n\, n!}$

( d ) $\sum_{n=1}^{\infty} \frac{3^n}{n^3\, 2^n}$

(e) $\sum_{n=3}^{\infty} \frac{1/n}{(\ln n)\sqrt{\ln^2 n - 1}}$

(f) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}+1}{n+1}$

(g) $\sum_{n=1}^{\infty} \frac{(-100)^n}{n!}$

(h) $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln^2 n)}$

( i ) $\sum_{n=1}^{\infty}(-1)^n \ln\left(1+\frac{1}{n}\right)$

**Question 2**

Find the Taylor series generated by

( a ) $f(x) = 2^x$ at $x = 1$

( b ) $f(x) = 2x^3 + x^2 + 3x - 8$ at $x = 1$

**Question 3**

Find the Maclaurin's polynomial of degree 3 for

( a ) $f(x) = (1+x)e^{2x}$

( b ) $f(x) = \ln(3 + e^x)$

**Question 4**

Evaluate the following determinants.

( a ) $\begin{vmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{vmatrix}$

( b ) $\begin{vmatrix} 1 & 3 & 2 & 5 \\ 0 & 0 & 3 & 2 \\ 1 & 5 & 4 & 0 \\ 1 & 2 & 1 & 1 \end{vmatrix}$

( c ) $\begin{vmatrix} 0 & -1 & 2 & 1 \\ -4 & 3 & -3 & 5 \\ 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 1 \end{vmatrix}$

**Question 5**

Let $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$.

Show that $x = 4$ is a root of $\det(\mathbf{A} - x\mathbf{I}) = 0$. Hence find the other roots.

**Question 6**

Solve the following systems using Cramer's rule.

( a ) $$\begin{aligned} 2x_1 + 4x_2 + 6x_3 &= 1 \\ 4x_1 + 6x_2 + 2x_3 &= 3 \\ 6x_1 + 2x_2 + 4x_3 &= 5 \end{aligned}$$

( b ) $\begin{array}{rcrcrcl} 2x_1 & + & 2x_2 & - & x_3 & = & 3 \\ 3x_1 & - & 4x_2 & + & 2x_3 & = & 1 \\ 8x_1 & - & x_2 & - & 3x_3 & = & 4 \end{array}$

**Question 7**

Let $\mathbf{A}$ be a square matrix.

( a ) If $\mathbf{A}^3 = \mathbf{0}$, show that $\mathbf{I} - \mathbf{A}$ is invertible.

( b ) Suppose $\mathbf{A}^3 - \mathbf{A} + \mathbf{I} = \mathbf{0}$. Show that $\mathbf{A}$ is invertible.

**Question 8**

Suppose $f_1(x), f_2(x), g_1(x),$ and $g_2(x)$ are differentiable functions.

Show that, if $W = \begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}$, then $\dfrac{dW}{dx} = \begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$.

**Question 9** 2013/14 Semester 1 AMA1007 Examination Questions 6, 7, and 9.

**Question 10** 2013/14 Semester 2 AMA1007 Examination Questions 3 and 5.

**Question 10** 2013/14 Semester 2 AMA1007 Examination Questions 3 and 5.

**Question 11** Use the open source online software CoCalc to solve the linear system

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 8 \\ 1 & 3 & 4 & 7 \\ 1 & 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

. [Paste the CoCalc output inside the box.]

**END**