

AMA1007 / AMA1120 (Calculus and Linear Algebra)

Assignment 02

Students should submit their solutions **via Blackboard** :

- (1) Sign the covering declaration statement and write your answers with proper steps (but do not include rough work) within the designated area (i.e., inside the designated boxes). Plan your space properly and do not use any other paper.
- (2) Use **Microsoft Office Lens** on your mobile device to scan page-by-page into one single clear and readable PDF file, (pages must be in sequence following page numbering, and must be one full page per page scan, and all pages must be in the upright portrait orientation). This Microsoft Office Lens app provides an option to save a copy of your PDF file onto your **PolyU Connect OneDrive**, and you can then manipulate the file from your computer. You must not use any other scanner software or any other app other than Microsoft Office Lens. Check the ordering of pages to make sure it is in sequence.
- (3) Make sure your file is of file size no bigger than 3MB, and the
- (4) file name must be student's name with surname first.
- (5) Then, make submissions from your computer (do not make submissions via your mobile device), submissions must be made **by 5:00pm** on the due date to **Blackboard**.

Solutions with detailed workings, presented in a clear, decent, formal, precise and concise mathematical way, in simple but grammatically correct English are required. Sketch diagrams whenever necessary.

Covering declaration

By submitting this work through the online system, I affirm on my honour that I am aware of the Regulations on Academic Integrity in Student Handbook and

- (i) have not given nor received any unauthorized aid to/from any person or persons, and
- (ii) have not used any unauthorized materials in completing my answers to this submission.

Signature: _____

Name : _____

Student Number _____

Question 1

Find $f'(x)$ of the following functions.

(a) $f(x) = (\cos x)^{\cos x}$

(b) $f(x) = \cos^{-1}(-3x)$

(c) $f(x) = \ln(1+2^x)$

(d) $f(x) = \log_4(\tan^{-1} x^2)$

Question 2

Let p be a real number and define $y = \left(\frac{x+1}{x-1}\right)^p$ for $x > 1$.

(a) Show that $\frac{dy}{dx} = \frac{-2py}{x^2-1}$.

(b) For $n = 1, 2, \dots, \infty$, show that $(x^2 - 1)y^{(n+1)} + 2(nx + p)y^{(n)} + (n^2 - n)y^{(n-1)} = 0$ where

$$y^{(n)} = \frac{d^n y}{dx^n}. \text{ (Hint: Use Leibniz's rule)}$$

Question 3

Evaluate the following limits (if exist):

(a) $\lim_{x \rightarrow \pi/2} \frac{\cot x}{\cot 3x}$

(b) $\lim_{x \rightarrow 0} \frac{1}{x} \tan \frac{x}{2}$

(c) $\lim_{x \rightarrow \infty} \frac{x + \sqrt{x}}{x + 1}$

(d) $\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right)$

Question 4

Sketch the graph of each of the following function. Clearly indicate (if exist) the coordinates of local maxima/minima, inflection points, and equations of asymptotes.

(a) $f(x) = -x^4 + 6x^2 - 4$



(b) $f(x) = x\sqrt{8-x^2}$



(c) $f(x) = \frac{x^2 - 4}{x^2 - 3}$



(d) $f(x) = x^{2/3}(x-5)$



Question 5

A piece of wire of length 60cm is cut into two parts. One part is bent into a square, while the other part is bent into a circle. The length of a side of the square is x cm. Find the value of x which gives the least total area. What is the least total area?

Question 6

Find the dimensions of a right circular cylinder of maximum volume that can be inscribed in a sphere of radius 10 cm. What is the maximum volume?

Question 7

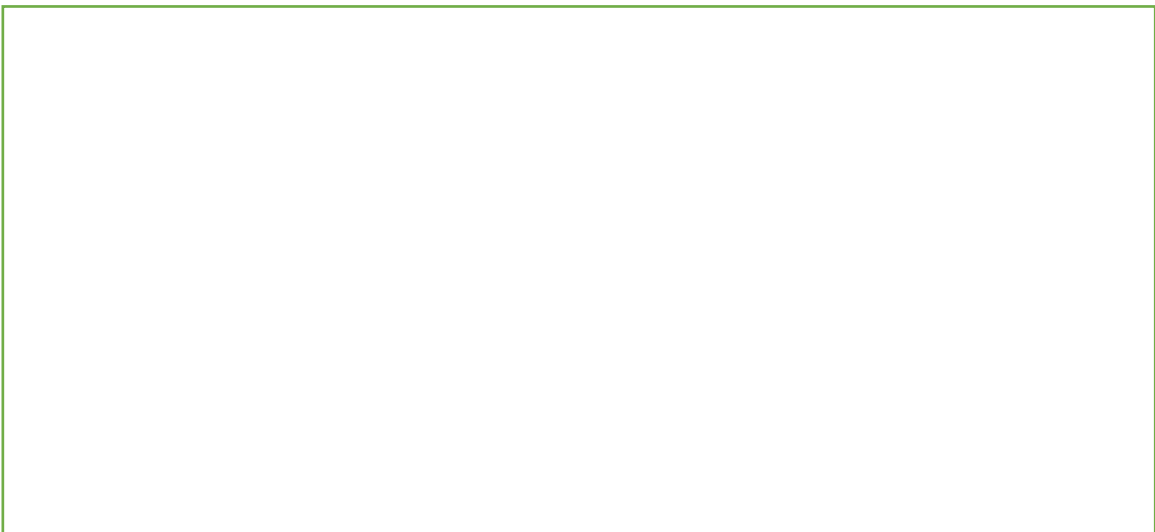
For what values of a , m , and b does the function

$$f(x) = \begin{cases} 3 & x = 0 \\ -x^2 + 3x + a & \text{for } 0 < x < 1 \\ mx + b & 1 \leq x \leq 2 \end{cases}$$

satisfy the hypotheses of the Mean Value Theorem on the interval $[0, 2]$?

**Question 8**

Assume that f and g are differentiable on $[a, b]$ and that $f(a) = g(a)$ and $f(b) = g(b)$. Show that there is at least one point between $x = a$ and $x = b$ where the tangents to the graphs of f and g are either parallel or the same line.



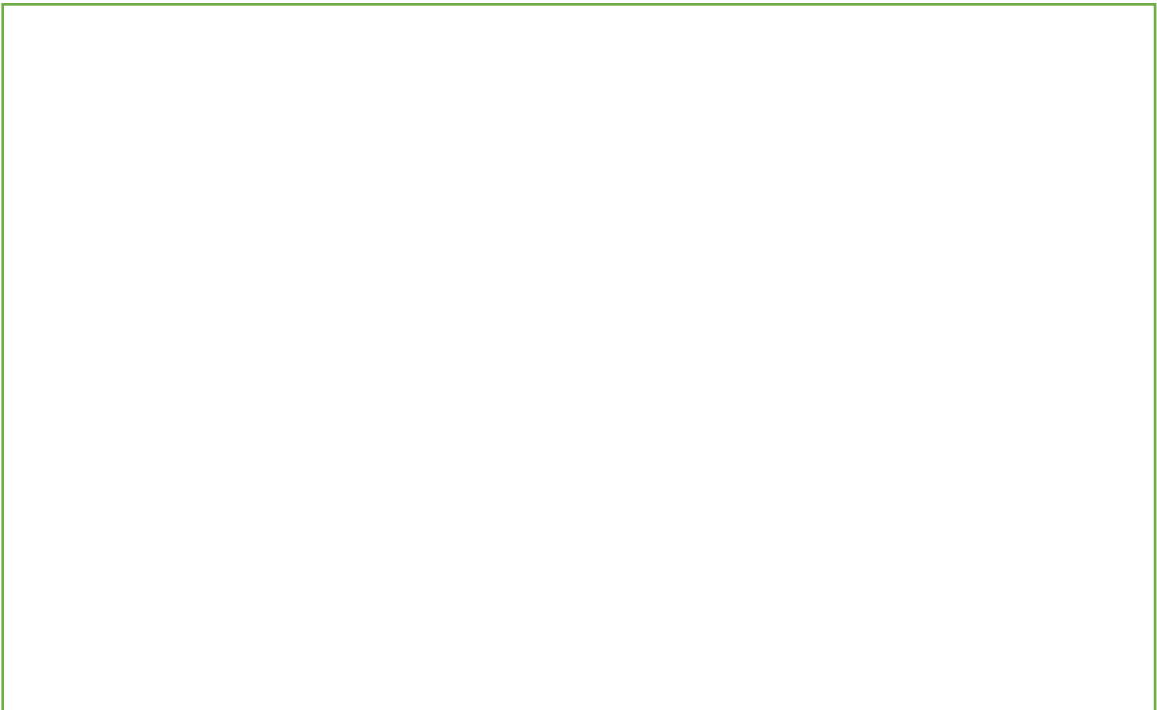
Question 9

Evaluate the following integral:

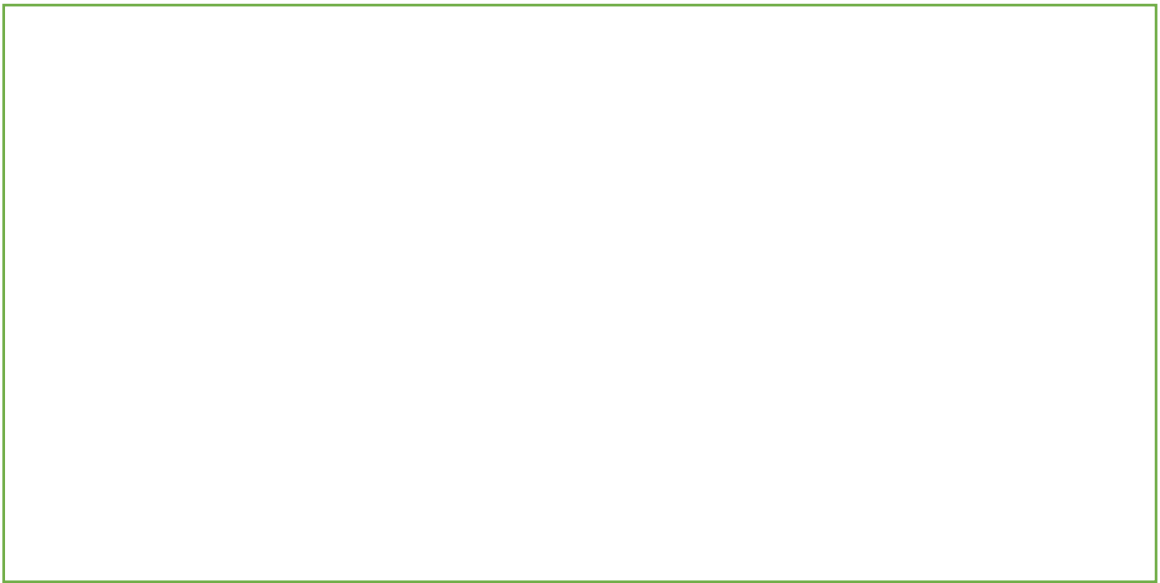
(a) $\int x^5 \sqrt{x^2 - 1} dx$



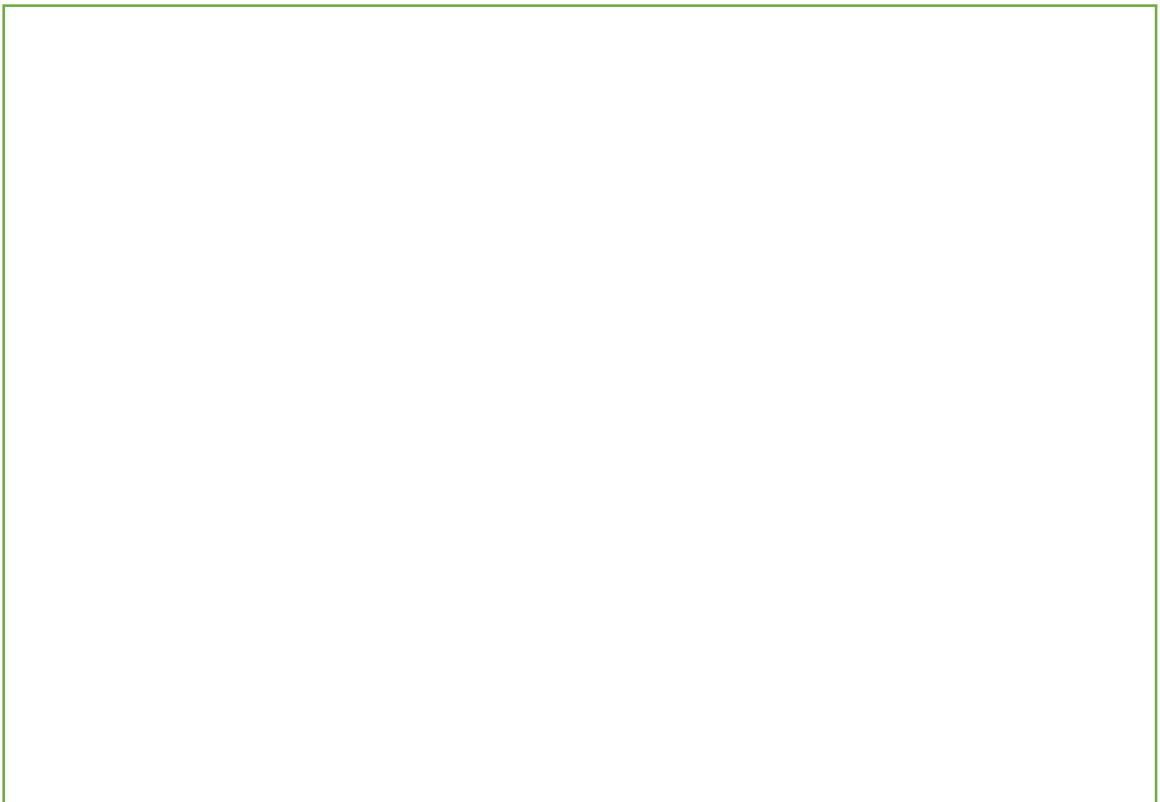
(b) $\int x \cos x^2 dx$



(c) $\int \frac{1}{\sqrt{4x-x^2}} dx$



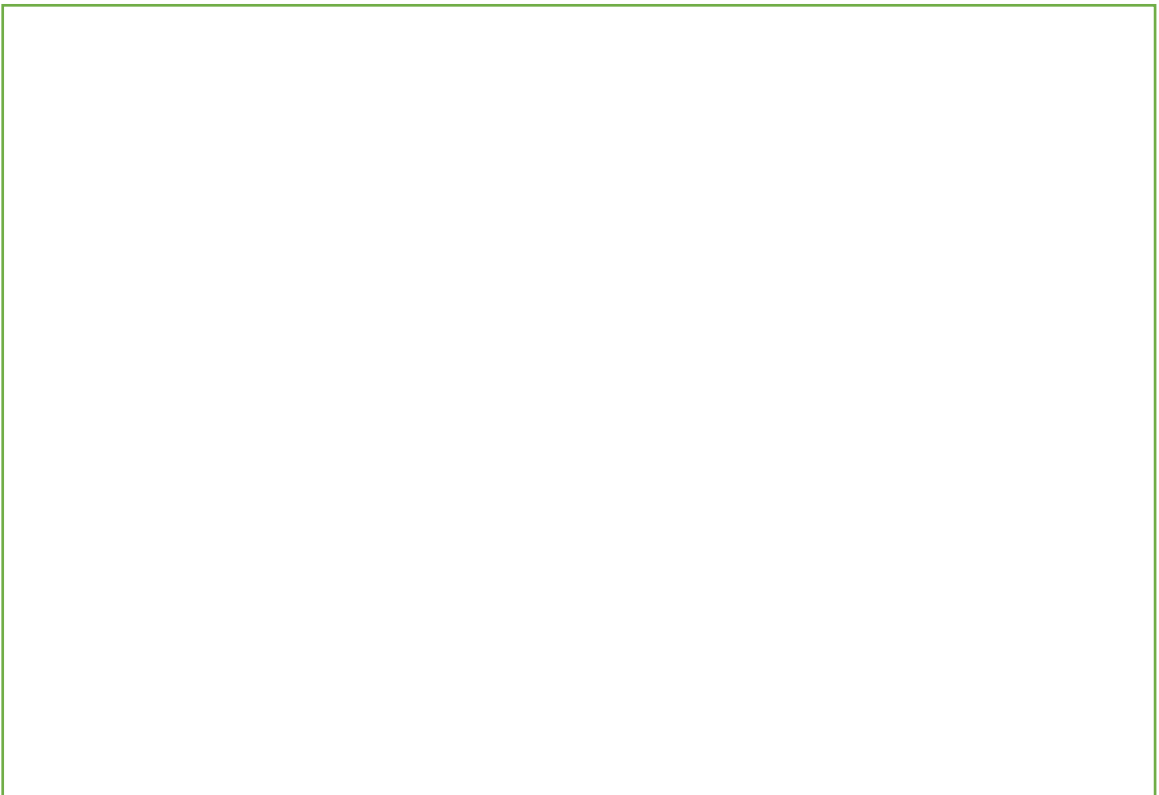
(d) $\int x^2 e^{4x} dx$



(e) $\int e^x \ln(1+e^x) dx$



(f) $\int \cos\sqrt{x} dx$



Question 10 2013/14 Semester 1 AMA1007 Examination Questions 4 and 5(a).





Question 11 2014/15 Semester 2 AMA1007 Examination Questions 4 and 5.





Question 12 Consider $g(x) = \frac{x + 1}{x^2 + 9}$.

Use the open source online software CoCalc to evaluate the integral $\int g(x) dx$, and plot the graph of $g(x)$ against x for $-15 \leq x \leq 15$. [Paste the CoCalc output inside the box.]



END