

Two Integration Examples:

Example 1. Evaluate $\int_0^1 \sqrt{\frac{(ax+b)^2}{cx}} dx$, where c is positive.

Solution

Note that

$$\sqrt{\frac{(ax+b)^2}{cx}} = \frac{ax+b}{\sqrt{c}\sqrt{x}} = \frac{a}{\sqrt{c}}x^{\frac{1}{2}} + \frac{b}{\sqrt{c}}x^{-\frac{1}{2}}.$$

Therefore

$$\begin{aligned} \int_0^1 \sqrt{\frac{(ax+b)^2}{cx}} dx &= \frac{a}{\sqrt{c}} \int_0^1 x^{\frac{1}{2}} dx + \frac{b}{\sqrt{c}} \int_0^1 x^{-\frac{1}{2}} dx \\ &= \frac{a}{\sqrt{c}} \frac{x^{3/2}}{3/2} \Big|_0^1 + \frac{b}{\sqrt{c}} \frac{x^{1/2}}{1/2} \Big|_0^1 \\ &= \frac{2a}{3\sqrt{c}} + \frac{2b}{\sqrt{c}}. \end{aligned}$$

Example 2. Suppose $y = f(x) = x^\beta$ where $x \geq 0$, and $\beta > 1$. Consider the inverse function $x = f^{-1}(y)$. Integrate the inverse with respect to y , from $y = 0$ to $y = 1$.

Solution:

$$\begin{aligned} \int_0^1 f^{-1}(y) dy &= \int_0^1 y^{1/\beta} dy \\ &= \frac{y^{(1/\beta)+1}}{(1/\beta)+1} \Big|_0^1 \\ &= \frac{1}{(1/\beta)+1} = \frac{\beta}{1+\beta}. \end{aligned}$$

