1. (a) Observe that

$$\theta = \angle POQ \;=\; \angle POR - \angle QOS.$$

Consider the right angle triangle POR:

$$\tan(\angle POR) = \frac{x}{a} \qquad \Longrightarrow \qquad \angle POR = \tan^{-1}\left(\frac{x}{a}\right)$$

Consider the right angle triangle QOS:

$$\tan(\angle QOS) = \frac{x}{a+b} \qquad \Longrightarrow \qquad \angle QOS = \tan^{-1}\left(\frac{x}{a+b}\right)$$

Therefore,

$$\begin{aligned} \theta = \angle POQ \;&=\; \angle POR - \angle QOS \;=\; \tan^{-1}\left(\frac{x}{a}\right) - \tan^{-1}\left(\frac{x}{a+b}\right) \\ &=\; \tan^{-1}\left(\frac{x}{10}\right) - \tan^{-1}\left(\frac{x}{40}\right). \end{aligned}$$

(b) Finding the maximum of $\theta(x)$ where

$$\theta(x) \;=\; \tan^{-1}\left(\frac{x}{10}\right) - \tan^{-1}\left(\frac{x}{40}\right).$$

By differentiating $\theta(x)$ with respect to x, we have

$$\begin{aligned} \theta'(x) = \frac{d\theta(x)}{dx} \;&=\; \frac{d}{dx}\left[\tan^{-1}(\frac{x}{10}) - \tan^{-1}(\frac{x}{40})\right] \\ &=\; \left[\frac{1}{1+(\frac{x}{10})^2}\right]\cdot\frac{d}{dx}\left[\frac{x}{10}\right] \;-\; \left[\frac{1}{1+(\frac{x}{40})^2}\right]\cdot\frac{d}{dx}\left[\frac{x}{40}\right] \\ &=\; \frac{\frac{1}{10}}{1+(\frac{x}{10})^2} - \frac{\frac{1}{40}}{1+(\frac{x}{40})^2} \;=\; \frac{\frac{1}{10}}{\frac{100+x^2}{100}} - \frac{\frac{1}{40}}{\frac{1600+x^2}{1600}} \\ &=\; \frac{10}{100+x^2} - \frac{40}{1600+x^2} \;=\; \frac{16000+10x^2-4000-40x^2}{(100+x^2)(1600+x^2)} \\ &=\; \frac{12000-30x^2}{(100+x^2)(1600+x^2)} \;=\; \frac{30(400-x^2)}{(100+x^2)(1600+x^2)}. \end{aligned}$$

The maximum occurs at x where x satisfies the 1st order necessary condition of optimality $\frac{d}{dx}\Big[\;\theta(x)\;\Big] = 0$, thus we have

$$\begin{aligned} (400-x^2) \;&=\; 0 \\ \Longrightarrow \qquad x^2 \;&=\; 400 \\ \Longrightarrow \qquad x \;&=\; 20 \qquad (\textit{reject} \quad x=-20). \end{aligned}$$

x should be non-negative, otherwise the flying camera will fly pass the banner. Observe that $\theta'(x) > 0$ when $0 \le x < 20$, and $\theta'(x) < 0$ when $x > 20$, we can conclude that $x = 20$ is global maximum for all $x \ge 0$.