THE HONG KONG POLYTECHNIC UNIVERSITY

Department of Applied Mathematics

Subject Code: AMA1007 **Subject Title:** Calculus and Linear Algebra

Session: Semester 1, 2012/2013

Date: January 9, 2013 **Time:** 12:30 - 14:30

Time Allowed: 2 hours

This question paper has 3 pages (including this page)

Instructions: This paper has **10** questions.

Attempt **ALL** questions in this paper.

Subject Examiners: Dr. LEE Heung Wing Joseph and Dr. LEE Yu Chung Eugene

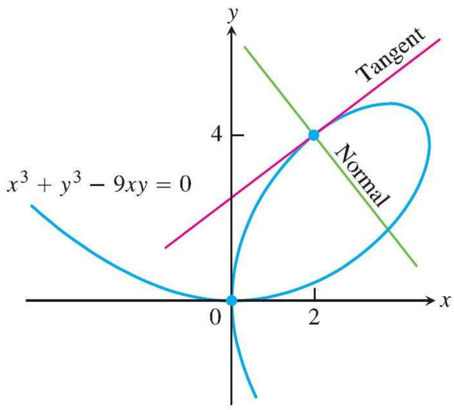
DO NOT TURN OVER THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1. Show that the straight line $\frac{x}{a}+\frac{y}{b}=2$ touches the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ tangentially at (a,b), for any integer $n>2$. **[5 points]**

2. Sketch the graph $y=\frac{x^2+4}{2x}$. **[15 points]**

3. Find all the eigenvalues and the assocoated eigen-vectors of $\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. **[10 points]**

4. Find the equation of the tangent and the normal to the folium of Descartes $x^3+y^3-9xy=0$ at $(2,4)$. **[10 points]**



5. Evaluate $\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx$. **[5 points]**

6. Evaluate $\int \frac{2x^3-4x^2-x-3}{x^2-2x-3}\,dx$. **[10 points]**

7. Consider $f(x)=\frac{1}{x\cdot[\ln(x)]^2}$ for $x\geq 2$.

   (a) Find $\lim_{n\to\infty}\int_2^n f(x)dx$. **[10 points]**

   (b) Determine if $\sum_{n=2}^{\infty} f(n)$ is convergent or not. **[5 points]**

8. Let A and B be square matrices with the same size.

   (a) Give an example in which $(A+B)^2\neq A^2+2AB+B^2$. **[5 points]**

   (b) Give a valid expression for $(A+B)^2$ for all choices of A and B. **[5 points]**

9. Consider a 9×9 lower triangular matrix A,

$$A = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & & \vdots \\ \vdots & & \ddots & 0 \\ a_{91} & a_{92} & \cdots & a_{99} \end{bmatrix}.$$

   (a) Find the determinant of A in terms of a_{ij}. **[5 points]**

   (b) State the condition in terms of a_{ij} that A is not invertible. **[5 points]**

10. Solve the homogeneous system $\mathbf{Ax} = \mathbf{0}$ with

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & -2 & 0 \\ -2 & 3 & -1 \\ -1 & 2 & 0 \\ 2 & -3 & 1 \end{bmatrix},$$

and describe the solution space (e.g. no solution, point solution, line solution, or plane solution etc.). **[10 points]**

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