

In [1]:

```
A=matrix([[2,1,0],[0,3,1],[1,0,4]])
show(A)
```

Out[1]:

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 1 \\ 1 & 0 & 4 \end{pmatrix}$$

In [2]:

```
b=vector([1,1,1]).column()
show(b)
```

Out[2]:

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

In [3]:

```
Ab=A.augment(b)
show(Ab)
```

Out[3]:

$$\begin{pmatrix} 2 & 1 & 0 & 1 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 4 & 1 \end{pmatrix}$$

In [4]:

```
# elementary matrix E1, multiply row 1 with 1/2
E1=identity_matrix(3);
E1 = E1.change_ring(QQ)
E1.rescale_row(0,1/2)
show(E1)
```

Out[4]:

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In [5]:

```
show(E1*Ab)
```

Out[5]:

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 4 & 1 \end{pmatrix}$$

In [6]:

```
# elementary matrix E2, minus row 1 from row 3  
E2=identity_matrix(3);  
E2 = E2.change_ring(QQ)  
E2.add_multiple_of_row(2, 0, -1)  
show(E2)
```

Out[6]:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

In [7]:

```
show(E2*E1*Ab)
```

Out[7]:

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 3 & 1 & 1 \\ 0 & -\frac{1}{2} & 4 & \frac{1}{2} \end{pmatrix}$$

In [8]:

```
# elementary matrix E3, multiply row 2 with 1/3  
E3=identity_matrix(3);  
E3 = E3.change_ring(QQ)  
E3.rescale_row(1,1/3)  
show(E3)
```

Out[8]:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In [9]:

```
show(E3*E2*E1*Ab)
```

Out[9]:

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & -\frac{1}{2} & 4 & \frac{1}{2} \end{pmatrix}$$

In [10]:

```
# elementary matrix E4, add 1/2 of row 2 to row 3
E4=identity_matrix(3);
E4 = E4.change_ring(QQ)
E4.add_multiple_of_row(2, 1, 1/2)
show(E4)
```

Out[10]:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{2} & 1 \end{pmatrix}$$

In [11]:

```
show(E4*E3*E2*E1*Ab)
```

Out[11]:

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{25}{6} & \frac{2}{3} \end{pmatrix}$$

In [12]:

```
# elementary matrix E5, multiply row 3 with 6/25
E5=identity_matrix(3);
E5 = E5.change_ring(QQ)
E5.rescale_row(2,6/25)
show(E5)
```

Out[12]:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{6}{25} \end{pmatrix}$$

In [13]:

```
show(E5*E4*E3*E2*E1*Ab)
```

Out[13]:

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{4}{25} \end{pmatrix}$$

In [14]:

```
# elementary matrix E6, minus 1/3 of row 3 from row 2  
E6=identity_matrix(3);  
E6 = E6.change_ring(QQ)  
E6.add_multiple_of_row(1, 2, -1/3)  
show(E6)
```

Out[14]:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 1 \end{pmatrix}$$

In [15]:

```
show(E6*E5*E4*E3*E2*E1*Ab)
```

Out[15]:

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{7}{25} \\ 0 & 0 & 1 & \frac{4}{25} \end{pmatrix}$$

In [16]:

```
# elementary matrix E7, minus 1/2 of row 2 from row 1  
E7=identity_matrix(3);  
E7 = E7.change_ring(QQ)  
E7.add_multiple_of_row(0, 1, -1/2)  
show(E7)
```

Out[16]:

$$\begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In [17]:

```
show(E7*E6*E5*E4*E3*E2*E1*Ab)
```

Out[17]:

$$\begin{pmatrix} 1 & 0 & 0 & \frac{9}{25} \\ 0 & 1 & 0 & \frac{7}{25} \\ 0 & 0 & 1 & \frac{4}{25} \end{pmatrix}$$

In [18]:

```
# double check with rref  
show(Ab.rref())
```

Out[18]:

$$\begin{pmatrix} 1 & 0 & 0 & \frac{9}{25} \\ 0 & 1 & 0 & \frac{7}{25} \\ 0 & 0 & 1 & \frac{4}{25} \end{pmatrix}$$

In [0]: