AMA1007   2013/2014 Semester 2 Examination

(Numerical Answers and Hints)

1. Answer: $\dfrac{1}{2}x^2 + x + \dfrac{4}{3}\ln|x-2| + \dfrac{5}{3}\ln|x+1| + C$

    (Hint: Use partial fractions.)

2. (a) Answer: π, (b) Answer: $\lim\limits_{b\to\infty} 2\pi \ln(b)$ Hence, it has an infinite surface area.

    (Hint: For each of (a) and (b), integrate an appropriate integrand from 1 to b and take the limit of $b \to \infty$.)

3. (a) Hint: Show that $\det(A - \lambda I) = -(\lambda - 1)^2 (\lambda - 10)$.

    (b) Answer: The required line in parametric form is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = s \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

    (Hint: Consider $\lambda = 10$.)

    (c) Answer: The required plane in parametric form is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1/2 \\ 0 \\ 1 \end{bmatrix}$

    (Hint: Consider $\lambda = 1$.)

4. (a) Answer: $\sinh(x);\ \sinh(x)$ (same)

    (b) Answer: $\cosh(x) = \displaystyle\sum_{n=0}^{\infty} \dfrac{x^{2n}}{(2n)!}$

    (c) Answer: $\dfrac{e - 1/e}{2}$

    (Hint: Substitute $y = \cosh(x) = \dfrac{e^x + e^{-x}}{2}$ into the integral $\displaystyle\int_0^1 \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}\, dx$.)

5. Answer: The radius of convergence about $x_0 = -1$ is 2. (Hint: Use the ratio test.)

6. (a) Answer: No. Give a counter example.

   (b) Answer: Yes. Give an explanation.

7. Answer: $$\int_0^{\sqrt[n]{a}} x^n dx = \frac{a^{1+\frac{1}{n}}}{n+1}$$

   $$\int_0^a \sqrt[n]{x}\, dx = a\sqrt[n]{a} - \frac{a^{1+\frac{1}{n}}}{n+1}$$

   Hint for the second integral:

   $$\int_0^a \sqrt[n]{x}\, dx = \int_{y=0}^{y=a} \sqrt[n]{y}\, dy$$

   Consider the region: $0 \le x \le \sqrt[n]{a}, 0 \le y \le a$ and the curve $y = x^n$.

8. Let x, y, and z be the prices of a cow, a sheep and a pig respectively.

   Answer: $x = 1200$, $y = 500$, $z = 300$