AMA1007 2013/2014 Semester 1 Examination

(Numerical Answers and Hints)

1. $\boldsymbol{P_0P_1}$, $\boldsymbol{P_1P_2}$, $\boldsymbol{P_2P_3}$, …., form a geometric sequence. What is the first term? What is the common ratio? Hence, each of the 3 sums required is a geometric series.

   The 3 sums are $\dfrac{a\sin\theta}{1-\cos\theta}$, $a\csc\theta$, and $a\cot\theta$ respectively.

2. (a) divergent, (b) divergent

   Hint: For each of (a) and (b), compare the given integral with an appropriate integral that is divergent by p-test.

3. (c) ≈ 0.7475

4. $c = \pm\dfrac{1}{\sqrt[4]{5}}$

5. (a) Hint: Apply the product rule of differentiation and the identity $\sin^2\theta + \cos^2\theta = 1$

   (b) Hint: Show that arc length is given by $L = \int_\alpha^\beta \sqrt{\left(\dfrac{dx}{d\theta}\right)^2 + \left(\dfrac{dy}{d\theta}\right)^2}\, d\theta$ and hence obtain the required expression.

   (c) The required arc length is 8. (Hint: Find $r'(\theta)$. Substitute $\alpha = 0$ and $\beta = 2\pi$ into the expression in (b) and simplify the integrand to ease integration with the given hint.)

6. Let x, y, and z be the measures of the first, second and third classes of corn.

   Answer: $x = \dfrac{37}{4}$, $y = \dfrac{17}{4}$, $z = \dfrac{11}{4}$

7. The inverse is $\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (Hint: Example 4.6 page-491)

8. Hint: Show that the scalar triple product $\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = 0$, and hence the four given points lie on the same plane.

9. The eigenvalues are $\lambda = 1,\ \lambda = -2,\ \lambda = -1$.

For $\lambda = 1$, $\begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

For $\lambda = -2$, $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

For $\lambda = -1$, $\begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

10. Answer: π (Hint: Evaluate the integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$.)