Erratum to: On the Equivalence of Inexact Proximal ALM and ADMM for a Class of Convex Composite Programming

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Abstract

The original paper [1] has a gap in the proof of Lemma 3.3. Here we make the corresponding corrections.

1 Introduction

The original paper [1] has a gap in the proof of Lemma 3.3. The reason is that there in the inequality (3.25), we have made the following estimation:

$$\|(\widehat{\Sigma}_h + \mathcal{S} - \Sigma_k)(w^{k+1} - w^k)\|^2 \leq \|\widehat{\Sigma}_h + \mathcal{S}\|\langle w^{k+1} - w^k, (\widehat{\Sigma}_h + \mathcal{S})(w^{k+1} - w^k)\rangle. \tag{R1}$$

However, such an estimation may not hold in general. Here, we will prove the following inequality (by adding a factor of 4 to the right hand of (R1))

$$\|(\widehat{\Sigma}_h + \mathcal{S} - \Sigma_k)(w^{k+1} - w^k)\|^2 \leq 4\|\widehat{\Sigma}_h + \mathcal{S}\|\langle w^{k+1} - w^k, (\widehat{\Sigma}_h + \mathcal{S})(w^{k+1} - w^k)\rangle. \tag{R2}$$

With this new inequality, all the conclusions in [1] remain true. The details for the changes will be provided in the next section. In the final section, we give the proof to (R2).

2 Revision details

Besides (R2), we make the following corrections to the original paper:

(1) In (3.23) of Lemma 3.3, we need revise the term $\frac{2\|\widehat{\Sigma}_h+\mathcal{S}\|}{\tau\sigma}\|w^{k+1} - w^k\|^2_\Theta$ to be $\frac{8\|\widehat{\Sigma}_h+\mathcal{S}\|}{\tau\sigma}\|w^{k+1} - w^k\|^2_\Theta$.

(2) The term $\frac{2\|\widehat{\Sigma}_h+\mathcal{S}\|}{\tau\sigma}\|\bar{w}^{k+1} - w^k\|^2_\Theta$ in (3.34) should be replaced by $\frac{8\|\widehat{\Sigma}_h+\mathcal{S}\|}{\tau\sigma}\|\bar{w}^{k+1} - w^k\|^2_\Theta$.

(3) Between (3.26) and (3.27), we should let $\zeta > 0$ be the smallest real number such that $\zeta\Xi \succeq 4\Theta$ instead of $\zeta\Xi \succeq \Theta$.

There is nothing more to be revised.

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3 Proof for (R2)

By using the assumption $\mathcal{S} \succeq -\frac{1}{2}\widehat{\Sigma}_h$ and $0 \preceq \Sigma_k \preceq \widehat{\Sigma}_h$, we have

$$0 \preceq \widehat{\Sigma}_h + \mathcal{S} - \frac{1}{2}\Sigma_k \preceq \widehat{\Sigma}_h + \mathcal{S} \quad \text{and} \quad 0 \preceq \frac{1}{2}\Sigma_k \preceq \widehat{\Sigma}_h + \mathcal{S},$$

which imply

$$\begin{aligned}
&\|(\widehat{\Sigma}_h + \mathcal{S} - \Sigma_k)(w^{k+1} - w^k)\|^2 \\
&= \|(\widehat{\Sigma}_h + \mathcal{S} - \tfrac{1}{2}\Sigma_k)(w^{k+1} - w^k) - \tfrac{1}{2}\Sigma_k(w^{k+1} - w^k)\|^2 \\
&\leq 2\|(\widehat{\Sigma}_h + \mathcal{S} - \tfrac{1}{2}\Sigma_k)(w^{k+1} - w^k)\|^2 + 2\left\|(-\tfrac{1}{2}\Sigma_k)(w^{k+1} - w^k)\right\|^2 \\
&= 2\|(\widehat{\Sigma}_h + \mathcal{S} - \tfrac{1}{2}\Sigma_k)(w^{k+1} - w^k)\|^2 + 2\left\|\tfrac{1}{2}\Sigma_k(w^{k+1} - w^k)\right\|^2 \\
&\leq 2\|\widehat{\Sigma}_h + \mathcal{S} - \tfrac{1}{2}\Sigma_k\|\langle w^{k+1} - w^k, (\widehat{\Sigma}_h + \mathcal{S} - \tfrac{1}{2}\Sigma_k)(w^{k+1} - w^k)\rangle + 2\|\tfrac{1}{2}\Sigma_k\|\langle w^{k+1} - w^k, \tfrac{1}{2}\Sigma_k(w^{k+1} - w^k)\rangle \\
&\leq 2\|\widehat{\Sigma}_h + \mathcal{S} - \tfrac{1}{2}\Sigma_k\|\langle w^{k+1} - w^k, (\widehat{\Sigma}_h + \mathcal{S})(w^{k+1} - w^k)\rangle + 2\|\tfrac{1}{2}\Sigma_k\|\langle w^{k+1} - w^k, (\widehat{\Sigma}_h + \mathcal{S})(w^{k+1} - w^k)\rangle \\
&\leq 2\|\widehat{\Sigma}_h + \mathcal{S}\|\langle w^{k+1} - w^k, (\widehat{\Sigma}_h + \mathcal{S})(w^{k+1} - w^k)\rangle + 2\|\widehat{\Sigma}_h + \mathcal{S}\|\langle w^{k+1} - w^k, (\widehat{\Sigma}_h + \mathcal{S})(w^{k+1} - w^k)\rangle,
\end{aligned}$$

where the last inequality follows from the simple fact that for any two self-adjoint and positive semidefinite linear operators $\mathcal{A}$ and $\mathcal{B}$, one has $\|\mathcal{A}\| \geq \|\mathcal{B}\|$ if $\mathcal{A} \succeq \mathcal{B}$. This completes the proof of (R2).

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References

[1] L. Chen, X.D. Li, D.F. Sun, and K.-C. Toh: On the equivalence of inexact proximal ALM and ADMM for a class of convex composite programming, Math. Program., 2021, 185: 111-161