





DEPARTMENT OF APPLIED MATHEMATICS 應用數學系

# 2016 Workshop on Stochastic Control and Financial Applications

# **Objective**

The purpose of this workshop is to bring together scholars working in stochastic control and related fields to review some recent development and explore exciting new directions in stochastic control theories and financial applications.

# Time and Venue

Time: 9:00-18:00, August 16-17, 2016 Venue: Room Y305, The Hong Kong Polytechnic University, Hong Kong

# **Speakers**

Xi-Ren Cao
Arturo Kohatsu Higa
Naoyuki Ichihara
Jian Song
Lixin Wu
Hailiang Yang

- Min Dai Ying Hu Lingfei Li Qingshuo Song Qi Wu Xunyu Zhou
- Xuefeng Gao Tomoyuki Ichiba Xianhua Peng Ruodu Wang Jie Xiong

# **Organizers**

Ying Hu, James Huang, Xun Li, Zuo Quan Xu, Xiang Yu.

## **Sponsors**

AMSS-PolyU Joint Research Institute Department of Applied Mathematics, The Hong Kong Polytechnic University

## Enquiries

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# All are welcome!

# 2016 Workshop on Stochastic Control and Financial Applications

## Schedule

	August 16	August 17
8:50 - 9:00	Welcome	
9:00 - 9:35	Arturo Kohatsu-Higa	Hailiang Yang
9:35 - 10:10	Ying Hu	Qingshuo Song
10:10 - 10:50	Coffee Break	Coffee Break
10:50 - 11:25	Ruodu Wang	Xunyu Zhou
11:25 - 12:00	Xuefeng Gao	Min Dai
12:00 - 14:00	Lunch	Lunch
14:00 - 14:35	Xi-Ren Cao	Lixin Wu
14:35 - 15:10	Naoyuki Ichihara	Lingfei Li
15:10 - 15:50	Coffee Break	Coffee Break
15:50 - 16:25	Jian Song	Tomoyuki Ichiba
16:25 - 17:00	Qi Wu	Xianhua Peng
17:00 - 17:35	Jie Xiong	
17:35 - 19:30	Dinner	

August 16, Morning session chair: Xun Li

August 16, Afternoon session chair: James Huang

August 17, Morning session chair: Zuo Quan Xu

August 17, Afternoon session chair: Xiang Yu

Venue: Room Y305, The Hong Kong Polytechnic University, Hong Kong

### An Introduction to an Alternative Approach To Optimization and Its Application to Stochastic Control

Xi-Ren Cao, Shanghai Jiao Tong University

#### Abstract

The direct-comparison (DC) based approach was proposed as an alternative to dynamic programming (DP) in optimization. With DP one partitions the time-state space horizontally in time, while with DC, one is in effect partitioning with the sample paths vertically. An analogy is the Riemann integration vs Lebesgue integration. The central piece of DC is the performance difference formula (PDF), which provides the difference of the performance measures under any two policies in the entire finite of infinite time horizon. DP works at a particular time instant and therefore is based on local information, while DC contains global information. Applying DC to stochastic control problems, we find that the PDF contains a term that measures the effect of the non-smooth points in [t, t+dt], which is at the order of square root of dt and cannot be captured by derivatives in DP. This feature allows us to derive the optimality condition at a class of non-smooth points and to study the control of degenerate diffusions and other problems. Viscosity solution is not needed. Other advantages of applications of the DC approach will also be discussed.

### Singular Control Approximation in Portfolio Selection with Capital Gains Tax

Min Dai, National University of Singapore

#### Abstract

We consider a singular stochastic control problem arising from continuous time investment and consumption with capital gains tax, where the associated Hamilton-Jacobi-Bellman (HJB) equation admits infinitely many solutions. In terms of an explicit construction, we show that the optimal strategy can be approximated by a sequence of sub-optimal strategies related to regular control. As far as we know, this is the first paper to explicitly construct such approximations in the singular control literature. Since the constructed sub-optimal strategies lead to a penalized approximation of the HJB equation, our result demonstrates that the penalty method can still work for numerical solutions, even if the HJB equation lacks uniqueness of solutions. We also show that the resulting value function corresponds to the minimal viscosity solution of the HJB equation. Our approach can be extended to general singular stochastic problems. This work is jointly with Baojun Bian and Xinfu Chen.

### Optimal Timing for Spread Crossing in a Limit Order Book

Xuefeng Gao, The Chinese University of Hong Kong

#### Abstract

We study when a precommitted trader converts a limit order to a market order in algorithmic executions of orders. We formulate the problem as an optimal stopping problem. We present structural properties of the optimal strategy and show how it depends on market conditions. We also study the optimal spread crossing problem under a regime switching model for the fundamental value of an asset. Our numerical experiments illustrate how the the optimal spread crossing time depends on the stock price and the belief of the trader on the state of the market. This is a joint work with Nan Chen and Xiang Ma.

### Backward Stochastic Differential Equations with Mean Reflection

Ying Hu, Université Rennes 1

#### Abstract

We study a new type of backward stochastic differential equation, where the distribution of the Y-component of the solution is required to satisfy an additional constraint, written in terms of the expectation of a loss function. This constraint is imposed at any deterministic time t and is typically weaker than the classical pointwise one associated to reflected backward stochastic differential equations. Focusing on solutions (Y, Z, K) with deterministic K, we obtain the well-posedness of such equation, in the presence of a natural Skorokhod type condition. Such condition indeed ensures the minimality of the enhanced solution, under an additional structural condition on the driver. Our results extend to the more general framework where the constraint is written in terms of a static risk measure on Y. In particular, we provide an application to the super hedging of claims under running risk management constraint. This is a joint work with Philippe Briand (Universit de Savoie) and Romuald Elie (Universit Paris-Est).

### Large Banking System with Defaults

Tomoyuki Ichiba, University of California Santa Barbara

#### Abstract

We shall consider a large system of diffusions with drifts of mean-field type and with self-exciting spikes for modelling systemic risk of banking system. Here self-exciting spikes depict banks' default and start of new banks. For the sake of simplicity we consider the case that after banks default, exactly the same number of new banks are introduced, so that the number of banks in the system is stable in time. We examine convergence of system to the McKean-Vlasov dynamics, derive stationary distributions, solve a stationary mean field game problem and then discuss related problems. This is joint work with Romuald Elie and Mathieu Lauriere.

### Generalized Principal Eigenvalues for Superquadratic Viscous Hamilton-Jacobi Equations

Naoyuki Ichihara, Aoyama Gakuin University

#### Abstract

This talk is concerned with the ergodic problem for superquadratic viscous Hamilton-Jacobi equations. We investigate some qualitative properties of the generalized principal eigenvalue with respect to a perturbation of the potential function. It turns out that certain phase transition phenomena take place with respect to such perturbation, and that different situations are observed according to the exponent of nonlinearity. A part of this talk is based on the joint work with Emmanuel Chasseigne (University of Tours).

### Probabilistic Interpretation of the Parametrix Method

Arturo Kohatsu-Higa, Ritsumeikan University

#### Abstract

The parametrix method has been used for many purposes and many different variations of it have been introduced in the past. In this talk, I will present some of the probabilistic interpretations and some of its applications that I have found interesting for me in the recent past.

### Error Analysis of Finite Difference and Markov Chain Approximations for Option Pricing

Lingfei Li, The Chinese University of Hong Kong

#### Abstract

Mijatovic and Pistorius (Math. Finance, 2013) proposed an efficient Markov chain approximation method for pricing European and barrier options in general one-dimensional Markovian models, however sharp convergence rates of this method for realistic financial payoffs which are non-smooth are rarely available. In this paper, we solve this problem for general one-dimensional diffusion models, which play a fundamental role in financial applications. For such models, the Markov chain approximation method is equivalent to the method of lines using central difference. Our analysis is based on the spectral representation of the exact solution and the approximate solution. By establishing the convergence rate for the eigenvalues and the eigenfunctions, we obtain sharp convergence rates for the transition density and the price of options with non-smooth payoffs. In particular, we have shown that for call/put-type payoffs, convergence is second order, while for digital-type payoffs, convergence is only first order in general. Furthermore, we provide theoretical justification for two well-known smoothing techniques that can restore second order convergence for digital-type payoffs and explain oscillations observed in the convergence for options with non-smooth payoffs. As an extension, we also establish sharp convergence rates for European options in a rich class of Markovian jump models constructed from diffusions via subordination. The theoretical estimates are confirmed by numerical examples.

### EM Algorithm and Stochastic Control

Xianhua Peng, Hong Kong University of Science and Technology

#### Abstract

We propose an algorithm called EM-Control (EM-C) algorithm to solve multi-period finite-time horizon stochastic control problems, where the optimal policy is not necessarily stationary. Generalizing the idea of the EM algorithm, the EM-C algorithm sequentially update the control parameters in each time period in a time-backward manner. Similar to the EM algorithm, the EM-C algorithm has monotonicity of performance improvement in every iteration, and hence has good convergence properties. We apply the EM-C algorithm to solve stochastic control problems in real business cycle and monopoly pricing of airline tickets, which demonstrates the effectiveness of the algorithm. This is a joint work with Steven Kou and Xingbo Xu.

### Temporal Asymptotics for Fractional Parabolic Anderson Model

Jian Song, The University of Hong Kong

#### Abstract

In this talk, I will first review some recent developments on the parabolic Anderson model which is described by a class of linear stochastic partial differential equations with multiplicative Gaussian noise. Then, I will present my recent work joint with X. Chen, Y. Hu and X. Song. In this work, we consider fractional parabolic equation of the form  $\frac{\partial u}{\partial t} = -(-\Delta)^{\frac{\alpha}{2}}u + u\dot{W}(t,x)$ , where  $-(-\Delta)^{\frac{\alpha}{2}}$  with  $\alpha \in (0,2]$  is a fractional Laplacian and  $\dot{W}$  is a Gaussian noise colored in space and time. The precise moment Lyapunov exponents for the Stratonovich solution and the Skorohod solution are obtained.

### Solvability of Dirichlet problem with Nonlinear Integro-differential Operator

Qingshuo Song, The City University of Hong Kong

#### Abstract

This talk studies the solvability of a class of Dirichlet problem associated with non-linear integro-differential operator. The main ingredient is Perron's method and the probabilistic construction of continuous supersolution via the identification of the continuity set of the exit time operators under Skorohod topology.

### **XVAs:** From Pricing to Management

Lixin Wu, Hong Kong University of Science and Technology

#### Abstract

Valuation adjustments, so-called XVAs, have been a central issue in financial markets in recent years. There exist different and even opposing views on the definition and adoption of some of the XVAs from the viewpoints of pricing, booking or accounting. In this talk, we work under a framework of replication pricing and redefine or reformulate XVAs. We will also discuss the management of the XVA risks.

### Diversification of Portfolio Tail Risk and CCP Margin

Qi Wu, The Chinese University of Hong Kong

#### Abstract

We develop explicit and accurate asymptotic expansions of the portfolio Value-at-Risk (VaR) and portfolio Expected Shortfall (ES) for a large family of multivariate elliptical distributions. The family includes distributions of exponential type such as Kotz distributions, and power type such as the multivariate Student t-distribution. Our results imply that, for a given portfolio, the difference between its ES and VaR depends on the tail heaviness of the joint asset return distribution. In particular, for assets exhibiting exponential tail decay, the ratio between ES and VaR is asymptotically zero, whereas for assets exhibiting power type tail decay, the portfolio ES is strictly larger than its VaR. Meanwhile in the context of joint portfolio margining, the benefits of merging multiple sub-portfolios into a single one, measured in terms of the amount of the risk reduction, depends crucially on the dispersion of the joint asset return distribution, and it is true for VaR and ES.

### Quantile-based Risk Sharing

Ruodu Wang, University of Waterloo

#### Abstract

We address the problem of risk sharing among players using a two-parameter class of quantile-based risk measures, the so-called Range-Value-at-Risk (RVaR), as their preferences. The family of RVaR includes the Value-at-Risk (VaR) and the Expected Shortfall (ES), the two popular and competing regulatory risk measures, as special cases. We first establish an inequality for RVaR aggregation, showing that a special form of subadditivity is satisfied by RVaR. Then, the risk sharing problem is solved by explicit construction. Three relevant issues in the optimal allocations are investigated: extra sources of randomness, comonotonicty, and model uncertainty. We show that in general, a robust optimal allocation exists if and only if none of the underlying risk measures is a VaR. Practical implications of main results for risk management and policy makers are discussed, including gambling behaviour, moral hazard, regulatory arbitrage, and model misspecification. In particular, in the context of regulatory capital reduction, we provide some general guidelines on how a regulatory risk measure can lead to certain desirable or undesirable properties of risk sharing among firms, and show many novel advantages of ES from the perspective of a regulator.

### An introduction to stochastic filtering and optimal control

Jie Xiong, University of Macau

#### Abstract

In this talk, we will first introduce the theory of nonlinear filtering. Then, we will introduce the stochastic maximum principle for optimal control problems. A few examples from mathematical finance will be considered. Finally, we will solve a coupled filteringcontrol problem explicitly.

### Geometric Stopping of a Random Walk and Its Applications to Valuing Equity-linked Death Benefits

Hailiang Yang, The University of Hong Kong

#### Abstract

We study discrete-time models in which death benefits can depend on a stock price index, the logarithm of which is modeled as a random walk. Examples of such benefit payments include put and call options, barrier options, and lookback options. Because the distribution of the curtate-future-lifetime can be approximated by a linear combination of geometric distributions, it suffices to consider curtate-future-lifetimes with a geometric distribution. In binomial and trinomial tree models, closed-form expressions for the expectations of the discounted benefit payment are obtained for a series of options. They are based on results concerning geometric stopping of a random walk, in particular also on a version of the Wiener-Hopf factorization. This is a joint paper with Hans U. Gerber and Elias S.W. Shiu.

### Time Inconsistency, Self Control and Portfolio Choice

Xunyu Zhou, Columbia University

#### Abstract

Time inconsistency arises when one's preferences are not aligned over time; thus timeinconsistent dynamic control is essentially a self control problem. In this talk I will introduce several classes of time-inconsistent dynamic optimisation problems together with their economic motivations, and highlight the ways to address the time inconsistency. I will then provide a solution to a continuous-time portfolio choice model under the rank-dependent utility which is inherently time inconsistent.