PolyU-HKBU-SDU JRC Workshop on
Applied Mathematics with Financial Applications
3 February 2018

Objective
With recent advances in finance technology, it is essential to develop a deeper understanding of the portfolio management and trading process. The aim of this workshop is to bring together researchers in financial mathematics from Shangdong University (SDU) and PolyU to present and discuss recent developments in financial mathematics, both from a theoretical and a practical point of view. This workshop will encourage collaboration between the two Universities.

Details
Date: 3 February 2018 (Saturday)
Venue: Room TU/801, PolyU
Enquiry: Miss Natalie Cheung  Tel: (852) 2766 6938; Email: Natalie.cheung@polyu.edu.hk
Sponsor: The PolyU-HKBU-SDU Joint Research Center on Financial Mathematics
Website: http://www.polyu.edu.hk/ama/joint_research_center/

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<td>PolyU</td>
<td>Multigrid methods for Maxwell’s equations and a fourth-order curl problem based on Hodge decomposition</td>
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<td>Li Xun</td>
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<td>Lu Siyu</td>
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<td>How to efficiently construct a large-scale valid and optimal correlation matrix?</td>
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<td>Wang Guangzhen</td>
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<td>Backward Stochastic Differential Equations and Optimal Switching Problem with Two-Time-Scale Markov Chains: Weak Convergence and Stock Trading</td>
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All are welcome
# PolyU-HKBU-SDU JRC Workshop on Applied Mathematics with Financial Applications

Feb 3, 2018

**Venue**: TU801

## Session 1  Chair: Yiu Ka Fai Cedric

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<td>10:00-10:30</td>
<td>Sun Defeng (PolyU)</td>
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<td>Yu Xiang (PolyU)</td>
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<td>Cedric Yiu (PolyU)</td>
<td>Optimal portfolio and insurance problems under a value-at-risk constraint</td>
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Backward Stochastic Differential Equations and Optimal Switching Problem with Two-Time-Scale Markov Chains:
Weak Convergence and Stock Trading

Zhen Wu
School of Mathematics
Shandong University, Jinan

Abstract: This talk is concerned with backward stochastic differential equations (BSDEs) and optimal switching problem coupled by a continuous-time finite-state Markov chain which has a two-time-scale structure, i.e., the states of the Markov chain can be divided into a number of groups such that the chain jumps rapidly within a group and slowly between the groups. In this talk, we give some convergence results as the fast jump rate goes to infinity, which can be used to reduce the complexity of the original problem. This method is also referred to as singular perturbation.

The first result is the weak convergence of the BSDEs with two-time-scale Markov chains. It is shown that the solution of the original BSDE system converges weakly under the Meyer-Zheng topology to that of an aggregated BSDE system. Then we focus on the optimal switching problem for regime switching model with two-time-scale Markov chains. We obtain the optimal switching strategy by virtue of dynamic programming principle and prove the convergence of the value function under the two-time-scale structure. Finally, as an application of the theoretical results, an example concerning the stock trading problem in a regime switching market is provided. Both the optimal trading rule and convergence result are numerically demonstrated in this example.

Maximum principle for partially observed mean-field stochastic differential equation

Guangchen Wang
School of Control Science and Engineering
Shandong University, Jinan

Abstract: This talk is concerned with an optimal control problem driven by mean-field stochastic differential equation, where the information available to the controller is represented by a noisy observation of the state. A maximum principle for optimality is derived and is used to solve a financial problem.
How to efficiently construct a large-scale valid and optimal correlation matrix?

Sun Defeng
Department of Applied Mathematics,
The Hong Kong Polytechnic University

Abstract: In finance, risk management, and many other areas, one often has to deal with invalid correlation matrices. Mathematically, a given symmetric matrix is an invalid correlation matrix if and only if its smallest eigenvalue is negative assuming that all its diagonal entries are ones. The question is how to construct a reasonably good correlation matrix from the given invalid one. Statistically, one could argue that such an invalid correlation matrix is not properly formulated and one should use better statistical methods to reconstruct a valid one. However, in practice, it is more often than not that the reformulated correlation matrix contains quite a number of negative eigen-values albeit of small magnitudes. There are a number of reasons contributing to this phenomenon: insufficient/missing raw data, non-synchronous data, human factors, and so on. In this talk we aim to construct an optimal valid correlation matrix of dimensions up to 10,000 by 10,000 from the observed one by using modern non-smooth optimization theory, in particular, on the second order sparsity of the metric projector over the cone of symmetric and positive semi-definite matrices. Computer codes in Matlab/R/Python for solving the correlation matrix problems will be made available to all the participants.

Continuous-time mean-variance portfolio selection with random horizon in an incomplete market

Lu Siyu
Department of Mathematics
Southeast University

This talk is concerned with a continuous-time mean-variance portfolio selection problem with random market parameters and random time horizon in an incomplete market. This problem will be formulated as a linearly constrained stochastic linear quadratic (LQ) optimal control problem. The solvability of this LQ problem will be reduced to the global solvability of two backward stochastic differential equations (BSDEs). We shall apply the martingales of bounded mean oscillation, briefly called BMO-martingales, to provide a direct and simplified proof of the solvability of the two BSDEs. We also derive closed-form expressions for both the optimal portfolios and the efficient frontier in terms of the solutions of the two BSDEs.

This is a joint work with Profs Z. Wu and Z. Yu.
Multigrid methods for Maxwell's equations and a fourth-order curl problem based on Hodge decomposition

Jintao Cui
Department of Applied Mathematics,
The Hong Kong Polytechnic University

In this work, we investigate the numerical solution for two-dimensional Maxwell's equations on graded meshes. The approach is based on the Hodge decomposition. The solution \( u \) of Maxwell's equations is approximated by solving standard second order elliptic problems. The quasi-optimal error estimates for both \( u \) and \( \text{curl} \, u \) in the \( L_2 \) norm are obtained on graded meshes. We then prove the uniform convergence of the W-cycle and full multigrid algorithms for the resulting discrete problems. The performance of these methods is illustrated by numerical results. Similar numerical approach can also be applied to solve a fourth order curl problem. We will report some preliminary results for the multigrid algorithms.

Time consistent behavioral portfolio policy for dynamic mean–variance formulation with constraints

Li Xun
Department of Applied Mathematics,
The Hong Kong Polytechnic University

When one considers an optimal portfolio policy under a mean-risk formulation, it is essential to correctly model investors’ risk aversion which may be time variant or even state dependent. In this work, we propose a behavioral risk aversion model, in which risk aversion is a piecewise linear function of the current excess wealth level with a reference point at the discounted investment target (either surplus or shortage), to reflect a behavioral pattern with both house money and break-even effects. Due to the time inconsistency of the resulting multi-period mean–variance model with adaptive risk aversion, we investigate the time consistent behavioral portfolio policy by solving a nested mean–variance game formulation. We derive a semi-analytical time consistent behavioral portfolio policy which takes a piecewise linear feedback form of the current excess wealth level with respect to the discounted investment target. Finally, we extend the above results to time consistent behavioral portfolio selection for dynamic mean–variance formulation with a cone constraint.
Optimal Consumption under Non-addictive Habit Formation In Incomplete Markets

Yu Xiang
Department of Applied Mathematics,
The Hong Kong Polytechnic University

Abstract: This paper studies the optimal consumption under a non-addictive habit formation preference in semimartingale market models. The utility function is defined on the whole real line which allows the non-negative consumption to fall below the habit formation level. To avoid the path-dependence in this problem, we propose to work on a set of auxiliary processes. However, the non-negative consumption constraint becomes a path-dependent constraint on auxiliary primal elements. In the formulation of the dual problem, the stochastic Lagrange multipliers appear. It is revealed in this paper that the optimal consumption can be obtained via the construction of the stochastic Lagrange multiplier using the optimal solution from the auxiliary unconstrained dual problem. An endogenous stopping time $\tau^*$ is provided such that it is optimal for the individual to forgo consumption completely before $\tau^*$. After the stopping time $\tau^*$, the path-dependent constraint on the auxiliary process ceases to bind which implies that our optimal non-negative consumption can be constructed using the solution of an auxiliary unconstrained utility maximization problem.
Joint work with Erhan Bayraktar, University of Michigan.

Optimal portfolio and insurance problems under a value-at-risk constraint

K.F. Cedric Yiu
Department of Applied Mathematics
The Hong Kong Polytechnic University

In this talk, the portfolio selection problem will be considered. For ordinary investors or insurers who invest in the financial market, they need to manage portfolios continuously. At the same time, they need to fulfill the regulatory requirement governed by the value-at-risk constraints. In this talk, this problem is addressed. For the investment, the goal is to maximize the expected utility of terminal wealth. By using the principle of dynamic programming, the Hamilton-Jacobi-Bellman (HJB) equation can be derived. We will examine a few scenarios with different stochastic processes such as regime switching, and discuss how to solve the resulting HJB equation. Furthermore, we will investigate the impacts of the risk constraint on the optimal strategies.