Time	29 May 2017 (Monday)	
Venue	Y305	
8:30-		
8:50	Registration	
8:50-	On wing Address	
9:00	Opening Address	
	Morning Session I Chair: Yanping Lin	
9:00 -	Chi mana Sher (Dersen University United States)	
9:30	Chi-wang Shu (Brown University, United States) Discontinuous Galerkin Methods for Weakly Coupled Hyperbolic Multi-domain Problems	
9:30 -	Dongwoo Sheen (Seoul National University, Korea)	
10:00	Finite Element Methods for Periodic Problems – Revisited	
10:00 -	Yuesheng Xu (Sun Yat-Sen University, China)	
10:30	Fast Fourier-Galerkin Methods for Solving Boundary Integral Equations	
10:30 -	Tea Reception	
10:50	-	
	Morning Session II Chair: Chi-wang Shu	
10:50 -	Zhilin Li (North Carolina State University, United States)	
11:20	An Augmented IIM & Preconditioning Technique for Jump Embedded Boundary Conditions	
11:20 -	Dexuan Xie (University of Wisconsin, Milwaukee, United States)	
11:50	Recent Advances in Protein Electrostatic Modeling and Fast Finite Element Solvers	
11:50 -	Do Young Kwak (KAIST, Korea)	
12:20	A New Finite Element for Interface Problems Having Robin Type Jump	
12:20 - 14:00	Lunch [4/F, Communal Building]	
	Afternoon Session Chair: Zhilin Li	
14:00 -	Amiya Kuma Pani (Indian Institute of Technology Bombay, India) Semi-discrete Finite Element Analysis of Time Fractional Diffusion Problems with Nonsmooth	
14:30	Initial Condition using Energy Arguments	
14:30 -	Weiwei Sun (City University of Hong Kong, Hong Kong)	
15:00	A New Mixed Formulation and Efficient Numerical Solution of Ginzburg- Landau Equa Under the Temporal Gauge	
15:00	Shangyou Zhang (University of Delaware, United States) A P4 Bubble Enriched P3 Divergence-free Finite Element on Triangular Grids	
15:30		
15:30 -	Tea Reception	
16:00 18:00 -		
10.00 -	Banquet [King's Cuisine, 8/F, Harbour Plaza Metropolis]	

Time	30 May 2017 (Tuesday)			
Venue				
	Morning Session I Chair: Yuesheng Xu			
9:00 -	Jie Shen (Purdue University, United States)			
9:30	Efficient and Accurate Spectral Methods for Pdes With Singular Solutions			
9:30 -	Zhimin Zhang (Beijing CSRC, China & Wayne State University, United States)			
10:00	Recent Development on Superconvergence Theory in Computational Mathematics			
10:00 -	Xiu Ye (University of Arkansas, Little Rock, United States) A Posteriori Error Analysis on Polytopal Meshes and Simple Methods for the Problems With Non-divergence Forms			
10:30				
10:30 - 10:50	Tea Reception			
	Morning Session II Chair: Jie Shen			
10:50 -	Tao Lin (Virginia Tech, United States)			
11:20	High Degree Immersed Finite Element Spaces by a Least Squares Method			
11:20 -	Steven Wenbin Liu (University of Kent, United Kingdom) Some Recent Advances in Computation of Stochastic Optimal Control			
11:50				
12:00 - 14:00	Lunch [Kong Chiu Lau]			

## Abstracts

### **Invited** Talks

#### A New Finite Element for Interface Problems Having Robin Type Jump

Do-Young Kwak Department of Mathematical Sciences, Korea Advanced Institute of Science and Technology, Korea E-mail: kdy@kaist.ac.kr

Abstract. We propose a new finite element method for solving second order elliptic interface problems whose solution has a Robin type jump along the interface. We cast the problem into a new variational form and introduce a finite element method to solve it using a uniform grid. We modify the  $P_1$ -Crouzeix-Raviart element so that the shape functions satisfy the jump conditions along the interface. We note that there are cases that the Lagrange type basis cannot be used because of the jump in the value. Numerical experiments are provided.

#### An Augmented IIM & Preconditioning Technique for Jump Embedded Boundary Conditions

Zhilin Li CRSC & Mathematics, North Carolina State University, USA School of Mathematical Sciences, Nanjing Normal University, China E-mail: zhilin@math.ncsu.edu

Abstract. A second-order accurate augmented method is proposed and analyzed for a general elliptic PDE with a general boundary condition using the jump embedded boundary conditions (JEBC) formulation. First of all, the existence and uniqueness of an interface problem with given are discussed. Then, the well-posedness theory is extended to the interface problems with JEBC. In the proposed numerical method, one novel idea is to preconditioning the PDE first so that the coefficient of the highest derivative is of O(1). The second idea is to introduce two augmented variables corresponding to the jump in the solution and its normal derivative along the boundary to get an interface problem. For a piecewise constant coefficient, the fast Poisson solver then can be utilized in a rectangular domain. The augmented variables can be determined from a Schur complement system. We also propose two preconditioning techniques for the GMRES iterative method for the Schur complement; one is from the flux jump condition, and the other one is from the algebraic preconditioner based on the interpolation scheme in the augmented algorithm. The presented numerical results show that the proposed method has not only obtained second order accurate solutions in the  $L^{\infty}$  norm globally, but also second order accurate normal derivatives at the boundary from each side of the interface. The proposed preconditioning technique can speed up 50-90% compared with the method without preconditioning.

This is a joint work with Philippe Angot from Aix-Marseille University.

#### High Degree Immersed Finite Element Spaces by a Least Squares Method

Tao Lin Department of Mathematics, Virginia Tech, USA E-mail: tlin@vt.edu

Abstract. We present a least squares framework for constructing p-th degree immersed finite element (IFE) spaces for typical second-order elliptic interface problems. This least squares formulation enforces interface jump conditions including extended ones already proposed in the literature, and it guarantees the existence of the p-th IFE shape functions on interface elements. The uniqueness of the proposed p-th degree IFE shape functions is also discussed. Computational results are also presented to demonstrate the approximation capabilities of the proposed p-th IFE spaces

#### Some Recent Advances in Computation of Stochastic Optimal Control

Steven Wenbin Liu Kent Business School, University of Kent, UK E-mail: W.B.Liu@kent.ac.uk

Abstract. In this talk we introduce some recent research on developing computational methods for stochastic optimal control. Firstly, we study stochastic finite element and stochastic collocation methods for optimal control governed by PDEs with random field. In particular, we are interested in adaptive methods with mesh-free setting and state constrained stochastic optimal control. We further study optimal control governed by stochastic ODEs with while-noise. We have progressed in two directions: One is to develop algorithms based on non-Ito integral and another is to utilize some newly developed computation methods for the backward SDE.

#### Semi-discrete Finite Element Analysis of Time Fractional Diffusion Problems with Nonsmooth Initial Condition using Energy Arguments

Amiya Kuma Pani Department of Mathematics, Indian Institute of Technology Bombay, India E-mail: amiya.pani08@gmail.com

Abstract. In this talk, the conforming piecewise-linear finite element method (FEM) is applied to approximate the solution of time-fractional diffusion equations with variable diffusivity on bounded convex domains. Standard energy arguments do not provide satisfactory results for such a problem due to the low regularity of its exact solution. Using a delicate energy analysis, a priori optimal error bounds in  $L^2$ ,  $H^1$ , and quasi-optimal in  $L^{\infty}$ -norms are derived for the semidiscrete method for cases with smooth and nonsmooth initial data. The main tool of our analysis is based on a repeated use of an integral operator and use of a  $t^m$  type of weights to take care of the singular behavior of the continuous solution at t = 0. The generalized Leibniz formula for fractional derivatives is found to play a key role in our analysis. The present analysis can be extended to other types of fractional in time evolution problems.

#### Finite Element Methods for Periodic Problems Revisited

Dongwoo Sheen

Department of Mathematics and Interdisciplinary Program in Computational Sciences & Engineering, Seoul National University, Korea E-mail: dongwoosheen@gmail.com

Abstract. Periodic problems occur frequently in nature and industrial design analysis. Recently up-scaling via homogenization or multiscale approaches is one such typical example. Classical treatments of periodic problems lead to either singular linear algebraic systems or overdetermined systems, and there seem to have been many satisfactory solutions. However, when looking at reducing the degrees of freedom, we are led to find a couple of interesting features related with equivalents linear systems. We will discuss in this talk how to minimize such linear systems that can be solved efficiently for conforming and nonconforming finite element methods.

#### Efficient and Accurate Spectral Methods for Pdes With Singular Solutions

Jie Shen School of Mathematical Sciences, Xiamen University, China Department of Mathematics, Purdue University, USA E-mail: shen7@purdue.edu

**Abstract.** The usual spectral methods will provide high-order accuracy for problems with smooth solutions. However, they may not work well for problems with singular solutions due to various facts such as corner singularities, non-matching boundary conditions, non-smooth coefficients.

If the form of the singular expansion for the solution is known, we develop a Muntz Galerkin method which is based on specially tuned Muntz polynomials to deal with the singular behaviors of the underlying problems, and show that it provides optimal error estimates. On the other hand, if the Muntz Galerkin method is not applicable or efficient, we present a new extended spectral-Galerkin method which allows us to split it into two separate problems: one is to find an approximation for the smooth part by a usual spectral method, the other is to determine an approximation to the singular part with k terms by solving a  $k \times k$  system. So the new method is very easy to implement, very efficient and is capable of providing very accurate approximations for a class of singular problems.

We will present ample numerical results for a variety of problems with singular solutions, including fractional PDEs, to demonstrate the effectiveness of our approaches.

#### Discontinuous Galerkin Methods for Weakly Coupled Hyperbolic Multi-domain Problems

Chi-Wang Shu Division of Applied Mathematics, Brown University, USA E-mail: shu@dam.brown.edu

**Abstract.** In this talk, we will describe our recent development and analysis of a discontinuous Galerkin (DG) method to solve weakly coupled hyperbolic multi-domain problems. Such problems involve transfer type boundary conditions with discontinuous fluxes between different domains, calling for special techniques to prove stability of the DG methods. We first prove both stability and error estimates for our DG methods on simple models, and then apply them to a biological cell proliferation model. Numerical results are provided to illustrate the good behavior of our DG methods. This is a joint work with Qingyuan Liu and Mengping Zhang.

#### A New Mixed Formulation and Efficient Numerical Solution of Ginzburg- Landau Equations Under the Temporal Gauge

Weiwei Sun

Department of Mathematics, City University of Hong Kong, Hong Kong E-mail: maweiw@math.cityu.edu.hk

**Abstract.** In this talk we present our recent work on which we introduced a new numerical approach to the time dependent Ginzburg–Landau (GL) equations under the temporal gauge (zero electric potential gauge). The approach is based on a mixed formulation of the GL equations, which consists of two parabolic equations for the order parameter  $\psi$  and the magnetic field  $\sigma = \operatorname{curl} \mathbf{A}$ , respectively and a vector ordinary differential equation for the magnetic potential **A**. A fully linearized Galerkin FEM is presented for solving the mixed GL system. The new approach offers many advantages on both accuracy and efficiency over existing methods. In particular, the equations for  $\psi$ and  $\sigma$  are uniformly parabolic and therefore, the method provides optimal-order accuracy for the two physical components  $\psi$  and  $\sigma$ . Since in the temporal direction, a fully linearized backward Euler scheme is used for  $\psi$  and  $\sigma$  and a forward Euler scheme is used for A, respectively, the system is fully decoupled and at each time step, the three variables  $\psi$ ,  $\sigma$  and A can be solved simultaneously. Moreover, we present numerical comparisons with two commonly-used Galerkin methods for the GL equations under the temporal gauge and the Lorentz gauge, respectively. Our numerical results show that the new approach requires less iterations for solving the linear systems arising at each time step and the computational cost for the vector ODE seems negligible. Several numerical examples with complex geometries are also investigated.

#### Recent Advances in Protein Electrostatic Modeling and Fast Finite Element Solvers

Dexuan Xie Department of Mathematical Sciences, University of Wisconsin-Milwaukee, USA E-mail: dxie@uwm.edu

Abstract. Calculation of electrostatics for a protein (or other biomolecules) in an ionic solvent is a fundamental task in structural biology, computational biochemistry, biophysics, and mathematical biology. The Poisson-Boltzmann equation (PBE) is one commonly used dielectric continuum model for such calculation. It has been applied to protein study, rational drug design, and many other bioengineering applications. To reflect polarization correlation among water molecules and ionic size effects, size modified PBE, nonlocal PBE, and Poisson-Fermi models have been developed as variants of PBE. In this talk, I will report the recent advances that we made on these new models and their fast finite element solvers. I will also introduce our new SMPBS (Size Modified Poisson-Boltzmann Solvers) web server (smpbs.math.uwm.edu), which was published on the Journal of Computational Chemistry in March, 2017. Our research projects were partially supported by the National Science Foundation, USA, through grants DMS-0921004 and DMS-1226259.

#### Fast Fourier-Galerkin Methods for Solving Boundary Integral Equations

Yuesheng Xu School of Data and Computer Science, Sun Yat-Sen University, China Department of Mathematics, Syracuse University, USA E-mail: xuyuesh@sysu.edu.cn

**Abstract.** We shall present recent advances in fast solutions of the Fourier Galerkin Method for solving boundary integral equations for boundary value problems of elliptic PDEs. Theoretical results regarding optimal convergence estimates, linear computational complexity and numerical stability will be presented. Numerical results will be shown to demonstrate the theoretical results.

#### A Posteriori Error Analysis on Polytopal Meshes and Simple Methods for the Problems With Non-divergence Forms

Xiu Ye

Department of Mathematics and Statistics, University of Arkansas at Little Rock, USA E-mail: xxye@ualr.edu

Abstract. The goal of this talk is twofold. First, a posteriori error estimators have been developed for both the weak Galerkin and the discontinuous Galerkin finite element methods. The most existing a posteriori error analysis only work on simplicial elements even for the polygonal and polyhedral finite element methods. Our new residual type estimators can be applied to general meshes such as hybrid mesh, polytopal mesh and mesh with hanging node. In addition, these estimators consist fewer terms and are easy to compute. Second, we present simple finite element methods for solving some nonclassic problems such as second order elliptic equations in non-divergence form, Cauchy Problem and simple hyperbolic problem. Error analysis have been provided and extensive numerical examples have been tested.

#### A P4 Bubble Enriched P3 Divergence-free Finite Element on Triangular Grids

Shangyou Zhang Department of Mathematical Sciences, University of Delaware, USA E-mail: szhang@udel.edu

Abstract. On triangular grids, the continuous  $P_k$  plus discontinuous  $P_{k-1}$  mixed finite element is stable for polynomial degree k greater than or equal to 4. When k is 3, the inf-sup condition fails and the mixed finite element converges at an order that is two orders lower than the optimal order. We enrich the continuous P3 by adding some P4 divergence-free bubble functions, to be exact, one P4 divergence-free bubble function each component each edge. We show that such an enriched P3-P2 mixed element is inf-sup stable, and converges at the optimal order.

#### Recent Development on Superconvergence Theory in Computational Mathematics

Zhimin Zhang Beijing Computational Science Research Center, China & Department of Mathematics, Wayne State University, USA E-mail: zmzhang@csrc.ac.cn

Abstract. Superconvergence phenomenon is well understood for the h-version finite element method and researchers in this old field have accumulated a vast literature during the past 40 years. However, the relevant systematic study for discontinuous Galerkin, finite volume, and spectral methods is lacking. We believe that the scientific community would also benefit from the study of superconvergence phenomenon of those methods. Recently, some efforts have been made to expand the territory of the superconvergence. In this talk, I will summarize some recent development on superconvergence study for these methods. At the same time, some current issues and unsolved problems will also be addressed.

# **Address Book**

Number	Name	Institution
1	Xiaojun Chen	The Hong Kong Polytechnic University
2	Do Young Kwak	KAIST
3	Zhilin Li	North Carolina State University
4	Tao Lin	Virginia Tech
5	Yanping Lin	The Hong Kong Polytechnic University
6	Steven Wenbin Liu	University of Kent
7	Amiya Kuma Pani	Indian Institute of Technology Bombay
8	Zhonghua Qiao	The Hong Kong Polytechnic University
9	Dongwoo Sheen	Seoul National University
10	Jie Shen	Purdue University
11	Chi-wang Shu	Brown University
12	Weiwei Sun	City University of Hong Kong
13	Dexuan Xie	University of Wisconsin, Milwaukee
14	Yuesheng Xu	Sun Yat-Sen University
15	Xiu Ye	University of Arkansas, Little Rock
16	Shangyou Zhang	University of Delaware
17	Xu Zhang	Mississippi State University
18	Zhimin Zhang	Beijing CSRC and Wayne State University