2nd Workshop on
Optimization and Risk Management

Venue: Room M1603, Senate Room, Li Ka Shing Tower, Polyu

4 October

Time	Speakers
09:30-10:30	Terry Rockafellar Random Variables and Convex Analysis
10:30-10:40	Group photo
10:40-11:00	Coffee break
11:00-11:40	Bingsheng He PPA Like Contraction Methods for Convex Optimization in the Frame of Variational Inequalities
11:40-12:20	Houduo Qi Computing the Nearest Euclidean Distance Matrix with Low Embedding Dimensions
12:30-13:40	Lunch
14:00-14:40	Shinji Mizuno On the Number of Solutions Generated by the Simplex Method for LP
14:40-15:00	Xiaoqi Yang First- and Second-Order Necessary Conditions via Lower Order Exact Penalty Functions
15:00-15:20	Cedric Yiu Optimal Inventory Strategies under Value-at-risk Constraint
15:20-15:40	Coffee break
15:40-16:20	Xiaojiao Tong Stochastic Programming Arising from Economic Dispatch with Reliability Risk Constraint in Smart Grid
16:20-16:40	James Huang A Mixed Linear Quadratic Optimal Control Problem with a Controlled Time Horizon
16:40-17:00	Xiaojun Chen Residual Minimization for Stochastic Variational Inequalities

5 October

Time	Speakers
09:30-10:10	Yong Shi Error Correction Models in Binary Classification Problem Based on Multiple-Criteria and Multiple-Constraint Levels Linear Programming
10:10-10:30	Chi-kin Chan Environmental Performance-Impacts of Vendor-buyer Coordination
10:30-10:50	Xun Li A Mean-field Linear-quadratic Optimal Control Problem in Infinite Horizon
10:50-11:10	Coffee break
11:10-11:50	Naihua Xiu Low-Rank Solutions of Continuous and Discrete Lyapunov Equations Over Symmetric Cone
11:50-12:30	Defeng Sun Finding the Low Rank Correlation Matrix from a few Noisy Observations
12:40-14:00	Lunch
14:00-16:00	Visit AMA and free discussion in HJ610

Random Variables and Convex Analysis

Terry Rockafellar

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Uncertain costs or hazards in stochastic optimization are modeled as random variables, and the characteristics of the distributions of those random variables are critical then in managing risk. The theory of risk measures is an important tool for that purpose which relies heavily on convex analysis. However, there is a more fundamental connection with convex analysis which has largely escaped attention and yet is the key to basic properties and developments. This is the association of the distribution function of a random variable with a maximal monotone relation which in turn is the subdifferential of a convex function. Quantiles of the random variable correspond to the subdifferential of the conjugate convex function. The discovery of the minimization formula for value-at-risk and conditional value-at-risk was made through this connection.

PPA Like Contraction Methods for Convex Optimization in the Frame of Variational Inequalities

Bingsheng He

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The first order optimal conditions of the linear constraint convex programming is a mixed monotone variational inequality in primal and dual variables. For solving variational inequalities, the proximal point algorithm (PPA) is a classical method. In this talk, we show that with appropriate choices of proximal parameters, the application of the customized PPA to the linearly constrained convex programming can result in easy proximal subproblems. In particular, under some practical assumptions on the objective function, these proximal subproblems become explicit in the sense that they all have closed-form solutions or can be efficiently solved up to a high precision. We thus present some implementable contraction methods with explicit proximal regularization for linearly constrained convex programming, and their global convergence is proved easily under the analytic framework of contraction type methods. Some novel applications and numerical experiments are reported.

On the Number of Solutions Generated by the Simplex Method for LP

Shinji Mizuno and Tomonari Kitahara

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Tokyo Institute of Technology, Japan

The linear programming problem (LP) is a fundamental optimization model in risk management. In this talk, we show upper bounds for the number of distinct solutions generated by the simplex method for solving LP. The upper bounds are polynomial of the number of variables, the number of constraints, and the ration between the maximum and minimum positive elements of all the basic feasible solutions of LP. Moreover the upper bounds are small for some special linear programming problems including network optimization problems, LP with a totally unimodular matrix, and the Markov decision problem. We also show that the upper bounds are almost tight by using a simple variant of Klee-Minty's LP.

Computing the Nearest Euclidean Distance Matrix with Low Embedding Dimensions

Houduo Qi and Xiaoming Yuan

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Euclidean distance embedding appears in many high-profile applications including wireless sensor network localization, where not all pairwise distances among sensors are known or accurate. The classical Multi-Dimensional Scaling (cMDS) generally works well when the partial or contaminated Euclidean Distance Matrix (EDM) is close to the true EDM, but otherwise performs poorly. A natural step preceding cMDS would be to calculate the nearest EDM to the known matrix. A crucial condition on the desired nearest EDM is for it to have a low embedding dimension and this makes the problem nonconvex. There exists a large body of publications that deal with this problem. Some try to solve the problem directly and some are the type of convex relaxations of it. In this paper, we propose a numerical method that aims to solve this problem directly. Our method is strongly motivated by the majorized penalty method of Gao and Sun for low-rank positive semi-definite matrix optimization problems. The basic geometric object in our study is the set of EDMs having a low embedding dimension. We establish a *zero* duality gap result between the problem and its Lagrangian dual problem, which also motivated the majorization approach adopted. Numerical results show that the method works well for the Euclidean embedding of Network coordinate systems and for a class of large scale sensor.

Error Correction Models in Binary Classification Problem Based on Multiple-Criteria and Multiple-Constraint Levels Linear Programming

Bo Wang and Yong Shi

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In classification problems, the accuracy is a key issue to evaluate the effectiveness of an algorithm. For the two-class classification problem, the accuracy can be more precisely interpreted by two types of errors. That is, evaluate the accuracy through two types of errors respectively in addition to the measure of the rate of accuracies. Increasing the rate of accuracies, in fact, is equally important to correcting two types of errors. This paper introduces an improvement of the accuracy evaluation in data mining problems when the algorithm of multiple-criteria and multiple-constraint levels linear programming (MC2LP) is used. It explores an MC2LP model that can correct two types of error according to different practical requirements. Furthermore, taking account of the cost of this modification, some details of the new model are discussed.

Finding the Low Rank Correlation Matrix from a Few Noisy Observations

Defeng Sun

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In this talk, we aim to find a nearest correlation matrix of exact low rank from m independent noisy observations of entries under a general sampling scheme. Since the nuclear norm (trace) of a correlation matrix is a constant, the widely used nuclear norm regularization technique can no longer be applied to achieve this goal in the noisy setting. Here, we propose a new convex optimization approach by introducing a rank correction term based on a reasonable initial estimator. This convex optimization problem can be easily written as an H-weighted least squares semidefinite programming problem, which can be efficiently solved, even for large-scale cases. We show that our approach possesses the desired rank consistency for a broad selection of rank correction functions. Additionally, we also provide non-asymptotic bounds on the estimation error that validates the superiority of our approach.

Stochastic Programming Arising from Economic Dispatch with Reliability Risk Constraint in Smart Grid

Xiaojiao Tong

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In this talk, we consider a class of stochastic programming arising from economic dispatch (ED) in smart grid. The background of power system operation with reliability risk is introduced. Then based on the conditional value-at-risk (CVaR) management, we study the single-stage ED optimization model and the solution algorithm. Furthermore, according to the ED decision pattern of modern smart grid, we set up two types of three-stage stochastic programming with respect to the different case of random information giving. Numerical example for the single-stage ED problem is proposed to validate the model and solution method.

Low-Rank Solutions of Continuous and Discrete Lyapunov Equations Over Symmetric Cone

Ziyan Luo and Naihua Xiu

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Lyapunov equation is famous in control and system theory. In this talk, we will show that low-rank solutions of both continuous and discrete Lyapunov equations over symmetric cone, including nonnegative cone, second-order cone and semidefinite cone, are equivalent to symmetric cone programming problems. Therefore, they are polynomial-time solvable.

Environmental Performance-Impacts of Vendor-buyer Coordination

Chi Kin Chan and Y.C.E. Lee and J.F. Campbell

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Effective coordination is essential in the successful operation of supply chains. Without such coordination, vendors and buyers will act independently to make decisions that optimize their own objectives (e.g., maximize their respective profits or minimize their costs). This non-coordinated approach is unlikely to be optimal when one considers the supply chain as a whole. Most vendor-buyer coordination research concentrates only on an objective of minimizing the total system costs, but with the increasing environmental awareness in business and the general public, many organizations acknowledge that strategies and practices which incorporate environmental considerations are essential to acquire a competitive advantage. The objectives of this research are concerned not only with the economic impact of vendor-buyer coordination on the organizations involved, but also with the wider effects on society, such as the effects of pollution on the environment. This research demonstrates how to incorporate environmental issues into vendor-buyer coordination. Firstly, we will study the impacts of cost-minimizing vendor-buyer coordination models on environmental performance when compared with independent (non-coordinated) optimization. Secondly, we will develop a single-vendor multi-buyer coordination model that includes both costs and environmental performance measures in its objective function. A numerical example and a real case study show that both the total system cost and the environmental performance can be simultaneously improved.

Residual Minimization for Stochastic Variational Inequalities

Xiaojun Chen

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This talk presents a new expected residual minimization formulation for a class of stochastic variational inequalities by using the gap function. The objective function of the expected residual minimization problem is nonnegative and Lipschitz continuous. Moreover, it is convex for some stochastic linear variational inequalities, which helps us guarantee the existence of a solution and convergence of approximation methods. We propose a globally convergent (a.s.) smoothing sample average approximation (SSAA) method to minimize the expected residual function. We show that the residual minimization problem and its SSAA problems have minimizers in a compact set and any cluster point of minimizers and stationary points of the SSAA problems is a minimizer and a stationary point of the expected residual minimization problem (a.s.). Our examples come from applications involving traffic flow problems. We show that the conditions we impose are satisfied and that the solutions, efficiently generated by the SSAA procedure, have desirable properties.

A Mixed Linear Quadratic Optimal Control Problem with a Controlled Time Horizon

James Huang and Xun Li and Jiongmin Yong

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Herein, we study a class of mixed linear quadratic (MLQ) optimal control problems. The controlled stochastic system consists of two diffusion states which are of inconsistent time-horizons. By "mixed," we mean there are two control variables $(u(\cdot), \tau)$ which are framed in some combined way: on one hand, the standard control variable $(u(t))_{0\leq t\leq T}$ enters the drift and diffusion coefficients of both state equations, on the other hand, the decision maker can also choose some stopping time τ upon which the second state is initialized with initial condition depending on the first state. The motivation of MLQ from two-stage project management is presented and its difference to the stochastic impulse control is also specified. We verify that the solution of MLQ problem is equivalent to sequentially solve one random-duration linear quadratic (RLQ) problem and one optimal time (OT) problem to associated Riccati equations. In particular, the optimal cost functional can be represented by two term-coupled stochastic Riccati equations (SREs). By "term-coupled", the terminal condition of first SRE depends on the initial condition of the second SRE. Some optimality condition for MLQ is therefore obtained using the equivalence among MLQ, RLQ and OT. In case of one dimension and deterministic coefficients, the optimal control of MLQ is derived explicitly.

A Mean-field Linear-quadratic Optimal Control Problem in Infinite Horizon

Xun Li

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A linear-quadratic (LQ, for short) optimal control problem is considered for mean-field stochastic differential equations with constant coefficients in an infinite horizon. The stabilizability of the control system is studied followed by the discussion of the well-posedness of the LQ problem. The optimal control can be expressed as a linear state feedback involving the state and its mean, through the solutions of two algebraic Riccati equations. The solvability of such kind of Riccati equations is investigated by means of semi-definite programming method.

Optimal Inventory Strategies under Value-at-risk Constraint

K.F.C. Yiu and S.Y. Wang and K.L.Mak

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In this work, we consider the raw material inventories of a manufacture as a part of investment. Then the inventory control problem can be treated as an optimal portfolio problem. A portfolio consisting of the risky raw material inventory and the risk free bank account is studied and the VaR of the portfolio is analyzed and imposed as a risk control constraint. The objective function is to maximize the utility of total portfolio value. In this model, the ordering cost is assumed to be fixed and the selling cost is proportional to the value. The optimality conditions and transaction regions are derived by using stochastic optimal control theory and the method of Lagrange multiplier. Under this formulation, the optimal inventory level is reviewed and adjusted continuously. A numerical method is proposed and the results illustrate how the material price, inventory level and VaR are interrelated.

First- and Second-Order Necessary Conditions via Lower Order Exact Penalty Functions

Xiaoqi Yang

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In this talk we study first- and second-order necessary conditions for nonlinear programming problems from the viewpoint of exact penalty functions. We first establish necessary and sufficient conditions for a penalty term to be of KKT-type by using the kernel of the subderivative of the penalty term. This is done by showing that at a local minimum of the penalty term the polar cone of the subderivative kernel is the same as the positive hull of its regular subdifferential. We then derive a second-order necessary condition by assuming a second-order constraint qualification which requires that the second-order linearized tangent set is included in the closed convex hull of the kernel of the parabolic subderivative of the penalty term. In particular, for an $l_{\frac{2}{3}}$ lower-order penalty term, by assuming the nonpos- itiveness of a sum of a second-order derivative and a third-order derivative of the original data and applying a third-order Taylor expansion, we obtain the second-order necessary condition.