Optimization and Statistics

Terry Rockafellar

Department of Mathematics
University of Washington

Abstract. Much of statistics is concerned with estimating the values of uncertain quantities from available data, whether historical or experimental. Estimates are obtained by minimizing some error expression and fall therefore into a specially structured category of optimization. On the other hand, in applications of optimization more generally, the objective or constraint functions may depend on databases and require statistical methodology to be pinned down in a tractable form.

This raises important issues in stochastic optimization, where decisions can only shape the probability distributions of future "costs", and preferences in terms of so-called measures of risk must come in. It appears that such preferences should then influence the error expression used in estimation. Such interplay between optimization and statistics is leading to a greatly broadened theory of regression which draws on tools of convex analysis and techniques of numerical optimization but also supports models of reliable design in engineering.

Risk Aversion and Stochastic Programming: A New Kind of Stochastic Equilibrium

Daniel Ralph[1] and Yves Smeers[2]

[1] University of Cambridge
Judge Business School
[2] Universit*é* catholique de Louvain
Centre for Operations Research & Econometrics

Abstract. Consider a simple situation where two or more firms have production plants, and forecast their future net profits, or net costs, in each of several different scenarios. They also have an idea of the likelihood of each scenario, in fact have a list of likely probability density functions (PDFs) that seem reasonable; call this list their "risk set". Firms are risk averse and give a value today of their future output by taking the worst average over all PDFs in their risk set; call this their "coherent risk measure" in the terminology of Artzner et al 1999.

We introduce risk sharing via a perfectly competitive market for risk. For example, firms can trade financial instruments that act as hedges against what they consider to be bad outcomes. Our main result is that the equilibrium problem of pricing risk reduces to evaluating a new, system risk function that is also a coherent risk measure. It's interesting properties:

- 1. The system risk set includes a PDF that simultaneously characterises system risk and the risk attitude of each player: each player behaves as though it is risk neutral, ie, values future scenarios according to a straight average using this shared PDF.

- 2. The system risk problem is a convex optimization problem. If the firms' risk sets are polyhedral, eg, conditional value at risk is used, then risk can be priced by solving a linear program.

This is an example of a new kind of stochastic equilibrium problem in that the solution determines not only the optimal hedging strategy for each firm and the price of financial instruments used for hedging, but also an equilibrium PDF that describes risk neutral behaviour at equilibrium. In contrast, traditional stochastic optimization and stochastic equilibrium problems require a single PDF to be provided as data, and all agents to agree on this in advance.

Close the Gaps: A Learning-while-Doing Algorithm for a Class of Single-Product Revenue Management Problems

Yinyu Ye

Department of Management Science and Engineering
Stanford University

Abstract. In this work, we consider a retailer selling a single product with limited on-hand inventory over a finite selling season. Customer demand arrives according to a Poisson process, the rate of which is influenced by a single action taken by the retailer (such as price adjustment, sales commission, advertisement intensity, etc.). The relation between the action and the demand rate is not known in advance. The retailer will learn the optimal action policy "on the fly" as she maximizes her total expected revenue based on observed demand reactions. Using the pricing problem as an example, we propose a dynamic "learning-while-doing" algorithm to achieve a near optimal performance. Furthermore, we prove that the convergence rate of our algorithm is almost the fastest among all possible algorithms in terms of asymptotic "regret" (the relative loss comparing to the full information optimal solution). Our result closes the performance gaps between parametric and non-parametric learning and between a post-price mechanism and a customer-bidding mechanism. Important managerial insights from this research are that the value of information on 1) the parametric form of demand function and 2) each customer's exact reservation price are rather marginal. It also suggests the firms would be better off to perform concurrent dynamic learning and doing, instead of learning-first and doing-second in practice.

Incentive Mechanisms for the Co-Ordination of a Single-Vendor Multi-Buyer Supply Chain

Chi-kin Chan

Department of Applied Mathematics
The Hong Kong Polytechnic University

Abstract. Co-ordination between various parties in a supply chain is essential in nowadays business environments. An effective supply chain co-ordination can reduce the total system cost when compared with independent optimization, i.e. vendor and buyers act independently to minimize their own costs. However, a general result that applies in all analyses of co-ordinated supply chain models is that the cost to most of the buyers is significantly increased. The vendor is motivated to seek co-ordination in the whole supply chain but the buyers are not. Hence, the interest is in examining what incentive mechanisms are needed from the vendor to motivate the buyers to change their policies to allow the savings from co-ordination to be achieved. In this research, we consider a co-ordinated single-vendor multi-buyer supply chain model by synchronizing ordering and production cycles. Further, we propose two incentive mechanisms which can guarantee that a buyer's total relevant cost of co-ordination will not be increased when compared with independent optimization. More importantly, the vendor does not require any cost information from the buyers when applying these mechanisms.

Expected Residual Minimization for Stochastic Variational Inequalities

Xiaojun Chen

Department of Applied Mathematics
The Hong Kong Polytechnic University

Abstract. The stochastic variational inequality (SVI) has been used widely in engineering and economics, as an effective mathematical model for a number of equilibrium problems involving uncertain data. We present an expected residual minimization (ERM) formulation for a class of SVI, including the complementarity problem as a special case. The objective of the ERM-formulation is Lipschitz continuous and semismooth which helps us guarantee the existence of a solution and convergence of approximation methods. Moreover, this minimization problem is convex for linear SVI if the expected matrix is positive semi-definite. We propose a globally convergent (a.s.) smoothing sample average approximation (SSAA) method to minimize the residual function. We show that the ERM problem and its SSAA problems have minimizers in a compact set and any cluster point of minimizers and stationary points of the SSAA problems is a minimizer and a stationary point of the ERM problem (a.s.). We illustrate the ERM and SSAA by examples from traffic equilibrium assignment problems.

Network Equilibrium Under Cumulative Prospect Theory and Endogenous Stochastic Demand and Supply

Agachai Sumalee

Department of Applied Mathematics
The Hong Kong Polytechnic University

Abstract. In this presentation we consider a network whose travel demands and road capacities are endogenously considered to be random variables. With stochastic demand and supply the route travel times are also random variables. In this scenario travelers choose their routes under travel time uncertainties. Several evidences suggest that the decision making process under uncertainty is significantly different from that without uncertainty. Therefore, we apply the decision framework of cumulative prospect theory (CPT) to capture this difference. We first formulate a stochastic network model whose travel demands and link capacities follow lognormal distributions. The stochastic travel times can then be derived under a given route choice modeling framework. For the route choice, we consider a modeling framework where the perceived value and perceived probabilities of travel time outcomes are obtained via transformations following CPT. We then formulate an equilibrium condition similar to that of User Equilibrium in which travelers choose the routes that maximizes their perceived utility values in the face of transformed stochastic travel times. Conditions are established guaranteeing existence (but not uniqueness) of this equilibrium. We then proposes a solution algorithm for the proposed model which is then tested with a test network.

Piecewise Linear Multi-Criteria Programs: The Continuous Case and Its Discontinuous Generalization

Yaping Fang[1], Kaiwen Meng[2] and Xiaoqi Yang[2]
[1]Department of Mathematics
Sichuan University
[2]Department of Applied Mathematics
The Hong Kong Polytechnic University

Abstract. In this paper we study piecewise linear multi-criteria programs, that is, multi-criteria programs with either a continuous or discontinuous piecewise linear objective function and a polyhedron set constraint. We obtain an algebraic representation of a semi-closed polyhedron and apply it to show that the image of a semi-closed polyhedron under a continuous linear function is always one semi-closed polyhedron. We establish that the (weak) Pareto solution/point set of a piecewise linear multi-criteria program is the union of finitely many semi-closed polyhedra. We propose an algorithm for finding the Pareto point set of a continuous piecewise linear bi-criteria program and generalize it to the discontinuous case. We apply our algorithm to solve the discontinuous bi-criteria portfolio selection problem with an l_∞ risk measure and transaction costs and show that this algorithm can be improved by using an ideal point strategy.

Optimal Portfolio and Insurance Problems with Risk Constraint

Liu Jingzhen and Cedric Yiu
Department of Applied Mathematics
The Hong Kong Polytechnic University

Abstract. We consider the risk-constrained portfolio selection problems arising from an ordinary investor or an insurer who can invest her surplus into financial market. For an insurer, with value-at-risk (VaR) imposed as a dynamic risk constraint, the portfolio selection problem is to minimize the ruin probability. A closed-form solution can be found by solving the associated Hamilton-Jacob-Bellman (HJB) equation. For an ordinary investor, we consider the risk-constrained utility maximizing problem with a jump diffusion model or a regime switching model. Conditional value-at-risk (CVaR) and maximal value-at-risk (MVaR) are imposed as the risk constraint in the two models, respectively. The associated HJB equations can be treated using numerical techniques.